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Boson-exchange superconductor model with a Van Hove singularity

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Abstract

A realistically-shaped electron–boson interaction Eliashberg spectrum $\alpha^2F(\omega)$ is shown to be crucial in determining superconducting transition temperatures when the Fermi-energy shift from a Van Hove saddle-point singularity lies between the minimum and maximum boson frequencies in $\alpha^2F(\omega)$.

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A Van Hove scenario (VHS) consisting of a saddle-point singularity at energy ϵ_{VH} in the vicinity of the Fermi energy ϵ_F in Eliashberg superconductors is the simplest model of a rapidly-varying electronic density of states (EDOS) $N(\epsilon)$ [1–12]. Eliashberg superconductivity is caused by charge carriers pairing mediated via some kind of *bosonic excitations* in the correlated electron-ion system. Although the analogous Migdal theorem (allowing neglect of vertex corrections to many-body-perturbation-theoretic Feynmann diagrams) has been proved only for *phonon* mediation, Eliashberg equations are still useful as a first approximation [13,14]. Hopefully, vertex corrections, if important, can be included within effective kernels without modifying the form of the basic equations [13–15].

The behavior of the superconducting transition temperature, T_c, as a function of the shift $t \equiv |\epsilon_F - \epsilon_{VH}|$, was considered [1] in a modified Eliashberg theory that includes a non-constant EDOS [16–18]. It was found that for large t, i.e., $t > \omega_{max}$, T_c is insensitive to the EDOS singularity and identical to the familiar McMillan T_c-formula. But T_c changes radically if a peak in $N(\epsilon)$ is close to ϵ_F. Qualitatively, T_c depends only *weakly*, in either extreme $t < \omega_{min}$ or $t > \omega_{max}$, on the fine-structure of the frequency distribution of the assumed electron–boson interaction spectrum function $\alpha^2F(\omega)$ of Eliashberg theory. Any dependence on the $\alpha^2F(\omega)$ fine structure enters the T_c-equation only via an average boson frequency ω_{ln}, the mass-renormalization factor λ, and possibly through the first few moments of $\alpha^2F(\omega)$, [17] – the Coulomb pseudopotential μ_* being taken as zero. Thus, two different spectra leading to identical λ and ω_{ln} parameters results in *nearly* equal $T_c(t)$ for either $t < \omega_{min}$ and $t > \omega_{max}$. However, the more interesting and realistic case when ϵ_F is shifted from ϵ_{VH} by some finite value [10] such that $\omega_{min} < t < \omega_{max}$ has not been studied.

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In this Letter we elucidate the effect upon T_c of a *realistically*-structured $\alpha^2F(\omega)$, for t *lying within the interval* $(\omega_{\min}, \omega_{\max})$. We ignore all complications arising from possible structural instabilities in a VHS (for an excellent review see [7]), and assume the simplest model for the EDOS, namely $N(\epsilon) = N_0 \ln(2W/|\epsilon - t|)$ over the entire range of ϵ [1,5]. This model may be relevant for any type intermediate boson such as excitons, plasmons, paramagnons, etc., e.g., in cuprate superconductors where ϵ_F is closely pinned to the ϵ_{VH} of $N(\epsilon)$ as now firmly established experimentally [7]. We need not assume that the singular $N(\epsilon)$ originates from the intrinsic band structure of CuO-planes. Indeed, flat regions of the band structure – leading to singularities in $N(\epsilon)$ – may arise from the complex electron-electron interactions in *high-T_c-oxides*, but we do not exclude that an intermediate boson necessary for the Eliashberg formalism would be provided by these interactions [19]. Furthermore, one need not specify precisely the interaction spectrum $\alpha^2F(\omega)$ which may be *any function of* ω over the interval $(\omega_{\min}, \omega_{\max})$ and zero otherwise. Effects of strong gap-anisotropy found in some high-T_c materials are not considered, although the symmetry of the order parameter may yet prove to be d-wave. If so, the present work would merely suggest the need of more careful consideration of the possible consequences from the 'ϵ_F *to* ϵ_{VH}-*vicinity*' in the corresponding Eliashberg theory. For *s-wave-type symmetry* these effects could be qualitatively included in the treatment given below within the spirit of [20] (see also [21]) – i.e., by means of a mean-square anisotropy factor a^2.

The Migdal-Eliashberg theory including all the complexities of band structure has been developed by Garland [22]. Ignoring interband and anisotropy effects modifies the relevant equations in a way that incorporates the energy dependence of $N(\epsilon)$ and leads to equations very similar to those of standard Eliashberg theory. A popular method of analytically solving the Eliashberg equations valid for general $N(\epsilon)$ [16–18] is described in [1] where the basic gap $\Delta(i\omega_n)$ and mass renormalization $\tilde{Z}(i\omega_n)$ functions are related, near $T = T_c$, through

$$\Psi(i\omega_n) = \Psi^\circ(i\omega_n) + \nu\pi T_c \sum_{m=-\infty}^{+\infty} K(i\omega_n, i\omega_m)\,\Psi(i\omega_m), \tag{1}$$

where $\Psi(i\omega_n) \equiv \Delta(i\omega_n)\tilde{Z}(i\omega_n)/\Delta(0)Z(0)$, and ω_m are the Matsubara frequencies. The free term and the kernel in (1) are determined by specific $\alpha^2F(\omega)$ and $N(\epsilon)$ which enter through known functions $I(\omega_n, \omega_m)$ and $\tilde{N}(|\tilde{\omega}_n|)$ determined in [1] (see, also [16–18]), namely

$$\Psi^\circ(i\omega_n) = I(\omega_n, 0) + \kappa\big[I(\omega_n, 0) - 1\big],$$

$$K(i\omega_n, i\omega_m) = \big[I(\omega_n, \omega_m) - I(\omega_n, 0)\,I(0, \omega_m)\big]\,\frac{Z(0)}{\tilde{Z}(i\omega_m)}\,\frac{\tilde{N}(|\tilde{\omega}_m|)}{|\omega_m|}.$$

Here $\tilde{\omega}_n = \omega_n\tilde{Z}(i\omega_n)$, $\lambda = 2\int_0^\infty \frac{d\omega}{\omega}\alpha^2F(\omega)$, and $\nu = \lambda/Z(0)$, while $Z(0) = 1 + \lambda$ is the renormalization factor associated with $N(\epsilon) = \text{const}$. Then, T_c emerges as an eigenvalue of the equation $\Psi(0) = 1$.

Note, however, that *the exact transformation* used to arrive at (1) was suggested in [23] and is essentially a regularization procedure for integral equations with divergent kernels. Following Refs. [1,24,25] one generates a series for $\Psi(i\omega_n)$ from $\Psi^\circ(i\omega_n)$, which is a zero-order approximation that will be successful depending on the nature of the sum in (1) and provided the constant ν is small. For intermediate-coupling $\lambda < 1.5$ ($\nu < 0.6$) and $\omega_{ph} > \pi T_c$, it was shown [24,25] that the zero-order approximation already leads to rather accurate expressions for T_c. The exact value of κ is proportional to the weak pseudopotential μ_* [1], which for simplicity is initially taken as zero. As to the function $\tilde{Z}(\omega_n)$, following [14] we neglect its frequency-dependence and put

$$\tilde{Z}(0) = Z(0) + \frac{1}{\ln(2W/t)}\int_0^\infty \frac{d\omega^2}{t^2 - \omega^2}\ln\left(\frac{\omega}{t}\right)\alpha^2F(\omega_2)$$

as found in [1]. Results obtained from this analytic expression for the renormalization factor compare excellently with exact numerical calculations that were performed for $\tilde{Z}(0)$ based on the original Eliashberg equations.

The eigenvalue equation for T_c is then [1]

$$\frac{1}{g} = \frac{1}{\lambda^2}\int_0^\infty d\omega_1^2 \alpha^2 F(\omega_1)\int_0^\infty d\omega_2^2 \alpha^2 F(\omega_2)\frac{[P(\omega_1)-P(\omega_2)]}{\omega_2^2-\omega_1^2}, \tag{2}$$

where $g = \lambda/\tilde{Z}(0)$ and

$$P(\omega) = \frac{1}{\ln\left(\frac{2W}{t}\right)}\frac{1}{\omega^2}\left[\ln\left(\frac{1.13\omega}{T_c}\right)\ln\left(\frac{2W}{\sqrt{|t^2-\omega^2|}}\right) - \tfrac{1}{2}\ln^2\left(\frac{1.13t}{T_c}\right) + S_t(\omega,T_c)\right],$$

for which we assumed that $\pi T_c < t$ and $\pi T_c < \omega$ for any ω contained in $\alpha^2 F(\omega)$. Similarly [1,26] one has

$$S_t(\omega,T_c) = S_t^\circ(\omega,T_c) + \theta(t-\omega)\,\Delta S_t(\omega,T_c),$$

where the Heaviside unit step-function $\theta(t-\omega)$ was introduced to write

$$S_t^\circ(\omega,T_c) = \ln\left(\frac{1.13\omega}{T_c}\right)\ln\left(\frac{1.13\sqrt{|t^2-\omega^2|}}{T_c}\right) + \delta_1(\omega),$$

$$\Delta S_t(\omega,T_c) = \ln\left(\frac{t}{\omega}\right)\ln\left(\frac{1.13\sqrt{|t^2-\omega^2|}}{T_c}\right) - \tfrac{1}{2}\ln^2\left(\frac{1.13\omega}{T_c}\right) + \delta_2(\omega).$$

Two remarks are in order: a) $\delta_1(\omega)$ and $\delta_2(\omega)$ do not depend on T_c – as a result they have no effect on the T_c-equation functional form, and lead to small corrections to T_c [1]; and b) due to a specific form for $S_t(\omega,T_c)$, T_c enters the eigenvalue Eq. (2) only via $\ln[1.13 f(\omega,t)/T_c]$. Hence, the T_c-dependence expressed as integrals over ω may be factored out since $\ln[1.13 f(\omega,t)/T_c] = \ln(1.13t/T_c) + \ln[(f(\omega,t)/t)]$. Then, (2) becomes

$$a(t)\ln^2\left(\frac{1.13t}{T_c}\right) + b(t)\ln\left(\frac{1.13t}{T_c}\right) + c(t) = 0, \tag{3}$$

where the coefficients $a(t)$, $b(t)$ and $c(t)$ are functionals of the spectral density of interaction. Their exact expressions, at least for $t < \omega_{\min}$ and $t > \omega_{\max}$, are known [1], e.g., $a(t) = 1/2$ or 0 for $t < \omega_{\min}$ or $t > \omega_{\max}$, respectively. In general, they are double integrals over $\alpha^2 F(\omega)$ and can be determined only if the spectrum $\alpha^2 F(\omega)$ is specified. However, one can perform in (2) the partial integrations *for any* $\alpha^2 F(\omega)$ by introducing the *partial interaction spectra* $S_1(\omega)$ and $S_2(\omega)$ split at $\omega = t$, and defined as

$$S(\omega) \equiv \alpha^2 F(\omega) = S_1(\omega)\theta(t-\omega) + S_2(\omega)\theta(\omega-t). \tag{4}$$

The integrals in (2) then factorize as

$$\langle(\ldots)\rangle_{S(\omega)} = \frac{\lambda_1(t)}{\lambda}\langle(\ldots)\rangle_{S_1(\omega)} + \frac{\lambda_2(t)}{\lambda}\langle(\ldots)\rangle_{S_2(\omega)},$$

via the *partial interaction parameters*

$$\lambda_1(t) = 2\int_0^t \frac{d\omega}{\omega} S(\omega)$$

and

$$\lambda_2(t) = 2\int_t^\infty \frac{d\omega}{\omega} S(\omega),$$

where by definition

$$\langle \dots \rangle_{X(\omega)} \equiv \int_0^\infty \left(d\omega^2/\omega^2\right) X(\omega)(\dots) \Big/ \int_0^\infty \left(d\omega^2/\omega^2\right) X(\omega), \quad X = S, S_1 \text{ or } S_2.$$

The spectra $S_1(\omega)$ and $S_2(\omega)$ given by (4) lead to physically different channels of charge-carrier Cooper pairings, i.e., to electron-like and hole-like sections simultaneously in different directions of the iso-energy surface $\epsilon = \epsilon_{\rm VH}$. Because $\epsilon_{\rm F}$ is shifted from $\epsilon_{\rm VH}$ by an amount t, absorption or emission by an electron of a phonon belonging to the spectrum $S_1(\omega)$ constrains an electron (hole) to an electron- (hole-) like orbit. Similarly, a phonon of energy $\omega > t$ [i.e., a phonon from $S_2(\omega)$] may convert an electron into a hole, and vice versa. This is in addition to the conventional channel associated with $S_1(\omega)$ which does not mix electron- and hole-like excitations.

The coefficients in (3) thus become

$$a(t) = \frac{1}{2}\frac{\lambda_2^2(t)}{\lambda^2} - \frac{1}{\lambda^2}\int_0^t \frac{d\omega_1^2}{\omega_1^2} S(\omega_1) \int_t^\infty \frac{d\omega_2^2}{\omega_2^2} S(\omega_2) \frac{\omega_1^2}{\omega_2^2 - \omega_1^2},$$

$$b(t) = \ln\left(\frac{2W}{t}\right) + \frac{\lambda_2(t)}{\lambda}\ln\left(\frac{\omega_2(t)}{t}\right) - \frac{1}{2}\left(\eta^\circ - \frac{\lambda_1^2(t)}{\lambda^2}\eta(t)\right) + \Delta b(t),$$

$$c(t) = \ln\frac{2W}{t}\left(\ln\frac{\omega_{\ln}}{t} - \frac{\eta^\circ}{2} - \frac{1}{g}\right) + \Delta c(t).$$

The *analytic* expressions for $\Delta b(t)$ and $\Delta c(t)$, whose contributions correspondingly to $b(t)$ and $c(t)$ are less than 5–10%, will be reported elsewhere. Here, $\omega_{\ln} \equiv \exp\langle \ln\omega \rangle_{S(\omega)}$ is the mean-logarithmic frequency of the total spectrum. The partial frequencies $\omega_1(t)$ or $\omega_2(t)$ are given by the expression for $\omega_{\ln}$ but with $S_1(\omega)$ or $S_2(\omega)$ instead of $S(\omega)$. The quantity η° – as well as $\omega_{\ln}$ – follow from standard Eliashberg theory with constant EDOS [24,25], and is just $\eta^\circ = \langle\langle \omega^2/(\omega^2 - \omega'^2)\rangle_{S(\omega')}\rangle_{S(\omega)}$. The factor $\eta(t)$ is obtainable by replacing the total $S(\omega)$ by $S_1(\omega)$ in η°. The latter quantity, as well as $\eta(t)$, are weakly-structure-dependent parameters of the interaction spectrum and thus result in differing asymptotic values of $T_{\rm c}$ for small and large t.

Eq. (3) then leads to the final expression

$$T_{\rm c} = 1.13t \exp\left[\frac{b(t) - \sqrt{b^2(t) - 4a(t)c(t)}}{2a(t)}\right], \tag{5}$$

so that $T_{\rm c}$ is clearly determined by the filling-parameter-dependent $a(t)$, $b(t)$ and $c(t)$ which in turn are functionals of $S_1(\omega)$ and $S_1(\omega)$. It depends principally on *the partial interaction parameters* $\lambda_1(t)$, $\omega_1(t)$ and $\lambda_2(t)$, $\omega_2(t)$ connected via the total λ and average frequency $\omega_{\ln}$ through

$$\lambda = \lambda_1(t) + \lambda_2(t), \quad \omega_{\ln} = \left[\omega_1(t)\right]^{\frac{\lambda_1(t)}{\lambda}} \left[\omega_2(t)\right]^{\frac{\lambda_2(t)}{\lambda}}.$$

The form of (5) reveals that the contribution of any ω to the eigenvalue equation for T_c depends on the location of ϵ_F in $N(\epsilon)$. For either small or large $t \equiv |\epsilon_F - \epsilon_{VH}|$ the form of the three coefficients (3) simplify considerably, and (5) is then expressible in terms of the ω_{ln} and λ as

$$T_c^{(1)} = \frac{1.13\omega_{ln}}{e^{\eta^\circ/2}}\left(\frac{2W}{t}\right)\exp\left(-\sqrt{\left(\ln\left(\frac{2W}{t}\right)+\frac{1}{g}\right)^2+\left(\ln\left(\frac{\omega_{ln}}{t}\right)-\frac{\eta^\circ}{2}\right)^2-\frac{1}{g^2}}\right) \quad (\text{small } t),$$

$$T_c^{(2)} = \frac{1.13\omega_{ln}}{e^{\eta^\circ/2}}\exp\left(-\frac{1}{g}\right) \quad (\text{large } t),$$

both of which were obtained in [1] and closely resemble those given in the VHS of the BCS theory [8,12]. Note that T_c also depends on the full electronic bandwidth $2W$ being a decreasing function of the ratio $2W/\omega_{ln}$. As this ratio diverges for fixed t one obtains the familiar McMillan T_c-formula [1]. As t increases, the 'evolution' of T_c (5) from $T_c^{(1)}$ to $T_c^{(2)}$ is entirely determined by the behavior of the coefficients (3), with $a(t)$ the highest-order term of $\ln(1.13t/T_c)$ being fixed *solely* by the interaction-spectrum parameters. This coefficient changes sign for some t lying within $(\omega_{min}, \omega_{max})$. By contrast, $b(t)$ and $c(t)$ being of opposite sign depend mainly on the electronic structure of the system.

To qualitatively understand the effect upon T_c of any (*non-δ-function-like*) fine-structure in $\alpha^2F(\omega)$ one can model the latter by simple functions reflecting *general* features of some realistic spectrum. Initially $\alpha^2F(\omega)$ was modeled by *single peaks* of the form $S_i(\omega) = A_i\omega^n/[(\omega-\omega_{0i})^2 + B_i^2]$, with parameters A_i, B_i and ω_{0i}, modulating the shapes of $\alpha^2F(\omega)$, and fitted to yield the same ω_{ln} and λ for each case 'i'. This guaranteed *nearly* equal values of T_c for either extreme $t \ll \omega_{ln}$ or $t \gg \omega_{ln}$. Fig. 1 shows that narrowing the spectrum gives slightly higher T_c in either extreme – a well-known result for constant EDOS [17]. However, the variation of T_c turns out to be *non-monotonic* in the intermediate region and passes through a *minimum* T_c^{min}. Fixing λ and ω_{ln}, T_c^{min} decreases with narrowing $\alpha^2F(\omega)$ eventually making superconductivity *disappear altogether* over some interval of intermediate t values near ω_{ln}. Broadening $\alpha^2F(\omega)$ leads to T_c a smoother function of t, while its minimum shifts towards larger filling-parameter values approaching T_c°, the value corresponding to constant EDOS, from below. Calculations for a smaller set of ν were repeated by fixing ω_{ln}, but changing the

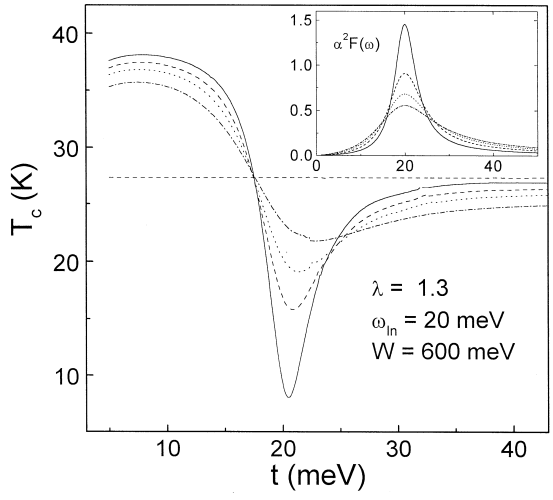

Fig. 1. T_c vs t for different shape-types of $\alpha^2F(\omega)$; single-peak results shown in inset. The set of parameters (A, B, ω_0) were chosen (in meV) as: (0.65,3,19.8) full curve; (1.12,5,19.4) dashed; (1.62,7,18.9) dotted; and (2.2,9,18.2) dot-dashed. All parameter sets yield the same λ and ω_{ln}. Horizontal dashed-line is T_c° as defined in text.

height of the (single-peak) curve in $\alpha^2F(\omega)$ so as to ensure different λ's. The T_c-enhancing effect of the VHS over that of constant EDOS increased significantly by *decreasing* λ [1,27], but the characteristic non-monotonic behavior of T_c vs t *remained unchanged*. Because the iterative solution of (1) is '*exact*' for small ν, we conclude that this non-trivial variation of T_c is an *inherent* property of *real* superconductors, i.e., systems with varying $N(\epsilon)$.

Next, $\alpha^2F(\omega)$ was put in the form of two superposed peaks, motivated by the fact that T_c itself depends critically on the details of variation of $a(t)$ (5) which in turn is quite sensitive to the structure in $\alpha^2F(\omega)$. The strongest variation in T_c then occurs near the characteristic peaks of $\alpha^2F(\omega)$, while far from these peaks T_c changes slightly [1]. Thus, for the two-peak-structured $\alpha^2F(\omega)$ we expect a 'ladder-like' behavior in the evolution of T_c between the two extremes $t < \omega_{\min}$ and $t > \omega_{\max}$, the exact nature of which is then dictated by the details of $\alpha^2F(\omega)$. In fact, when the peaks in $\alpha^2F(\omega)$ are narrowed, thus rendering sharper individual peaks, for t varying over the considerable part of the interval $(\omega_{\min}, \omega_{\max})$ the magnitude of T_c is more concentrated near two different but nearly constant values T_{c1} and T_{c2}. The width over which T_c changes from T_{c1} to T_{c2} is given by the width of the low-frequency part of $\alpha^2F(\omega)$, but the magnitude of $T_{c1} - T_{c2}$ is fixed by the interaction parameters $\lambda^{(1)}$ and $\lambda^{(2)}$ associated with each peak, Fig. 2. As in the case of a single peak, Fig. 1, T_c passes through a minimum $T_c^{\min}$ and at $t \gg \omega_{\ln}$ approaches T_c°. Decreasing λ does not eliminate the non-monotonic behavior of the evolution of T_c between the extremes $t \ll \omega_{\ln}$ and $t \gg \omega_{\ln}$. The reason for the slight T_c-reduction at very small shifts in Figs. 1 and 2 is discussed in [1], as well in pioneering work by Pickett [16]. BCS-like theories predict maximum T_c when ϵ_F and ϵ_{VH} coincide; however, additional work is needed when $t \leq \pi T_c$. Also, one finds that non-zero values of μ_* drastically decrease T_c for large t [1], but as shown in [12] for small t (viz., in the high-temperature region) the influence of μ_* is significantly reduced.

A clue in understanding the origin of this nontrivial T_c behavior as a function of t follows by proposing a Fourier transform $V(\omega)$ for the indirect pairwise boson-mediated interaction potential that is a complicated function and *changes sign* for some frequencies – thus resulting in a *depairing effect* for the Cooper pairs. In BCS-like theories this interaction potential is mimicked by a negative constant or zero depending on whether ω is or is not, respectively, in the interval $(-\omega_{\max}, \omega_{\max})$. The reason for the depairing component of $V(\omega)$ is clarified by considering physically-different pairing channels $S_1(\omega)$ and $S_2(\omega)$ in (4), as discussed above, and by assuming for the moment that there is only an $S_1(\omega)$-type channel (different from zero provided $\omega \leq t$).

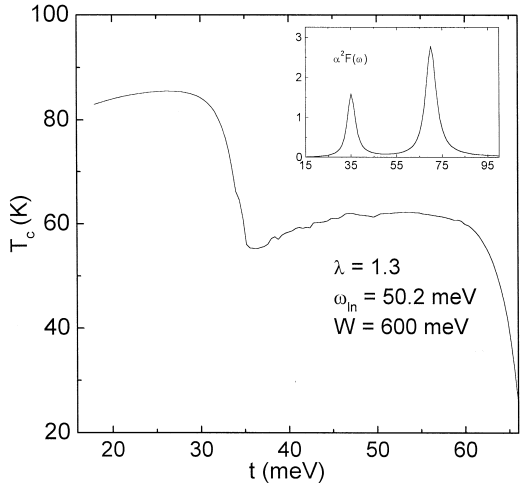

Fig. 2. T_c vs t for a two-peak-structured $\alpha^2F(\omega)$. Inset figure is for $\alpha^2F(\omega) = A_1\omega^n/[(\omega-\omega_{01})^2 + B_1^2] + A_2\omega^n/[(\omega-\omega_{02})^2 + B_2^2]$ with $A_1 = 0.2$, $B_1 = 2.1$, $\omega_{01} = 35$ meV and $A_2 = 0.31$, $B_2 = 2.8$, $\omega_{02} = 70$ meV.

Then, by applying BCS-theory it is easy to find a repulsive contribution to the total $V(\omega)$ originating from the exchange of two electrons via phonons of frequencies $\omega > t$.

The following argument may shed additional light into the origin of the non-monotonic T_c vs t-behavior. Since the EDOS varies sharply (even infinitely so at distances of order $t = |\epsilon_F - \epsilon_{VH}|$ from ϵ_F) pairing due to an individual boson of energy close to t is much stronger than that due to phonons with energies differing significantly from t, there being a large number of final states for electrons to be scattered into via phonons of energy $\sim t$. In other words, due to the logarithmic $N(\epsilon)$ the main contribution in the basic equations to the gap energy (which is proportional to T_c) comes from phonons with energies concentrated near t. On the other hand, as shown by several studies in conventional superconductors, e.g., [14], the gap function is roughly slowly-varying for frequencies less than ω_{max}, *decreases rapidly* near the upper edge of the phonon spectrum, and changes sign near ω_{max}. In fact, in many analytic studies of conventional superconductors, the gap function is simulated by single- or two-step shapes. Assuming now a very narrow electron-phonon interaction spectrum 'located' near some ω_0 (i.e., a spectrum for which $\omega_{max} \sim \omega_0$) and placing t near ω_{max} where the magnitude of the gap is significantly reduced, a reduction in T_c ensues.

To conclude, the behavior of T_c as a function of t depicted in special cases in Figs. 1 and 2 is useful in interpreting observations in several oxide superconductors. In $Bi_2Sr_2Cu_1O_x$ ($T_c \simeq 10K$) [28] and Sr_2RuO_4 ($T_c \simeq 0.1K$) [29,30] T_c is *unexpectedly* small in spite of ϵ_{VH} lying very close to ϵ_F. Assuming that the shift of ϵ_F from ϵ_{VH} in these compounds corresponds to values of t such that T_c falls near its minimum value (see, e.g., Fig. 1) then Eq. (5) adequately describes experiment. Moreover, for a two-peak-shaped $\alpha^2F(\omega)$, assuming an inter-electron (or -hole) interaction via bosons from the higher-frequency parts of $\alpha^2F(\omega)$ to be significant, the two-plateau structure, Fig. 2, for T_c is obtained which is well-established in empirical T_c vs x curves for $Y_1Ba_2Cu_3O_{7-x}$. Finally, the origin of so-called anomalous isotope effects in high-T_c-materials might be clarified, given the novel element that the partial $\lambda_1(t)$ and $\lambda_2(t)$ *depend* on the ionic mass (unlike the total λ *in the harmonic approximation*) and thus lead to non-vanishing terms in the isotopic-shift factor, as will be examined in detail elsewhere.

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