ELSEVIER

10 August 1998

Physics Letters A 245 (1998) 55-61

PHYSICS LETTERS A

Bose–Einstein condensation with a BCS model interaction

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Received 5 June 1997; revised manuscript received 17 November 1997; accepted for publication 5 May 1998
Communicated by A.R. Bishop

Abstract

A simple model of a boson–fermion mixture of unpaired fermions plus linear-dispersion-relation Cooper pairs that includes pair-breaking effects leads to Bose–Einstein condensation for dimensions greater than unity, at critical temperatures substantially greater than those of the BCS theory of superconductivity, for the same BCS model interaction between the fermions. © 1998 Published by Elsevier Science B.V.

PACS: 05.30.Fk; 05.30.Jp; 64.90.+b
Keywords: Boson condensation; Pairing

1. Introduction

It is widely believed that superconductivity is a kind of Bose–Einstein (BE) condensate, the idea going back at least to the 1940's [1]. Recent experimental observations [2] of BE condensation (BEC) in ultracold alkali-atom gas clouds, as well as the 1996 Nobel Prize [3] for the discovery of superfluid phases in liquid helium-three, have spurred even greater interest in the phenomenon, e.g., Ref. [4].

We first consider an ideal quantum gas of permanent (i.e., number-conserving) bosons with a general dispersion relation in d dimensions. For the more familiar (i.e., *quadratic*-dispersion-relation) bosons there exists a non-zero absolute temperature T_c below which a macroscopic occupation emerges for a *single* (of infinitely many) quantum state only if $d > 2$ [5,6]. (The $d = 2$ case, in fact, displays the *same* [7] smooth, singularity-free temperature-dependent specific heat for either bosons or fermions.) The BE distribution summed over all states yields the total number of bosons N_B, each of mass m, of which, say $N_{B,0}(T)$ are in the lowest state ε_k (= 0 in the thermodynamic limit). Explicitly

$$N_B = N_{B,0}(T) + \sum_{k \neq 0} \frac{1}{e^{\beta(\varepsilon_k - \mu_B)} - 1}, \tag{1}$$

where $\beta \equiv 1/k_B T$ and $\mu_B \leqslant 0$ is the chemical potential. For $T > T_c$, $N_{B,0}(T)$ is negligible compared with N_B; for $T < T_c$, $N_{B,0}(T)$ becomes a sizeable fraction of N_B. At precisely $T = T_c$, $N_{B,0}(T_c) \simeq 0$ and $\mu_B \simeq 0$, while at $T = 0$ the last term in (1) vanishes so that $N_B = N_{B,0}(0)$ (viz., absence of any exclusion principle).

The sum in (1) can be converted to an integral over *positive* $k \equiv |\boldsymbol{k}|$, where $\boldsymbol{k}$ is a d-dimensional vector, as follows. The volume of a hypersphere of radius

0375-9601/98/$ – see front matter
PII S0375-9601(98)00377-6

R in $d \geqslant 0$ dimensions is given [8] by $V_d(R) = \pi^{d/2}R^d/\Gamma(1+d/2)$. For $d=3$ this becomes $4\pi R^3/3$; for $d=2$ it is the area πR^2 of a circle of radius R; for $d=1$ it is just the "diameter" $2R$ of a line of "radius" R; and for $d=0$ it is unity. Using this for $d>0$ the summation in (1) over our d-dimensional vector $\boldsymbol{k}$ becomes

$$\sum_{k\neq 0} \longrightarrow \frac{2\pi^{d/2}}{\Gamma(d/2)}\left(\frac{L}{2\pi}\right)^d \int \mathrm{d}k\, k^{d-1} \tag{2}$$

with the prefactor reducing as it should to 2, 2π and 4π for $d = 1$, 2 and 3, respectively. Let

$$\varepsilon_k = C_s k^s, \quad s>0, \tag{3}$$

be the boson excitation energy as a function of the wavenumber k, i.e., the bosonic dispersion relation. For ordinary bosons of mass m in vacuum, $s=2$ and $C_s = \hbar^2/2m$, while for a Cooper pair in the Fermi sea $s=1$ as will be discussed below. Defining the boson number density in d dimensions as $n_B \equiv N_B/L^d$, then (1) with $T \leqslant T_c$, becomes an elementary integral easily evaluated in terms of the usual Bose integrals [8] (with $z \equiv \mathrm{e}^{\mu_B/k_B T}$ the fugacity)

$$g_\sigma(z) \equiv \frac{1}{\Gamma(\sigma)}\int_0^\infty \mathrm{d}x \frac{x^{\sigma-1}}{z^{-1}\mathrm{e}^x - 1} = \sum_{l=1}^{\infty}\frac{z^l}{l^\sigma} \underset{z\to 1}{\longrightarrow} \zeta(\sigma). \tag{4}$$

The last identification holds when $\sigma > 1$, where $\zeta(\sigma)$ is the Riemann Zeta-function of order σ. The function $\zeta(\sigma) < \infty$ for $\sigma > 1$, while the series $g_\sigma(1)$ diverges for $\sigma \leqslant 1$.

The condensate fraction for $0 < T < T_c$ in d-dimensions is the fractional number $N_{B,0}(T)/N_{B,0}(0)$ of bosons in the $k=0$ state. Note that $\mu_B \simeq 0$ when $0<T<T_c$ since from (1) $N_{B,0}(T) = (\mathrm{e}^{-\beta\mu_B} - 1)^{-1}$ implies that $\mathrm{e}^{\beta\mu_B} = N_{B,0}(T)/[N_{B,0}(T)+1] < 1$, and approaches 1^- over this *entire* temperature range because $N_{B,0}(T)$ on cooling grows to a sizeable fraction of N_B which is macroscopic. Since $N_B = N_{B,0}(T) + N_{B,k>0}(T)$ and $N_B = N_{B,0}(0)$, from (1) and (2) one can write

$$\frac{N_{B,0}(T)}{N_{B,0}(0)} = 1 - [2^{d-1}\pi^{d/2}\Gamma(d/2)n_B]^{-1} \times \int_0^\infty \frac{\mathrm{d}k\, k^{d-1}}{\mathrm{e}^{\beta C_s k^s} - 1}. \tag{5}$$

Using (4) to evaluate the last integral gives

$$\frac{N_{B,0}(T)}{N_{B,0}(0)} = 1 - [2^{d-1}\pi^{d/2}\Gamma(d/2)n_B]^{-1} \times \frac{\Gamma(d/s)\, g_{d/s}(1)}{s(\beta C_s)^{d/s}}. \tag{6}$$

Since $N_{B,0}(T_c)/N_{B,0}(0) = 0$, one obtains the *general T_c formula*

$$T_c = \frac{C_s}{k_B}\left[\frac{s\,\Gamma(d/2)\,(2\pi)^d}{2\pi^{d/2}\,\Gamma(d/s)\,g_{d/s}(1)} n_B\right]^{s/d}. \tag{7}$$

Using (7), the condensate fraction (6) then reduces to

$$\frac{N_{B,0}(T)}{N_{B,0}(0)} = 1 - (T/T_c)^{d/s}. \tag{8}$$

These results are formally valid for all $d > 0$ and $s>0$. Note, however, that for $0 < d \leqslant s$, $T_c = 0$ since $g_{d/s}(1) = \infty$ for $d/s \leqslant 1$.

The case $s = d$ gives the celebrated harmonic series $g_1(1) = 1 + \frac{1}{2} + \frac{1}{3} + \ldots$ which diverges. Clearly, for $d < s$, the series $g_{d/s}(1)$ diverges even more severely. This implies that BEC does *not* occur for s-dispersion-relation bosons for $d \leqslant s$ dimensions which is consistent with the well-known fact that BEC does *not* occur for quadratic-dispersion-relation bosons for dimensions equal or smaller than two. For $s=2$, $C_s = \hbar^2/2m$ and $d=3$, (7) and (8) become

$$T_c = \frac{2\pi\hbar^2 n_B^{2/3}}{mk_B[\zeta(3/2)]^{2/3}} \simeq \frac{3.31\hbar^2 n_B^{2/3}}{mk_B}$$

$$\text{and} \quad \frac{N_{B,0}(T)}{N_{B,0}(0)} = 1 - (T/T_c)^{3/2}, \tag{9}$$

since $\zeta(3/2) \simeq 2.612$. These are the familiar results for BEC in 3D.

2. Linear-dispersion-relation Cooper-pairs as bosons

Consider a gas of fermions at zero absolute temperature with kinetic energies $\epsilon_{k_1} \equiv \hbar^2 k_1^2/2m^*$ and $\epsilon_{k_2} \equiv \hbar^2 k_2^2/2m^*$ interacting pairwise via the *BCS model interaction*

$$V_{kk'} = -V \quad \text{if } E_F - \hbar\omega_D < \epsilon_{k_1}, \epsilon_{k_2} < E_F + \hbar\omega_D, \\ = 0 \quad \text{otherwise}, \qquad (10)$$

with $V > 0$, $E_F \equiv \hbar^2 k_F^2/2m^*$ is the Fermi energy and $\hbar\omega_D$ the maximum energy of a vibrating-ionic-lattice phonon, where $V_{kk'}$ is the double Fourier transforms of the interaction in which $k = \frac{1}{2}|\boldsymbol{k}_1 - \boldsymbol{k}_2|$ and $k' = \frac{1}{2}|\boldsymbol{k}'_1 - \boldsymbol{k}'_2|$ are *relative* wavevectors, and m^* is the fermion effective mass.

Without abandoning the phonon mechanism modeled by (10), Refs. [9] suggest that superconductivity is perhaps a BEC in either 2D or 3D, of excited (i.e., nonzero center-of-mass momentum (CMM), $\hbar K > 0$) "pairons" pre-existing *above* T_c. At $T = 0$ *all* pairons are at rest ($K = 0$), while a mixture of both kinds ($K = 0$ and $K > 0$) is present for $0 < T < T_c$, a $\boldsymbol{K} = 0$ pairon being an ordinary Cooper pair. Pairons in d dimensions have "excitation energy" in their (positive) binding energy Δ_K given [10] for weak coupling by

$$\varepsilon_K \equiv \Delta_0 - \Delta_K \xrightarrow[K\to 0]{} a(d)v_F\hbar K, \qquad (11)$$

where [9] $a(1) = 1$, $a(2) = 2/\pi$, $a(3) = 1/2$, the pair binding energy for $K = 0$ is $\Delta_0 = 2\hbar\omega_D/(e^{2/\lambda} - 1)$, $\lambda \equiv g(E_F)V$ being a dimensionless coupling constant and $g(\varepsilon)$ is the number of fermionic states per unit energy for each spin. This result was first cited in Ref. [11] for 3D. On the other hand, collective modes in a superconductor have indeed been discussed since the late 1950's by Bogoliubov, Tolmachev, Shirkov, Nambu, Anderson, Rickayzen, and Bardasis and Schrieffer. A review of the early work by Martin is available [12], as is a more recent treatment by Belkhir and Randeria [13]. However, we do *not* deal here with "collective modes" but rather with (nonzero center-of-mass) "Cooper pairs" which can Bose–Einstein condense while collective modes cannot. Cooper pairs are *entities distinct* from collective modes such as zero-sound phonons or plasmons since they: (a) are bounded in number (before the thermodynamic limit is taken), and (b) are fixed in number as they carry a fixed constituent-fermion-number (namely two), while phonons or plasmons, say, *do not* share either property. Pairons in general are considered "bosonic" even though they do *not* obey Bose commutation relations. This is because for a given K they have *indefinite* occupation number since for each K there are, in the thermodynamic limit, an indefinite number of allowed (relative wavevector) k values, so that pairons do in fact obey the Bose–Einstein distribution. Thus, (3) and (7) with $s = 1$ and $C_s = a(d)\, v_F\hbar$ give the weak-coupling T_c-formula in d dimensions for *linear* dispersion-relation bosons

$$T_c = \frac{a(d)v_F\hbar}{k_B}\left[\frac{\pi^{(d+1)/2}n_B}{\Gamma((d+1)/2)g_d(1)}\right]^{1/d}. \qquad (12)$$

For the moment we have ignored the fact that pairons with $K > K_{01}$ break up, where $K_{01} = \Delta_0/a(d)\hbar v_F$ is determined from $\Delta_{K_{01}} \equiv 0$, in the linear approximation implied by (11). Numerical calculations for Δ_K [10] show that K_{01} is somewhat smaller but *of the order* of K_0, where the *exact* Δ_K gives K_0 through $\Delta_{K_0} = 0$; i.e., the linear approximation to Δ_K is quite good. Since $g_2(1) \equiv \zeta(2) = \pi^2/6 \simeq 1.64493$ and $g_3(1) \equiv \zeta(3) \simeq 1.20206$, (12) reduces to the T_c formulae of Ref. [9], $T_c = 1.244\hbar {k_B}^{-1} v_F n_B^{1/2}$ in 2D and $T_c = 1.009\hbar {k_B}^{-1} v_F n_B^{1/3}$ in 3D. Note from (12) that $T_c > 0$ for $d > 1$, a result of possible relevance in understanding even quasi-1D organic superconductivity [14] as a BEC. Organic superconductors include $(1+\epsilon)$D materials such as the Bechgaard salts, $(2+\epsilon)$D materials like the ET salts and fully-3D materials such as the alkali- and alkaline-earth-doped fullerene crystals called "fulleride" superconductors [15]. The $(1+\epsilon)$D and $(2+\epsilon)$D compounds consist of *coupled* parallel chains and planes, respectively, of atoms.

In d-dimensions, applying (2) to the number of fermions $N = 2\sum_k \theta(k_F - k)$, where $\theta(x)$ is the Heaviside step function, the fermion number density becomes

$$n \equiv \frac{N}{L^d} = \frac{k_F^d}{2^{d-2}\pi^{d/2} d\, \Gamma(d/2)}. \qquad (13)$$

On the other hand, the number of bosons $N_{B,0}(0)$ actually formed at $T = 0$ through interaction (10) is precisely $g(E_F)\hbar\omega_D$, where [6]

$$g(\varepsilon) \equiv \left(\frac{L}{2\pi}\right)^d \frac{\mathrm{d}^d k}{\mathrm{d}\varepsilon} = \left(\frac{m^*}{2\pi\hbar^2}\right)^{d/2} \frac{L^d \varepsilon^{(d/2)-1}}{\Gamma(d/2)}. \quad (14)$$

If *all* fermions were imagined paired, $n_B/n = 1/2$. However, since $n_B = g(E_F)\hbar\omega_D/L^d$, (13) and (14) show that in fact

$$n_B/n = d\hbar\omega_D/4E_F \equiv \nu d/4, \quad (15)$$

a fraction much less than $1/2$ since typically $\hbar\omega_D \ll E_F$. This allows rewriting (12) as

$$\frac{T_c}{T_F} = 2a(d)\left[\frac{\nu}{2\Gamma(d)\zeta(d)}\right]^{1/d}$$
(pure unbreakable-pairon gas). (16)

In particular, for $d = 2$ with $a(2) = 2/\pi$ and $n_B/n = \nu/2$ from (15), we have

$$\frac{T_c}{T_F} = \frac{4\sqrt{6}}{\pi^2}\sqrt{\frac{n_B}{n}} = \frac{4\sqrt{3}}{\pi^2}\sqrt{\nu} \simeq 0.702\sqrt{\nu}$$
(pure unbreakable-pairon gas) (2D). (17)

Treating the pairons now as *breakable*, namely that $\Delta_K < 0$ according to (11) for all $K > K_{01}$, implies from (1) that (2) must really be integrated over K only up to K_{01} in order to avoid $\Delta_K < 0$. Thus, for $d \geqslant 1$

$$\frac{T_c}{T_F} \simeq \nu a(d)^d (d-1)\left(\frac{\mathrm{e}^{2/\lambda}}{\nu}\right)^{d-1} \xrightarrow[\lambda\to 0]{} \infty$$
(pure breakable-pairon gas) (18)

while for $d = 1, T_c/T_F = 0$ for any λ. The infinite result is expected since for vanishingly small coupling K_{01} also vanishes, meaning that *all* the pairons are $K = 0$ bosons and the system is BE condensed at all finite T.

However, unpaired background fermions mixed together with the *breakable* pairons will "tame" this diverging T_c down to a finite value, as shown below. First let us remark that the BCS "condensation energy", i.e., the energy shift of the many-fermion ground state energy in the superconducting, E_s, relative to the normal, E_n, state, ultimately reduces [16] to

$$E_s - E_n = -g(E_F)\hbar\omega_D \frac{2\hbar\omega_D}{\mathrm{e}^{2/\lambda}-1}$$
$$= -N_{B,0}(0)\Delta_0 \xrightarrow[\lambda\to 0]{} -\tfrac{1}{2}g(E_F)\Delta^2, \quad (19)$$

where

$$\Delta \xrightarrow[\lambda\to 0]{} 2\hbar\omega_D\,\mathrm{e}^{-1/\lambda}$$

is the zero temperature BCS gap energy. The final term is familiar, but the term before that is not: it shows that the energy shift is just a superposition of $N_{B,0}(0)$ *point* pairon binding energies Δ_0, and valid for any coupling. Note the crucial distinction between Δ_0 and Δ leading to this simple result which suggests the model to be treated now.

3. Simple boson–fermion model

Consider an ideal mixture of unpaired fermions and breakable pairons with linear dispersion relation for which thermal pair-breaking is explicitly allowed. The total number of fermions N consists of N_1 non-interacting, i.e., unpairable fermions and N_2 interacting or pairable ones, namely

$$N = N_1 + N_2. \quad (20)$$

Specifically, at $T = 0$ N_1 is just the number of fermions *below* the spherical interaction shell in k-space of thickness $\hbar\omega_D$, as implied by (10), while N_2 is the number *within* the shell. Unpairable fermions obey the usual Fermi–Dirac distribution with fermionic chemical potential μ, while the pairable ones at any T are

$$N_2(T) = 2\int_{\mu-\hbar\omega_D}^{\mu+\hbar\omega_D} \frac{g(\varepsilon)\mathrm{d}\varepsilon}{\mathrm{e}^{\beta(\varepsilon-\mu)}+1}$$
$$\simeq 2g(\mu)\int_{\mu-\hbar\omega_D}^{\mu+\hbar\omega_D} \frac{\mathrm{d}\varepsilon}{\mathrm{e}^{\beta(\varepsilon-\mu)}+1} = 2g(\mu)\hbar\omega_D, \quad (21)$$

which in 2D is independent of T and exact since $g(\varepsilon)$ is constant. Moreover, if $\hbar\omega_D/\mu \ll 1$, Eq. (21) is a good approximation for $d \neq 2$. The relevant *number equation* for the pairable fermions is

$$N_2(T) = N_{20}(T) + 2[N_{B,0}(T) + N_{B,0<K<K_{01}}(T)], \quad (22)$$

where $N_{20}(T)$ is the number of pairable but unpaired fermions. Let $N_{B,K>0}(T)$ denote the total number of unbreakable bosonic pairs, considered before, with center-of-mass momentum (CMM) $K > 0$, i.e.,

$$N_{B,K>0}(T) = \sum_{K>0} N_{B,K}(T)$$
$$= N_{B,0<K<K_{01}}(T) + N_{B,K_{01}<K}(T),$$

while $N_{B,0}(T)$ is the number of bosonic Cooper pairs with zero CMM.

A simple boson–fermion model follows on identifying

$$N_{20}(T) = 2N_{B,K_{01}<K<K_{\max}}(T), \tag{23}$$

namely, by asserting that the pairable but unpaired fermions are precisely the broken bosons with $K_{01} < K < K_{\max}$ in the pure unbreakable-pairon gas model considered before, where $K_{\max} \equiv 2k_F\sqrt{1+\nu}$ is the largest value of K beyond which the interaction (10) vanishes [17]. Note that (23) vanishes as $T \to 0$ so that $N_2(0) \equiv 2N_{B,0}(0)$, or that at $T = 0$ all pairable fermions are paired. From Eqs. (21) and (23) the *number equation* (22) becomes

$$g(\mu)\hbar\omega_D = N_{B,0}(0)$$
$$= N_{B,0}(T) + N_{B,0<K<K_{01}}(T) + N_{B,K_{01}<K<K_{\max}}(T). \tag{24}$$

Hence, the number equation of this breakable-boson-fermion-mixture model is almost equivalent to that of the model [9] of a pure gas of unbreakable linear-dispersion-relation bosons. These have a condensate fraction (8) for $s = 1$, and a critical temperature T_c obtainable from $N_{B,0}(T_c)/N_{B,0}(0) = 0$ which lead precisely to (12). Using the same techniques as lead to (7), the temperature $T_c^{\rm mix}$ at which $N_{B,0}(T) \simeq 0$ and $\mu_B \simeq 0$ in (24) satisfies a transcendental equation, as opposed to the algebraic one that gave (7). The transcendental equation can be manipulated to give the expansion

$$T_c^{\rm mix} = T_c\left\{1 + \frac{[4a(d)\sqrt{1+\nu}\,T_F/T_c^{\rm mix}]^{d-1}}{d\,\Gamma(d)\zeta(d)} \times \exp[-4a(d)\sqrt{1+\nu}T_F/T_c^{\rm mix}] + \cdots\right\}, \tag{25}$$

where T_c is given by (16), assuming $\mu \simeq E_F$ in any d, and $\nu \ll 1$ for $d \neq 2$. In fact, using the 2D expression $\mu(T)/E_F = 1-(T/T_F)\ln[1+{\rm e}^{-\mu(T)/k_BT}]$ [18] at $T =$

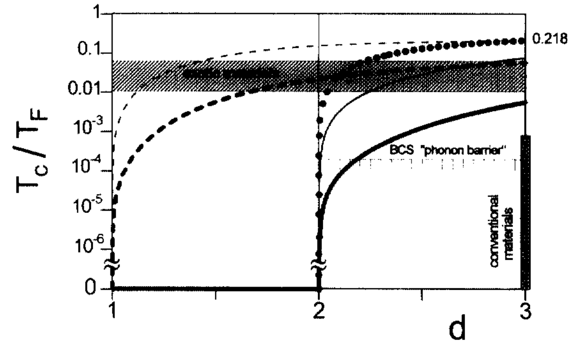

Fig. 1. Full curves refer to BEC critical temperature T_c (in units of T_F) in d dimensions according to (7) with $s = 2$, dashed curves to (12), where $n_B/n = d\Theta_D/4T_F$, $m = 2m^*$ as explained in text. Thin and thick curves refer to $\Theta_D/T_F = 0.05$ and 0.001, respectively. The dotted curve refers to (7) with $s = 2$, $n_B/n = \frac{1}{2}$ and $m = 2m^*$, namely *all* fermions imagined paired, the value of 0.218 at $d = 3$ being a familiar result. Light and dark crosshatchings comprise Uemura plot [19] data for *exotic* and *conventional* superconductors, respectively. The thin horizontal line marked BCS "phonon barrier" corresponds to the BCS T_c-formula with $\lambda \leqslant 1/2$, namely $T_c/T_F \leqslant (1.13e^{-2})\Theta_D/T_F \simeq 0.153\Theta_D/T_F$ for the case $\Theta_D/T_F = 10^{-3}$.

T_c as given by (17) for $\nu = 0.05$, $\mu(T_c) \simeq 0.9997E_F$. The correction term in (25) is minute: being about 2×10^{-7} in 2D using (17) with $\nu = 0.05$, and about 7×10^{-13} in 3D using (16) with $\nu = 0.001$. Thus, $T_c^{\rm mix} \simeq T_c$, the critical temperature for the pure unbreakable-boson gas. Curiously, in contrast to the infinite T_c (18) of a *pure* boson gas of breakable pairons, the mere presence of background unpaired fermions has driven T_c down to a *finite* value, in spite of all pairons in the mixture for weak coupling still being only $K = 0$ bosons.

Fig. 1 displays T_c/T_F *vs.* $1 \leqslant d \leqslant 3$ for: (a) "ordinary" quadratic-dispersion-relation bosons according to (3) with $s = 2$, $m = 2m^*$ and $n_B/n = \Theta_D d/4T_F \equiv \nu d/4$ for $\nu = 0.05$ (thin full curve), and $\nu = 0.001$ (thick full curve), as well as for $n_B/n = 1/2$ (*all* fermions imagined paired) (dotted curve); (b) linear-dispersion-relation Cooper pairons according to (3) with $s = 1$, $\nu = 0.05$ (thin dashed curve) and $\nu = 0.001$ (thick dashed curve). Fig. 2 exhibits the 2D critical temperature $T_c^{\rm mix} \simeq T_c$ (scaled in terms of T_F) for the simple boson–fermion model just discussed, which is really just (16) for $d = 2$ (full curve), compared with the maximal BCS result $T_c = 1.13\Theta_D{\rm e}^{-1/\lambda}$ for $\lambda = 1/2$, both as function of $\nu \equiv \Theta_D/T_F$. Empiri-

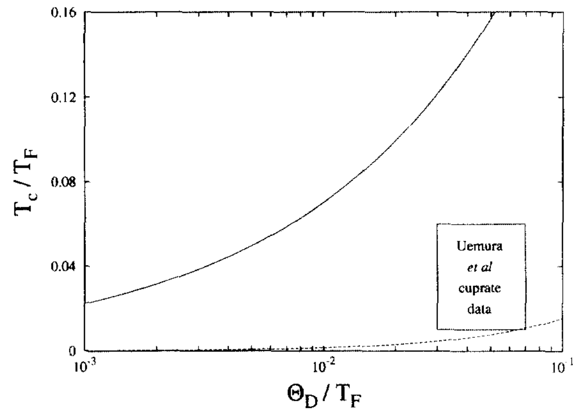

Fig. 2. Scaled critical BEC temperature T_c/T_F *vs.* Θ_D/T_F for the 2D simple boson–fermion model (equivalent to the 2D value in (17) as discussed in text, for $d = 2$) (full curve). The dashed curve is the maximal BCS result $T_c = 1.13\,\Theta_D\,e^{-1/\lambda}$ for $\lambda = 1/2$. The rectangle comprises empirical data for the cuprate superconductors [19].

cal [19,20] cuprate T_c values lie within the rectangle shown.

Lower predicted T_c values are expected from two refinements now under study: (a) reductions in n_B due to realistic non-spherical Fermi surfaces where "nesting" of some parts of the surface in general occur, thus necessarily reducing the number of pairable fermions and hence of pairs, and (b) interaction between (extended) pairons is expected to lower the T_c, as occurs say in liquid helium-four where interactions reduce [21] T_c by almost 30% from the ideal gas value.

4. Conclusions

Three BEC models have been discussed for a d-dimensional many-fermion system with a BCS model interaction: (a) a pure boson gas of unbreakable linear-dispersion Cooper pairons; (b) the same with *breakable* pairons; and (c) a (breakable) boson–fermion gas mixture in a simple model. Model (b) of breakable pairons in weak-coupling gives an *infinite* BEC critical temperature T_c, as expected since all bosons are in the zero center-of-mass-momentum state and thus BE condensed at all temperatures. Mixing those bosons with background unpaired fermions, model (c), however, gives finite T_c values which are substantially higher than those of BCS theory for the same interaction. In contrast to BCS theory, however, pairs are allowed to pre-exist *above* T_c before the weak-coupling limit is taken.

This same breakable-boson–fermion-mixture model might also serve as a useful zero-order picture of the superfluid transition [3] in liquid helium-three where the interaction is not (10) but rather an appropriate interatomic potential, e.g., [22], that bind a certain fraction of the fermions into bosonic Cooper pairs.

Acknowledgement

M.C. and M. de Ll. acknowledge partial support from grant PB95-0492 and SAB95-0312 (Spain). M. de Ll. thanks D.M. Eagles for correspondence, R. Escudero and A.A. Valladares for valuable discussions, and very especially R.M. Carter, N.J. Davidson, J.M. Getino and H. Rubio for critical readings of the manuscript. M.A.S. and M. de Ll. acknowledge partial support from UNAM-DGAPA-PAPIIT IN103894 (México).

References

[1] R.A. Ogg, Jr., Phys Rev. 69 (1946) 243;
V.L. Ginzburg, Usp. Fiz. Nauk. 48 (1952) 25; Fortschr. Phys. 1 (1953) 101;
M.R. Schafroth, Phys. Rev. 96 (1954) 1442;
M.R. Schafroth et al., Helv. Phys. Acta 30 (1957) 93;
J.M. Blatt, Theory of Superconductivity (Academic Press, New York, 1964), and references therein;
P.W. Anderson, Science 235 (1987) 1196;
P.W. Anderson et al., Phys. Rev. Lett. 58 (1987) 2790;
X.G. Wen, R. Kan, Phys. Rev. B 37 (1988) 595;
J.R. Schrieffer et al., Phys. Rev. Lett. 60 (1988) 944;
R. Friedberg, T.D. Lee, Phys. Rev. B 40 (1989) 6745;
R. Micnas et al., Revs. Mod. Phys. 62 (1990) 113;
N. Nagaosa, P.A. Lee, Phys. Rev. Lett. 64 (1990) 2450;
R. Friedberg et al., Phys. Lett. A 152 (1991) 417 and 423.

[2] M.H. Anderson et al., Science 269 (1995) 198;
G. Taubes, Science 269 (1995) 152;
K. Burnett, Science 269 (1995) 182;
C.C. Bradley et al., Phys. Rev. Lett. 75 (1995) 1687;
G.P. Collins, Phys. Today 48 [8] (1995) 17;
K.B. Davis et al., Phys. Rev. Lett. 75 (1995) 3969;
J.R. Ensher et al., Phys Rev. Lett. 77 (1996) 4984.

[3] G.B. Lubkin, Phys. Today 49 [12] (1996) 17; Sci. Am. 276 [1] (1997) 11.

[4] S. Curilef, Phys. Lett. A 218 (1996) 11;
M. R-Monteiro et al., Phys. Rev. Lett. 76 (1996) 1098.

[5] J.D. Gunton, M.J. Buckingham, Phys. Rev. 166 (1968) 152.

[6] R.M. Ziff et al., Phys. Rep. 32 (1977) 169.

[7] R.M. May, Phys. Rev. 135 (1964) 1515;
A. Haerdig, F. Ravndal, Eur. J. Phys. 14 (1993) 17;
S. Viefers et al., Am. J. Phys. 63 (1995) 369.
[8] R.K. Pathria, Statistical Mechanics (Pergamon, Oxford, 1972) pp. 177, 211, 501.
[9] S. Fujita et al., Found. Phys. 21 (1991) 1117;
S. Fujita, J. Supercond. 4 (1991) 297; 5 (1992) 83;
S. Fujita, S. Watanabe, J. Supercond. 5 (1992) 219;
S. Fujita, S. Godoy, Quantum Statistical Theory of Superconductivity (Plenum Press, New York, 1996).
[10] M. Casas et al., Physica C, in press.
[11] J.R. Schrieffer, Theory of Superconductivity (Benjamin, New York, 1964) p. 33.
[12] P.C. Martin, in: Superconductivity, ed. R.D. Parks, Vol. 1 (Marcel Dekker, New York, 1969).
[13] L. Belkhir, M. Randeria, Phys. Rev. B 49 (1994) 6829.
[14] D. Jérome et al., Physica B 206/207, (1995) 559;
N. Dupuis, G. Montambaux, Phys. Rev. B 49 (1994) 8993;
J.M. Williams et al., Science 252 (1991) 1501;
D. Jérome, Science 252 (1991) 1509;
A. Khurana, Phys. Today 43 [9] (1990) 17.
[15] A. Hebard, Phys. Today 45 [11] (1992) 26.
[16] A.S. Davydov, Phys. Reps. 190 (1990) 191, see esp. p. 208.
[17] V.C. Aguilera-Navarro et al., in: Cond. Matter Theories, ed. R.V. Panat, Vol. 12 (Nova, New York, 1997).
[18] N.W. Ashcroft, N.D. Mermin, Solid State Physics (Holt, Rinehart, Winston, New York, 1976) p. 53.
[19] Y.J. Uemura et al., Phys. Rev. Lett. 66 (1991) 2665; Nature 352 (1991) 605; Physica B 186–188 (1993) 223.
[20] D.R. Harshman, A.P. Mills, Phys. Rev. B 45 (1992) 10684;
R. Micnas et al., op. cit.
[21] F. London, Nature (London) 141 (1938) 643.
[22] S. Stringari, Phys. Lett. A 106 (1984) 267;
M. Barranco et al., Phys. Rev. B 54 (1996) 7394.