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Physica C 341–348 (2000) 151–152

PHYSICA C

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Cooper pair dispersion relation in two dimensions

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The Cooper pair binding energy *vs.* center-of-mass-momentum dispersion relation for Bose-Einstein condensation studies of superconductivity is found in two dimensions for a renormalized attractive delta interaction. It crosses over smoothly from a linear to a quadratic form as coupling varies from weak to strong.

For the attractive interfermion potential

$$V(r) = -v_0\delta(\mathbf{r}), \tag{1}$$

where $v_0 \geq 0$ is the interaction strength, one can apply the Lippmann-Schwinger as well as the Cooper-pair (CP) equation in two dimensions (2D) for two fermions of mass m with momenta wavevectors $\mathbf{k}_1$ and $\mathbf{k}_2$ in *free space* (i.e., vacuum) and in the momentum space above the *filled Fermi sea*, respectively. Combining these two equations so as to eliminate (the regularized, infinitesimally small [1]) v_0, one obtains the *renormalized* CP *equation*

$$\sum_k \frac{1}{B_2+\hbar^2k^2/m} - \sum_k{}' \frac{1}{\hbar^2k^2/m+\Delta_K-2E_F+\hbar^2K^2/4m} = 0, \tag{2}$$

where $\mathbf{k} \equiv (\mathbf{k}_1-\mathbf{k}_2)/2$ is the relative, $\mathbf{K} \equiv \mathbf{k}_1+\mathbf{k}_2$ the center-of-mass momentum (CMM), usually taken as zero in BCS theory, E_F the Fermi energy, $B_2 \geq 0$ the (single-bound-state) pair binding energy in vacuum, and $2E_F - \Delta_K$ the total energy of the CP with $\Delta_K \geq 0$ its binding energy now simply a function of B_2. The prime on the second summation denotes the restriction $|\mathbf{k} \pm \mathbf{K}/2| > k_F$. In 2D one finds that $\Delta_K = B_2$ exactly, but *only* for $K \equiv 0$. Expanding Eq. (2) for small but nonzero K and subtracting from the expression for $K = 0$ gives a small-CMM expansion valid for any dimensionless coupling $B_2/E_F = \Delta_0/E_F$, namely

$$\varepsilon_K \equiv (\Delta_0 - \Delta_K) = \frac{2}{\pi}\hbar v_F K + \left[1-\left\{2-\left(\frac{4}{\pi}\right)^2\right\}\frac{E_F}{B_2}\right]\frac{\hbar^2K^2}{2(2m)} + O(K^3), \tag{3}$$

where a nonnegative CP *excitation energy* ε_K has been defined, and the Fermi velocity v_F is given by $E_F/k_F = \hbar v_F/2$. It is this excitation energy that must be inserted into the Bose-Einstein (BE) distribution function to determine the critical temperature in a picture of superconductivity (or of superfluidity in, e.g., liquid ^{3}He) as a BE condensation (BEC) of CPs [2,3].

The leading term in (3) is *linear* in the CMM, followed by a *quadratic* term; similar results hold in 3D. In the strong coupling limit ($B_2 = \Delta_0 \gg E_F$) the quadratic term is exactly the CM kinetic energy of what was originally a CP (becoming

PII S0921-4534(00)00422-6

what is sometimes called a "local pair"), namely,

$$\lim_{\Delta_0 \gg E_F} \varepsilon_K = \frac{\hbar^2 K^2}{2(2m)}, \quad (4)$$

the familiar nonrelativistic kinetic energy of the composite pair of mass $2m$ and CMM K in vacuum. It is *this* dispersion relation that has been assumed in virtually all BEC studies in 3D of superconductivity (see, e.g., [3,4] among others)—but which in 2D is well-known to give *zero* transition BEC temperature T_c. However, recent BCS-Bose crossover picture root-mean-square radii calculations compared with empirical coherence lengths of several typical 2D-like cuprates [5] suggest these superconductors to be well within the BCS or weak-coupling regime, implying that the (nearly) linear relation is appropriate for them rather than the quadratic one.

Fig. 1 displays exact numerical results obtained from (2) for different couplings of a dimensionless CP excitation energy ε_K/Δ_0 as function of K/k_F. For weak enough coupling (and/or high enough density) the *exact* dispersion relation is practically linear over almost the entire interval of K/k_F up to breakup—despite the divergence in (3) of the quadratic and possibly higher-order terms as $B_2 \to 0$. For stronger coupling the quadratic dispersion relation (4) begins to dominate, i.e., as $B_2/E_F \to \infty$, meaning that either $B_2 \to \infty$ *or* $E_F \to 0$, the latter implying that the Fermi sea vanishes and the vacuum is recovered *as a limit*.

In conclusion, the CP problem with nonzero CMM evolves with weak to strong zero-range pairwise interaction to give a *linear* dispersion relation in the CMM that gradually *crosses over* to a *quadratic* relation with increasing coupling. These results will play a critical role in a model of superconductivity (or superfluidity) based on BE condensation of CPs, particularly in 2D where only the linear relation can give rise to *nonzero* T_c's.

Partial support from UNAM-DGAPA-PAPIIT (México) # IN102198, CONACyT (México) # 27828 E, DGES (Spain) # PB95-0492 and FAPESP (Brazil) is gratefully acknowledged.

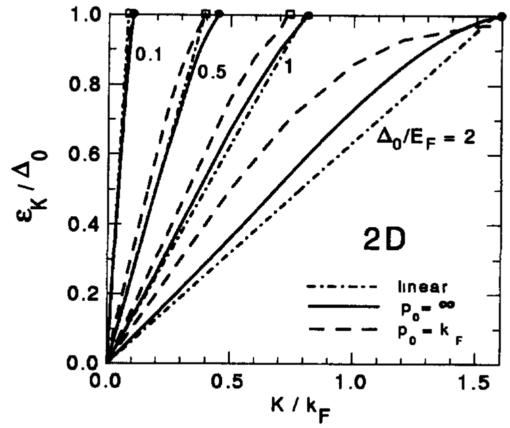

Figure 1. Dimensionless exact CP excitation energy ε_K/Δ_0 *vs* K/k_F in 2D (full curves) calculated from (2) for different B_2/E_F. Dot-dashed line is linear approximation (virtually coincident with the exact curve for all $B_2/E_F \lesssim 0.1$) while dashed curve is exact finite range result for a potential more general than (1) and whose double Fourier transform is $V_{pq} \equiv -(v_0/L^2)[(1+p^2/p_0^2)(1+q^2/p_0^2)]^{-1/2}$ (see Nozières and Schmitt-Rink [3]), which for $p_0 \to \infty$ becomes (1). Dots mark values of CMM wavenumber where the CP breaks up, i.e., where Δ_K turns negative.

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