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BEC-DRIVEN SUPERCONDUCTIVITY IN THE CUPRATES

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We apply to cuprates a three-fluid ideal boson-fermion statistical model of superconductivity in two dimensions (2D) derived from three extrema of the system Helmholtz free energy (subject to constant total fermion-number) for the BCS model interaction between fermions. The same interactions absent in BCS theory are neglected here. As the ensuing bosonic Cooper pairs move not in vacuum but in a Fermi sea we employ the correct linear—as opposed to the commonly-assumed quadratic—dispersion relation in the center-of-mass momentum (CMM). More importantly, pair breakup beyond a certain (very small) CMM is accounted for. Bose-Einstein condensation (BEC) critical temperatures of about 800 K result for moderate coupling with cuprate parameters.

A finite-temperature, $T \geq 0$, generalization of the zero center-of-mass momentum (CMM), $\mathbf{K} = \mathbf{k}_1 + \mathbf{k}_2 = 0$, Cooper pair[1] (CP) equation is

$$1 = \lambda \int_0^{\hbar\omega_D} d\xi (e^{-\beta\xi} + 1)^{-2} [2\xi + \Delta_0(T)]^{-1}, \tag{1}$$

whose numerical solution for the binding energy $\Delta_0(T) \geq 0$ is found to decrease monotonically for all $T \geq 0$, even *above* T_c. Here $\lambda \equiv g(E_F)V$ is the dimensionless coupling, where $V \geq 0$ is the BCS model interaction strength, nonzero only within a shell of energy width $\hbar\omega_D$ about the Fermi surface, $g(E_F) \equiv L^2 m/2\pi\hbar^2$ is the (constant) 2D electronic density of states for one spin at the Fermi energy $E_F \equiv \hbar^2 k_F^2/2m$, $\beta \equiv 1/k_B T$, and L is the many-fermion system size.

Following this line and extending results in Ref. 2 with $k_1^2, k_2^2 < k_\mu^2 + k_D^2$, where $k_\mu \equiv \sqrt{2m\mu/\hbar^2} \to k_F$ as $T \to 0$, and $k_D \equiv \sqrt{2m\omega_D/\hbar}$, one arrives at the 2D working equation for CP binding energy $\Delta_K(T)$, which is *linear*[2] in K for small K,

$$1 = \frac{4\lambda}{\pi}\int_{\theta_{min}}^{\pi/2} d\theta \int_{\tilde{\xi}_{min}(\theta)}^{\tilde{\xi}_{max}(\theta)} d\tilde{\xi}\,\tilde{\xi}\frac{[2\tilde{\xi}^2 + 2(1+\nu)\kappa^2 - 2 + \tilde{\Delta}_\kappa(\tilde{T})]^{-1}}{1+\exp(-\tilde{\beta}\{\tilde{\xi}^2 + (1+\nu)\kappa^2 + 2\sqrt{1+\nu}\,\kappa\tilde{\xi}\cos\theta - 1\})} \times [1+\exp(-\tilde{\beta}\{\tilde{\xi}^2 + (1+\nu)\kappa^2 - 2\sqrt{1+\nu}\,\kappa\tilde{\xi}\cos\theta - 1\})]^{-1} \quad (2)$$

where $\nu \equiv \hbar\omega_D/\mu$, μ is the fermion chemical potential, $\tilde{\xi}_{min}(\theta) \equiv \sqrt{1+\nu}\,\kappa\cos\theta + \sqrt{1-(1+\nu)\,\kappa^2\sin^2\theta}$, $\tilde{\xi}_{max}(\theta) \equiv -\sqrt{1+\nu}\,\kappa\cos\theta + \sqrt{(1+\nu)(1-\kappa^2\sin^2\theta)}$ and $\theta_{min} = 0$ if $2\kappa < 1 - \sqrt{(1-\nu)/(1+\nu)}$ and $= \cos^{-1}(\nu/4\sqrt{1+\nu}\,\kappa\sqrt{1+\nu/2-(1+\nu)\kappa^2})$ otherwise. Here, dimensionless quantities $\tilde{\xi} \equiv k/k_\mu$, with $\mathbf{k} = \frac{1}{2}(\mathbf{k}_1 - \mathbf{k}_2)$ the *relative* momentum wavenumber, $\tilde{\Delta}_\kappa(\tilde{T}) \equiv \Delta_K(T)/\mu$, $\tilde{T} \equiv k_B T/\mu$ or $\tilde{\beta} \equiv \mu\beta$, and $\kappa \equiv K/2\sqrt{k_\mu^2 + k_D^2}$ were introduced.

Minimizing the system free energy[3] gives the T-dependent number of composite (CP) bosons $N_B(T) = N_{B,0}(T) + N_{B,0<K<K_0}(T)$, where

$$N_{B,0<K<K_0}(T) \equiv \sum_{K>0}^{K_0} N_{B,K}(T) = \sum_{K>0}^{K_0} [e^{\beta\{\varepsilon_K(T) - \mu_B(T)\}} - 1]^{-1} \quad (3)$$

with $\varepsilon_K \equiv \Delta_0(T) - \Delta_K(T) \geq 0$ the CP excitation energy, $K_0 \sim 10^{-4} k_F$ the breakup CMM defined by $\Delta_{K_0}(T) \equiv 0$, and $\mu_B(T)$ the bosonic chemical potential for the CP-fermion binary mixture which vanishes (as for the more familiar pure-boson gas) for all $T \leq T_c$. Though their creation/annihilation operators for fixed $\mathbf{k}_1$ and $\mathbf{k}_2$ fail[4] to exactly satisfy the familiar boson commutation relations, CP's are bosonic because they do obey the Bose-Einstein (BE) distribution since an indefinitely large number of CP's with different relative $\mathbf{k}$ correspond to a given CMM $\mathbf{K}$. At the Bose-Einstein condensation (BEC) T_c both $N_{B,0}(T_c) \simeq 0$ and $\mu_B(T_c) \simeq 0$ so that (3) leads to the (implicit) T_c-equation

$$1 = \frac{\tilde{T}_c}{\nu}\ln\left[\frac{1+e^{-[\tilde{\Delta}_0(\tilde{T}_c)/2-\nu]/\tilde{T}_c}}{1+e^{-[\tilde{\Delta}_0(\tilde{T}_c)/2+\nu]/\tilde{T}_c}}\right] + \frac{8(1+\nu)}{\nu}\int_0^{\kappa_0} d\kappa\kappa[e^{\{\tilde{\Delta}_0(\tilde{T}_c)-\tilde{\Delta}_\kappa(\tilde{T}_c)\}/\tilde{T}_c} - 1]^{-1} \quad (4)$$

where $\kappa_0 \equiv K_0/2\sqrt{k_\mu^2 + k_D^2}$. Solving numerically for T_c in conjunction with (2) for $\tilde{\Delta}_\kappa(\tilde{T})$ and (1) for $\tilde{\Delta}_0(\tilde{T})$, we show results for $\lambda = 1/2$ in Fig. 1. For cuprates[5] $d \simeq 2.03$ is more realistic as this reflects inter-plane couplings, but results would be very similar to those given here for $d = 2$.

Finally, the root-mean-square radius of a CP at $T = 0$ in *any* d is[6] $x_{RMS}^{Coop} = 2\hbar v_F/\sqrt{3}\Delta_0(0)$, where $v_F = \sqrt{2E_F/m}$ is Fermi velocity. Overlap between CP's vanishes provided $x_{RMS}^{Coop} < R_0$, half the average center-to-center distance between pairs,

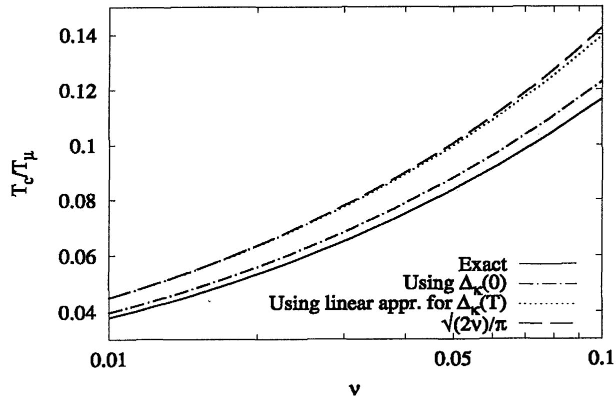

Fig. 1. Critical BEC temperature T_c in units of $T_\mu \equiv \mu/k_B$, where $\mu(T)$ is chemical potential for the free Fermi gas, resulting from (4) with $\lambda = 1/2$ for varying $\nu \equiv \hbar\omega_D/\mu$: with no approximations (full curve); using $\Delta_K(T)$ evaluated at $T = 0$ (dot-dashed); using the linear[2] (in K) approximation for $\Delta_K(T)$ (dotted); and compared with rough result $\sqrt{2\nu}/\pi$ found in Ref. 3 for $\lambda = 0^+$(dashed). Note that $T_c \sim 800$K for typical cuprate values of $\nu = 0.05$ and $T_\mu \simeq T_F \sim 10^4$K.

which in 2D is *defined* via $N_B(0)/L^2 = 1/\pi R_0^2$, where[3] $N_B(0) = \frac{1}{2}g(E_F)\Delta_0(0)$. This gives the *non-overlap condition* $\lambda > 2/\ln(1+3\nu) \simeq 14.3$ (for $\nu = 0.05$) which is only a rough estimate; the correct condition should be calculated at $T = T_c$.

To conclude, even for the (much-maligned) electron-phonon interaction in moderate coupling, critical BEC T_c's of around 800 K are found in 2D *under the same many-body dynamical conditions assumed in BCS theory*, namely, noninteracting CP's formed among fermions by the familiar BCS model interaction—conceivably applicable even for non-phononic mechanisms[7]. The present *interactionless* binary mixture statistical model will be accurate, however, only for strong enough coupling where CP's cease to overlap. Otherwise neglected interactions should be sizable, and expected to lower such high predicted T_c values.

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