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Cooper pairs as resonances

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Abstract

Using the Bethe–Salpeter (BS) equation, Cooper pairing can be generalized to include contributions from holes as well as particles from the ground state of either an ideal Fermi gas (IFG) or of a BCS many-fermion state. The BCS model interfermion interaction is employed throughout. In contrast to the better-known original Cooper pair (CP) problem for either two particles or two holes, the generalized Cooper equation in the IFG case has no real-energy solutions. Rather, it possesses two complex-conjugate solutions with purely imaginary energies. This implies that the IFG ground state is unstable when an attractive interaction is switched on. However, solving the BS equation for the BCS ground state reveals two types of *real* solutions: one describing moving (i.e., having nonzero total, or center-of-mass, momenta) CPs as resonances (or bound composite particles with a *finite* lifetime), and another exhibiting superconducting collective excitations analogous to Anderson–Bogoliubov–Higgs RPA modes. A Bose–Einstein-condensation-based picture of superconductivity is addressed.

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1. Introduction

The original Cooper pair (CP) equation [1] is a two-electron Schrödinger equation in momentum representation with a given two-body interaction (having some *attraction*) but includes ad hoc restrictions on the magnitudes of both electron wave vectors $\mathbf{k}_1$, $\mathbf{k}_2$, namely $k_1 > k_F$, $k_2 > k_F$, where k_F is the electron Fermi wave number for an ideal nonrelativistic many-electron system. One then seeks the energy eigenvalues of a CP bound state and its corresponding wave function.

Since there is no rigorous derivation of the original CP equation, several authors [2–5] reformulated the complete CP problem without neglecting holes, using the mathematically *exact* Bethe–Salpeter (BS) equation approach [6] applied to the system, in search of two-particle bound states in the presence of other system electrons. Such a treatment allows generalizing [7] several approaches in superconductivity theory, including the BCS, the BCS-Bose crossover, and the Bose–Einstein condensation pictures. But if the BS equation is based merely on the ideal Fermi gas (IFG) ground state one obtains purely imaginary solutions, suggesting that the ground state is

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unstable in the presence of attractive interactions of some kind. This is an *instability* of the IFG ground state with respect to the creation of two-particle (2p-) or two-hole (2h-) resonant states, a situation analogous to the classical problem of *hydrodynamic instability* [8].

2. BS equation based on BCS ground state

Consider, however, the generalized two-component, two-electron BS equation based *not* on the IFG ground state [2–5] but rather on the BCS ground state. We introduce the Bogoliubov–Valatin u, v transformation of electron Fermi operators $a_{\mathbf{k},\sigma}$ to new Fermi operators $\alpha_{\mathbf{k},\sigma}$, namely

$$a_{\mathbf{k},\sigma} = u_k \alpha_{\mathbf{k},\sigma} + 2\sigma v_k \alpha^{+}_{-\mathbf{k},-\sigma}, \tag{1}$$

where $\sigma = \pm 1/2$ is the spin projection for an electron state. The coefficients u_k, v_k are real and depend on k. The many-electron system hamiltonian is $H = H_0 + H_{\text{int}}$ where

$$H_0 = \sum_{\mathbf{k},\sigma} (\epsilon_k - E_{\text{F}}) a^{+}_{\mathbf{k},\sigma} a_{\mathbf{k},\sigma},$$

$$H_{\text{int}} = \frac{1}{2L^3} \sum'_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}'_1,\mathbf{k}'_2,\sigma_1,\sigma_2} v(|\mathbf{k}_1 - \mathbf{k}'_1|) a^{+}_{\mathbf{k}'_1\sigma_1} a^{+}_{\mathbf{k}'_2\sigma_2} a_{\mathbf{k}_2\sigma_2} a_{\mathbf{k}_1\sigma_1}, \tag{2}$$

where $\epsilon_k = \hbar^2 k^2/2m$ is the kinetic energy of an electron of mass m; $E_{\text{F}} = \hbar^2 k_{\text{F}}^2/2m$ is the Fermi energy; L is the system size; $v(q)$ the Fourier transform of the two-electron interaction potential; and the last sum is restricted by momentum conservation $\mathbf{k}'_1 + \mathbf{k}'_2 = \mathbf{k}_1 + \mathbf{k}_2$. This leads to the well-known BCS hamiltonian [4,5,9]

$$H_{\text{BCS}} = U_0 + \sum_{\mathbf{k},\sigma} E(k) \alpha^{+}_{\mathbf{k},\sigma} \alpha_{\mathbf{k},\sigma}, \tag{3}$$

with U_0 a generalized BCS ground-state energy

$$U_0 = 2\sum_{\mathbf{k}} (\epsilon_k - E_{\text{F}}) v_k^2 + L^{-3} \sum_{\mathbf{k},\mathbf{k}'} v(|\mathbf{k} - \mathbf{k}'|) \times v_k u_k v_{k'} u_{k'} + 2L^{-3} \sum_{\mathbf{k},\mathbf{k}'} v(0) v_k^2 v_{k'}^2 - L^{-3} \sum_{\mathbf{k},\mathbf{k}'} v(|\mathbf{k} - \mathbf{k}'|) v_k^2 v_{k'}^2,$$

where

$$u_k^2 = \frac{1}{2}\left[1 + \frac{A(k)}{E(k)}\right], \quad v_k^2 = \frac{1}{2}\left[1 - \frac{A(k)}{E(k)}\right],$$
$$2u_k v_k = -\frac{B(k)}{E(k)}, \quad E(k) \equiv \sqrt{A^2(k) + B^2(k)}. \tag{4}$$

Here $B(k)$ plays the role of the original BCS energy gap $\Delta(k)$, and

$$A(k) \equiv \epsilon_{\mathbf{k}} - E_{\text{F}} + 2L^{-3} v(0) \sum_{k'} v_{k'}^2 - L^{-3} \sum_{k'} v(|\mathbf{k} - \mathbf{k}'|) v_{k'}^2,$$
$$B(k) \equiv L^{-3} \sum_{k'} v(|\mathbf{k} - \mathbf{k}'|) u_{k'} v_{k'}. \tag{5}$$

For the new interaction hamiltonian we obtain

$$\begin{aligned}
H'_{\text{int}} &\equiv H - H_{\text{BCS}} \\
&= \frac{1}{2L^3} \sum'_{\mathbf{k}'_1,\mathbf{k}'_2,\mathbf{k}_1,\mathbf{k}_2,\sigma_1,\sigma_2} v(|\mathbf{k}_1 - \mathbf{k}'_1|) L(k_1,k'_1) L(k_2,k'_2) \\
&\quad \times \alpha^{+}_{\mathbf{k}'_1\sigma_1} \alpha^{+}_{\mathbf{k}'_2\sigma_2} \alpha_{\mathbf{k}_2\sigma_2} \alpha_{\mathbf{k}_1\sigma_1} \\
&\quad + \frac{1}{4L^3} \sum'_{\mathbf{k}'_1,\mathbf{k}'_2,\mathbf{k}_1,\mathbf{k}_2,\sigma_1,\sigma_2} v(|\mathbf{k}_1 - \mathbf{k}'_1|) M(k_1,k'_1) M(k_2,k'_2) \\
&\quad \times 2\sigma_1 \alpha^{+}_{\mathbf{k}'_1\sigma_1} \alpha^{+}_{-\mathbf{k}_1,-\sigma_1} 2\sigma_2 \alpha_{-\mathbf{k}'_2,-\sigma_2} \alpha_{\mathbf{k}_2\sigma_2} \\
&\quad + \frac{1}{4L^3} \sum'_{\mathbf{k}'_1,\mathbf{k}'_2,\mathbf{k}_1,\mathbf{k}_2,\sigma_1,\sigma_2} v(|\mathbf{k}_1 - \mathbf{k}'_1|) M(k_1,k'_1) L(k_2,k'_2) \\
&\quad \times 2\sigma_1 \alpha^{+}_{\mathbf{k}'_1\sigma_1} \alpha^{+}_{-\mathbf{k}_1,-\sigma_1} \alpha^{+}_{\mathbf{k}'_2\sigma_2} \alpha_{\mathbf{k}_2\sigma_2} \\
&\quad + \frac{1}{4L^3} \sum'_{\mathbf{k}'_1,\mathbf{k}'_2,\mathbf{k}_1,\mathbf{k}_2,\sigma_1,\sigma_2} v(|\mathbf{k}_1 - \mathbf{k}'_1|) L(k_1,k'_1) M(k_2,k'_2) \\
&\quad \times \alpha^{+}_{\mathbf{k}'_1\sigma_1} \alpha_{\mathbf{k}_1\sigma_1} 2\sigma_2 \alpha_{-\mathbf{k}'_2,-\sigma_2} \alpha_{\mathbf{k}_2\sigma_2} \\
&\quad + \frac{1}{8L^3} \sum'_{\mathbf{k}'_1,\mathbf{k}'_2,\mathbf{k}_1,\mathbf{k}_2,\sigma_1,\sigma_2} v(|\mathbf{k}_1 - \mathbf{k}'_1|) M(k_1,k'_1) M(k_2,k'_2) \\
&\quad \times 2\sigma_1 \alpha^{+}_{\mathbf{k}'_1\sigma_1} \alpha^{+}_{-\mathbf{k}_1,-\sigma_1} 2\sigma_2 \alpha^{+}_{\mathbf{k}'_2\sigma_2} \alpha^{+}_{-\mathbf{k}_2,-\sigma_2} \\
&\quad + \frac{1}{8L^3} \sum'_{\mathbf{k}'_1,\mathbf{k}'_2,\mathbf{k}_1,\mathbf{k}_2,\sigma_1,\sigma_2} v(|\mathbf{k}_1 - \mathbf{k}'_1|) M(k_1,k'_1) M(k_2,k'_2) \\
&\quad \times 2\sigma_1 \alpha_{-\mathbf{k}'_1,-\sigma_1} \alpha_{\mathbf{k}_1\sigma_1} 2\sigma_2 \alpha_{-\mathbf{k}'_2,-\sigma_2} \alpha_{\mathbf{k}_2\sigma_2}.
\end{aligned}$$

where $L(k,k') \equiv u_k u_{k'} - v_k v_{k'}$ and $M(k,k') \equiv u_k v_{k'} + u_{k'} v_k$.

To obtain the BS equation based on the BCS ground state consider the Feynman diagrams of perturbation theory based on this ground state, where Eq. (3) is the new unperturbed hamiltonian H_0'. We now have the usual arrowed electron lines labeled by $\mathbf{k}$, E, σ to which we associate the BCS unperturbed Green's function

$$\mathscr{G}_0(\mathbf{k}, E, \sigma) = \frac{\hbar}{\mathrm{i}} \frac{1}{-E + E(k) - \mathrm{i}\varepsilon}, \tag{6}$$

where $E(k)$ is given by Eq. (4). There exist four-line-end double vertices of six different kinds (see Fig. 1) where the interfermion interaction is denoted by dashed lines. To a double vertex type (a) of Fig. 1, with two outgoing line ends with indices $(\mathbf{k}_1', E_1', \sigma_1)$ and $(\mathbf{k}_2', E_2', \sigma_2)$ along with two incoming line ends with indices $(\mathbf{k}_1, E_1, \sigma_1)$ and $(\mathbf{k}_2, E_2, \sigma_2)$, we attach the factor

$$-L^{-3} v(|\mathbf{k}_1 - \mathbf{k}_1'|) L(\mathbf{k}_1, \mathbf{k}_1') L(\mathbf{k}_2, \mathbf{k}_2').$$

To a double vertex of type (b), with two outgoing line ends $(\mathbf{k}_1', E_1', \sigma_1)$ and $(\mathbf{k}_2', E_2', \sigma_2)$ along with two outgoing line ends $(-\mathbf{k}_1, -E_1, -\sigma_1)$ and $(-\mathbf{k}_2, -E_2, -\sigma_2)$, as well as to a double vertex of type (c) with two incoming line ends with indices $(-\mathbf{k}_1', -E_1', -\sigma_1)$ and $(-\mathbf{k}_2', -E_2', -\sigma_2)$ along with two incoming line ends with indices $(\mathbf{k}_1, E_1, \sigma_1)$ and $(\mathbf{k}_2, E_2, \sigma_2)$, we attach the factor

$$-L^{-3} v(|\mathbf{k}_1 - \mathbf{k}_1'|) 4\sigma_1\sigma_2 M(\mathbf{k}_1, \mathbf{k}_1') M(\mathbf{k}_2, \mathbf{k}_2').$$

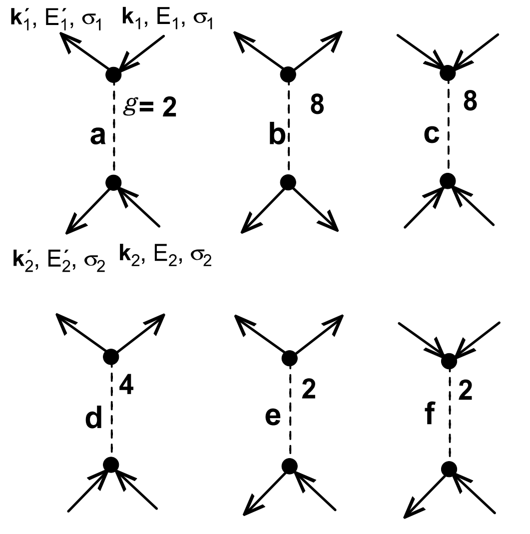

Fig. 1. Six different types of vertices with corresponding topological automorphism factors g.

3. Coupled BS equations

Because of different kinds of vertices in Fig. 1 we now have a system of *two* coupled BS equations. Fig. 2 shows their diagrammatic representation in the ladder approximation of the two-electron BS equation for the BCS ground state. Depicted are both the two-electron $\psi_+(\mathbf{k}E;\mathbf{K}\mathscr{E}_K)$ and two-hole $\psi_-(\mathbf{k}E;\mathbf{K}\mathscr{E}_K)$ bound-state functions. Here $\mathbf{K} \equiv \mathbf{k}_1 + \mathbf{k}_2$ is the total (or center-of-mass) wave vector and $\mathscr{E}_K$ is the total energy of the two electrons referred to $2E_\mathrm{F}$.

Using the diagramatic rules just described, the two-component BS equations for the bound state is

$$\begin{aligned}\psi_+(\mathbf{k}E) = &-\frac{1}{2\pi\mathrm{i}} \int_{-\infty}^{+\infty} \mathrm{d}E'\, L^{-3} \sum_{\mathbf{k}'} v(|\mathbf{k} - \mathbf{k}'|) \\ &\times L(\mathbf{K}/2 + \mathbf{k}, \mathbf{K}/2 + \mathbf{k}') L(\mathbf{K}/2 - \mathbf{k}, \mathbf{K}/2 - \mathbf{k}') \\ &\times (\mathrm{i}/\hbar)^2 \mathscr{G}_0(\mathbf{K}/2 + \mathbf{k}, \mathscr{E}_K/2 + E) \\ &\times \mathscr{G}_0(\mathbf{K}/2 - \mathbf{k}, \mathscr{E}_K/2 - E) \psi_+(\mathbf{k}'E') \\ &-\frac{1}{2\pi\mathrm{i}} \int_{-\infty}^{+\infty} \mathrm{d}E'\, L^{-3} \sum_{\mathbf{k}'} v(|\mathbf{k} - \mathbf{k}'|) \\ &\times M(\mathbf{K}/2 + \mathbf{k}, \mathbf{K}/2 + \mathbf{k}') M(\mathbf{K}/2 - \mathbf{k}, \mathbf{K}/2 - \mathbf{k}') \\ &\times (\mathrm{i}/\hbar)^2 \mathscr{G}_0(\mathbf{K}/2 + \mathbf{k}, \mathscr{E}_K/2 + E) \\ &\times \mathscr{G}_0(\mathbf{K}/2 - \mathbf{k}, \mathscr{E}_K/2 - E) \psi_-(\mathbf{k}'E'),\end{aligned} \tag{7}$$

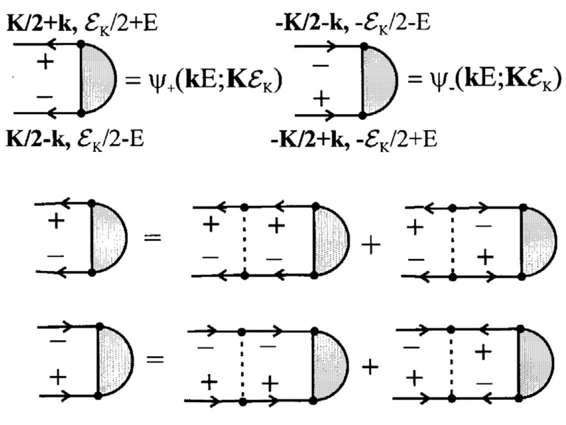

Fig. 2. Diagrammatic representation of the two-component coupled BS equations.

$$\psi_-(\mathbf{k}E) = -\frac{1}{2\pi i}\int_{-\infty}^{+\infty} dE' L^{-3}\sum_{\mathbf{k}'} v(|\mathbf{k}-\mathbf{k}'|)L(\mathbf{K}/2 + \mathbf{k}, \mathbf{K}/2+\mathbf{k}')L(\mathbf{K}/2-\mathbf{k}, \mathbf{K}/2-\mathbf{k}') \times (i/\hbar)^2 \mathscr{G}_0(-\mathbf{K}/2-\mathbf{k}, -\mathscr{E}_K/2 - E) \times \mathscr{G}_0(-\mathbf{K}/2+\mathbf{k}, -\mathscr{E}_K/2+E)\psi_-(\mathbf{k}'E') - \frac{1}{2\pi i}\int_{-\infty}^{+\infty} dE' L^{-3}\sum_{\mathbf{k}'} v(|\mathbf{k}-\mathbf{k}'|)M(\mathbf{K}/2 + \mathbf{k}, \mathbf{K}/2+\mathbf{k}')M(\mathbf{K}/2-\mathbf{k}, \mathbf{K}/2-\mathbf{k}') \times (i/\hbar)^2 \mathscr{G}_0(-\mathbf{K}/2-\mathbf{k}, -\mathscr{E}_K/2 - E) \times \mathscr{G}_0(-\mathbf{K}/2+\mathbf{k}, \mathscr{E}_K/2+E)\psi_+(\mathbf{k}'E'). \tag{8}$$

It can be shown that these equations coincide exactly with those used in the description of collective excitations in the BCS-Bogoliubov [10] microscopic theory of superconductivity.

4. CP solutions of BS equations

We now employ the BCS model interaction

$$v(|\mathbf{k}-\mathbf{k}'|) \Rightarrow -(k_F^2/k'^2)V\eta(k)\eta(k'), \tag{9}$$

where $V \geqslant 0$, and $\eta(k) = 1$ when $k_F - k_D < k < k_F + k_D$ and $= 0$ otherwise. Here $k_D \equiv m\omega_D/\hbar k_F$ with ω_D the Debye frequency, if $\hbar\omega_D \ll E_F$.

The detailed solution for the system of the two-component coupled BS equations of Fig. 2 is too cumbersome to present here, but it can be shown that they yield *two types* of independent solutions. The *first* solution of Eqs. (7) and (8) with Eq. (9) is just

$$\frac{Vk_F^2}{2\pi^2}\int_{k_F-k_D}^{k_F+k_D} dk \int_{-1}^{1} dt\, u(|\mathbf{K}/2+\mathbf{k}|)v(|\mathbf{K}/2-\mathbf{k}|) \times [u(|\mathbf{K}/2-\mathbf{k}|)v(|\mathbf{K}/2+\mathbf{k}|) - u(|\mathbf{K}/2+\mathbf{k}|) \times v(|\mathbf{K}/2-\mathbf{k}|)] \times \left[\frac{E(|\mathbf{K}/2+\mathbf{k}|)+E(|\mathbf{K}/2-\mathbf{k}|)}{-\mathscr{E}_K^2 + [E(|\mathbf{K}/2+\mathbf{k}|)+E(|\mathbf{K}/2-\mathbf{k}|)]^2}\right] = 1. \tag{10}$$

Here $t \equiv \cos\theta$, θ being the angle between $\mathbf{k}$ and $\mathbf{K}$ while $|\mathbf{K}/2+\mathbf{k}| = (k^2+Kkt+K^2/4)^{1/2}$ and $|\mathbf{K}/2-\mathbf{k}| = (k^2-Kkt+K^2/4)^{1/2}$.

As we see the numerator in Eq. (10) vanishes for $K=0$, so the denominator must vanish as $K \to 0$ as well. This gives $\mathscr{E}_K = \pm 2\Delta + O(K)$. To find the asymptotic solution for both small coupling and K we may neglect terms with Δ in u, v in the integrand. Thus we put $u(k) \simeq \theta(k_F - k) \equiv \theta_F(k)$, $v(k) \simeq \theta(k-k_F) \equiv \theta_G(k)$, where $\theta(k)$ is the usual Heaviside step function. Therefore,

$$u(|\mathbf{K}/2+\mathbf{k}|)v(|\mathbf{K}/2-\mathbf{k}|)[u(|\mathbf{K}/2-\mathbf{k}|)v(|\mathbf{K}/2+\mathbf{k}|) - u(|\mathbf{K}/2+\mathbf{k}|)v(|\mathbf{K}/2-\mathbf{k}|)] \simeq -\theta_F(|\mathbf{K}/2+\mathbf{k}|)\theta_G(|\mathbf{K}/2-\mathbf{k}|).$$

Thus we can rewrite Eq. (10) as

$$\frac{Vk_F^2}{4\pi^2}\int_{k_F-k_D}^{k_F+k_D} dk \int_0^1 dt\, \theta_F(|\mathbf{K}/2+\mathbf{k}|)\theta_G(|\mathbf{K}/2-\mathbf{k}|) \times \left[\frac{1}{-\mathscr{E}_K + E(|\mathbf{K}/2+\mathbf{k}|)+E(|\mathbf{K}/2-\mathbf{k}|)} + \frac{1}{\mathscr{E}_K + E(|\mathbf{K}/2+\mathbf{k}|)+E(|\mathbf{K}/2-\mathbf{k}|)}\right] = 1. \tag{11}$$

In Fig. 3 we show as shaded the region of integration over k and $t = \cos\theta$. The integral over the polar angle θ is restricted to from 0 to $\pi/2$ since the integrand in (10) vanishes for $t<0$. We first integrate over k from k_{min} to k_{max} which are solutions of $|\mathbf{K}/2+\mathbf{k}| = k_F$, and $|\mathbf{K}/2-\mathbf{k}| = k_F$, or

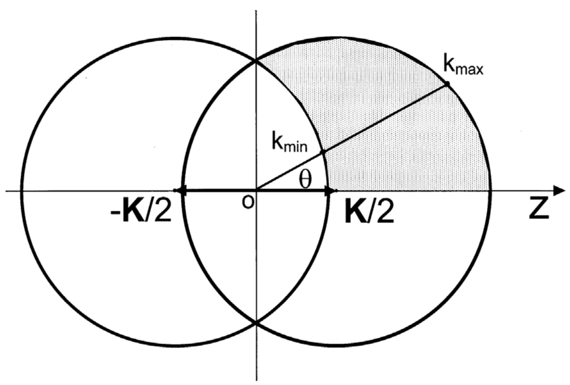

Fig. 3. Integration region in Eq. (11).

$$k_{\min} = -Kt/2 + \sqrt{K^2t^2/4 - K^2/4 + k_F^2},$$
$$k_{\max} = Kt/2 + \sqrt{K^2t^2/4 - K^2/4 + k_F^2}.$$

The restrictions $k_F - k_D \leqslant k \leqslant k_F + k_D$ are not illustrated in Fig. 3 as they are unimportant for us. As $K \to 0$ we then have

$$k_{\min} \simeq -Kt/2 + O(K^2),$$
$$k_{\max} \simeq Kt/2 + O(K^2),$$
$$E(|\mathbf{K}/2 + \mathbf{k}|) + E(|\mathbf{K}/2 - \mathbf{k}|) \simeq 2\Delta + O(K^2)$$

where we redefined the integration variable $k = k_F + xK$, with x the new variable. Consequently in seeking a solution of Eq. (11), for example of the form

$$\mathscr{E}_K = 2\Delta + c\hbar K + O(K^2),$$

where c is a constant, we obtain for small Δ and K

$$\frac{Vk_F^2}{4\pi^2\hbar}\int_0^1 dt \int_{k_F-Kt/2}^{k_F+Kt/2} \frac{dk}{cK} \simeq 1.$$

Thus $c = Vk_F^2/8\pi^2\hbar$. Finally one gets

$$\mathscr{E}_K = 2\Delta + \tfrac{1}{4}\lambda v_F\hbar K + O(K^2), \qquad (12)$$

together with its symmetric solution

$$\mathscr{E}_K = -2\Delta - \tfrac{1}{4}\lambda v_F\hbar K + O(K^2). \qquad (13)$$

where $\lambda \equiv N(0)V$ and $N(0) \equiv mk_F/2\pi^2\hbar^2$.

These two solutions describe *moving* 2*p*-CPs and 2*h*-CPs, respectively. A linear-in-K behavior of the moving pair binding energy was obtained for the original CP problem [11] but it was independent of the interaction coupling strength as it excluded 2*h*-CP contributions. More significantly, the binding energy there was *negative* as it refers to an infinite-lifetime composite particle, while in (12) it is *positive* as it describes a resonance in the continuum with a finite lifetime as evidenced by an imaginary contribution [12] appearing in higher order terms in K.

5. ABH-like mode solution of BS equations

The *second* solution of the coupled BS equation (7) and (8) follows from

$$\begin{aligned}
&\frac{Vk_F^2}{2\pi^2}\int_{k_F-k_D}^{k_F+k_D} dk \int_{-1}^{1} dt\, v(|\mathbf{K}/2+\mathbf{k}|)v(|\mathbf{K}/2-\mathbf{k}|) \\
&\times [u(|\mathbf{K}/2+\mathbf{k}|)u(|\mathbf{K}/2-\mathbf{k}|) + v(|\mathbf{K}/2+\mathbf{k}|) \\
&\times v(|\mathbf{K}/2-\mathbf{k}|)]\left[\frac{1}{\mathscr{E}_K + E(|\mathbf{K}/2+\mathbf{k}|)}\right. \\
&\left.+ E(|\mathbf{K}/2-\mathbf{k}|)\right] + \frac{Vk_F^2}{2\pi^2}\int_{k_F-k_D}^{k_F+k_D} dk \int_{-1}^{1} dt\, u(|\mathbf{K}/2+\mathbf{k}|) \\
&\times u(|\mathbf{K}/2-\mathbf{k}|)[u(|\mathbf{K}/2+\mathbf{k}|)u(|\mathbf{K}/2-\mathbf{k}|) \\
&+ v(|\mathbf{K}/2+\mathbf{k}|)v(|\mathbf{K}/2-\mathbf{k}|)] \\
&\times \left[\frac{1}{-\mathscr{E}_K + E(|\mathbf{K}/2+\mathbf{k}|) + E(|\mathbf{K}/2-\mathbf{k}|)}\right] \\
&= 1. \qquad (14)
\end{aligned}$$

Proceeding as before, in the $\lim K \to 0$ one now finds

$$\mathscr{E}_K = (v_F\hbar K/\sqrt{3})\left[1 + \frac{\hbar\omega_D}{4E_F}e^{-2/\lambda} + \cdots\right] + O(K^2), \qquad (15)$$

which is similar to the Anderson–Bogoliubov–Higgs (ABH) RPA excitation mode in the BCS theory of superconductivity [10,13–15].

6. Conclusions

We have presented a new many-fermion formalism based on a BS equation applied to the full BCS ground state which does *not* neglect the presence of holes. This leads in the ladder approximation to two types of solutions: the first referring to simple moving CPs consisting of two-electron (12) *or* two-hole (13) resonances. In addition, this formalism naturally provides a second solution (15) of an entirely different physical nature which is analogous to the ABH excitation mode. Bose–Einstein condensation can occur with the first type of objects but not with the second.

Acknowledgements

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