REVISTA MEXICANA DE FÍSICA **48 SUPLEMENTO 1**, 31 - 37 SEPTIEMBRE 2002

Natural Convection in a polar suspension with internal rotation

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Recibido el 19 de marzo de 2001; aceptado el 4 de junio 2001

In this paper the natural convection in a polar suspension fluid layer with internal rotation is investigated when polarization is present due to the influence of gravity. Constitutive equations obtained by means of extended irreversible thermodynamics [*J. Chem. Phys.* **96** (1992) 9102, and *J. Colloid and Interface Sci.* **178** (1996) 69] are used to describe the instability. This set of constitutive equations works to couple the velocity, spin and polarization fields. It is shown that the stationary instability of the polar suspension layer only depends on the relative magnitude of the Debye relaxation time and the polarization diffusity times. Besides, it is found that oscillatory convection can occur before the stationary one when important anti-symmetric polarization gradients coupling parameters are large enough.

Keywords: Natural convection; suspension; polarization; spin

En este artculo se investiga la convección natural en una capa de fluido formada por una suspensión polar en la que la polarización aparece debido a la influencia de la gravedad. Para describir la inestabilidad se hace uso de las ecuaciones constitutivas obtenidas por medio de la termodinámica irreversible extendida [J. Chem. Phys. 96 (1992) 9102 y J. Colloid and Interface Sci. 178 (1996) 69]. Este conjunto de ecuaciones tiene como fin acoplar los campos de velocidad, espín y polarización. Se demuestra que la inestabilidad estacionaria de la suspensión polar solo depende de la magnitud relativa del tiempo de relajamiento de Debye y de los tiempos de difusión de polarización. Además, se encuentra que la convección oscilatoria puede ocurrir antes que la estacionaria cuando parámetros importantes de acoplamiento de gradientes de polarización antisimétricos son suficientemente grandes.

Descriptores: Conveccin natural, suspensiones, polarización, espín

PACS: 44.27.+f; 44.35.+c; 47.27.Te; 47.55.kf; 47.50.+d

This paper is dedicated to Professor Leopoldo García-Colín Scherer on occasion of his 70th anniversary.

1. Introduction

Since almost half a century, suspensions have been the subject of intensive research due to their important applications in natural sciences as well as in industry. The increasing interest on their behavior is reflected in the number of papers published on this subject, reviewed extensively in Ref. 1. The different properties of suspensions should be present in the constitutive equations derived to describe their behavior. Internal rotation, or spin, of the colloidal particles is one of the main characteristics which appear when their friction with the carrier fluid is relevant. When a polar suspension is under an external field, like gravity or an electric field in dielectrics, it polarizes producing rotation on its particles which try to orient with the external field. In this way, the internal rotation is intimately related with the polar properties of the medium. Thus, the response of the polar suspension to the external field has influence on the magnitude and direction of the spin and, moreover, on the global fluid flow behavior according to the boundary conditions.

Constitutive equations for polar fluids with internal rotation have been put forward recently taking into account their viscoelastic properties. For polar fluids with dielectric characteristics constitutive equations were obtained by means of Extended Irreversible Thermodynamics by Dávalos-Orozco and del Castillo [2]. In case the center of mass of the suspended particle is displaced from its geometric center, it is said that it is polarized with respect to gravity, which plays the role of the external force. The constitutive equations for a viscoelastic polar suspension with spin having these properties were obtained by Dávalos-Orozco and del Castillo [3].

In this paper, the linear natural convection of a polar suspension fluid layer with internal rotation polarized by gravity is investigated using the constitutive equations obtained in [3]. The results of this research will give a light on the role played by all the parameters on fluid flow.

Simple boundary conditions are used to describe the suspension fluid layer instability when heated from below. Both, the stationary and oscillatory convective instabilities will be investigated. Here, all the viscoelastic properties of the suspension are neglected.

The structure of the paper is as follows. In Sec. 2, the equations of motion along with the constitutive equations for a fluid suspension are presented. These equations are linearized and made non dimensional in Sec. 3, where the stability analysis is developed to obtain an explicit expression for the marginal Rayleigh number. The corresponding numerical results for stationary and oscillatory convection are given in Sec. 4. Finally, Sec. 5 has the conclusions.

2. Equations of motion and constitutive relations

The system under investigation is a horizontal polar suspension fluid layer heated from below. The layer of thickness d is parallel to the (x, y)-plane and its lower boundary is located at $z = 0$. The suspension has a lower boundary with a temperature $T_0 + \Delta T$, higher than that of its upper boundary at $z = d$ which has a temperature T_0. The acceleration of gravity is parallel to the z-axis but in its negative direction. The suspension boundaries are supposed to be stress free and to have fixed temperatures, that is, the boundaries thermal properties are those of a very good conductor.

The constitutive equations of a viscoelastic polar suspension with internal rotation, were obtained in [3]. Note that here all the viscoelastic effects are ignored. Therefore, the coupled system of equations of motion and constitutive equations are

$$\rho\frac{d\vec{u}}{dt} = \nabla\cdot(\mathbf{Q}^s + \mathbf{Q}^a) - \nabla p + \rho\vec{g}, \quad (1)$$

$$I\frac{d\vec{\omega}}{dt} = \varepsilon\cdot\mathbf{Q}^a, \quad (2)$$

$$\frac{D\vec{P}}{Dt} - \delta_1(\chi_0\vec{g} - \vec{P}) - \delta_2\nabla\cdot\mathbf{Q}^s + \delta_4\nabla\cdot(\nabla\vec{P})^s + \delta_5\nabla\cdot(\nabla\vec{P})^a + \delta_6\nabla\cdot\mathbf{Q}^a = 0, \quad (3)$$

$$\mathbf{Q}^s - 2\delta_3(\nabla\vec{P})^s - 2\mu(\nabla\vec{u})^s = 0, \quad (4)$$

$$\mathbf{Q}^a - 2\delta_7(\nabla\vec{P})^a - 2\zeta\varepsilon\cdot\left(\frac{1}{2}\nabla\times\vec{u} - \vec{\omega}\right) = 0. \quad (5)$$

Here, ρ is the density, $\vec{u}$ is the velocity vector, p is the pressure, $\vec{g} = (0, 0, -g)$ is the acceleration of gravity, I is the mean moment of inertia of the suspended particles, $\vec{\omega}$ is the spin vector, $\vec{P}$ is the gravity polarization vector of the particle, χ_0 is the gravity polar susceptibility of the fluid, $\mathbf{Q}^s$ is the symmetric stress tensor, μ is the viscosity, $\mathbf{Q}^a$ is the anti-symmetric stress tensor, ζ is the vortex viscosity and ε is the three indexes alternating tensor. The operator d/dt is always the lagrangian time derivative. The operator D/Dt is the lagrangian, the corrotational or the codeformational time derivative, depending on the model assumed. Here, in the absence of viscoelastic effects, it is assumed to be the Lagrangian time derivative. The coefficients $\delta_2, \delta_3, \delta_4, \delta_5, \delta_6$ and δ_7 which appear in front of the gradients of $\vec{P}$, $\mathbf{Q}^s$, and $\mathbf{Q}^a$ give the magnitude of the coupling of the spatial inhomogeneities of polarization and the stresses. Besides, they are important in the coupling with the velocity field. The coefficient δ_1, is the inverse of the time scale representing polarization decay in the absence of couplings. Here, it is supposed that this representative time is the same as that for spin decay in the absence of polarization and velocity fields. That is

$$\tau_D = \frac{1}{\delta_1} = \frac{I}{4\zeta}, \quad (6)$$

where τ_D is the Debye relaxation time and the last result is obtained from Eq. (2) after substitution of $\mathbf{Q}^a$ in Eq. (5).

The above equations are coupled to the heat diffusion equation through the velocity field and the changes of density due to the temperature gradient imposed to the fluid layer. That is

$$\frac{dT}{dt} = k_T\nabla^2 T, \quad (7)$$

where T is the temperature, k_T is the heat diffusivity.

The continuity equation for an incompressible fluid is:

$$\nabla\cdot\vec{u} = 0. \quad (8)$$

These equations are linearized in the following section where the stability analysis is done.

3. Instability analysis

The instability of the suspension fluid layer is investigated under the Boussinesq approximation. This means that the density variation due to temperature is taken into account only in the gravity term.

The system of equations given in the previous section has the following hydrostatic solutions. From the imposed temperature gradient, the resulting temperature profile obtained from the heat diffusion equation is $\bar{T} = T/\Delta T = 1 - z + T_0/\Delta T$. If β is the coefficient of thermal volume expansion, the fluid density varies as

$$\rho = \rho_0(1 - \beta T) = \rho_0\left[1 - \Delta T\beta\left(1 - z + \frac{T_0}{\Delta T}\right)\right],$$

where ρ_0 is a reference density. It follows that the hydrostatic pressure, obtained from the z-component of Eq. (1) is $p = p_0 - g\rho_0 z[1 + \Delta T\beta(z/2 - 1 - T_0/\Delta T)]$. Besides, it is supposed that in the hydrostatic state the fluid polarization is $\vec{P}_0 = \chi_0\vec{g}$, where $\vec{g}$ is a uniform constant field.

Now, Eqs. (1)–(5), (7), and (8) are perturbed and linearized after substitution of the stress tensors. They are made nondimensional using d for distance, d^2/k_T for time, k_T/d for velocity, k_T/d^2 for spin angular velocity, $\rho_0 k_T^2/d^2$ for pressure, $-2\delta_6\zeta/d$ for polarization and ΔT for temperature. In this way, after subtracting the hydrostatic solutions, the linear equations for the convection instability are:

$$\frac{1}{\Pr}\frac{\partial\vec{u}}{\partial t} = -\nabla p + R\theta\hat{k} + N_1\nabla^2\vec{P}_p + N_2\nabla(\nabla\cdot\vec{P}_p) + (1+M)\nabla^2\vec{u} + 2M\nabla\times\vec{\omega}, \quad (9)$$

$$\frac{\partial\vec{\omega}}{\partial t} = \frac{1}{\tau}\left(\frac{1}{2}\nabla\times\vec{u} - \vec{\omega}\right) + 2N_3\nabla\times\vec{P}_p, \quad (10)$$

$$\frac{\partial\vec{P}_p}{\partial t} + \frac{1}{\tau}\vec{P}_p - N_4\nabla^2\vec{P}_p - N_5\nabla(\nabla\cdot\vec{P}_p) - N_6\nabla^2\vec{u} - \nabla\times\vec{\omega} = 0, \quad (11)$$

$$\frac{\partial\theta}{\partial t} - w = \nabla^2\theta, \quad (12)$$

$$\nabla\cdot\vec{u} = 0. \quad (13)$$

Here $\vec{u} = (u, v, w)$, p, $\vec{P}_p = (P_{p1}, P_{p2}, P_{p3})$, and $\vec{\omega} = (\omega_1, \omega_2, \omega_3)$ are the perturbations of velocity, pressure, polarization and spin, respectively. In Eq. (12) use has been made of the fact that $d\bar{T}/dz = -1$. The nondimensional parameters are defined as follows. $\Pr = \mu/\rho_0 k_T$ is the Prandtl number, $R = g\beta\Delta T d^3/\nu k_T$ is the Rayleigh number, $M = \zeta/\mu$ is the vortex and fluid viscosities ratio, and $\tau = \tau_D/(d^2/k_T)$, $N_1 = -2\delta_6\zeta(\delta_3 + \delta_7)/\rho_0\nu k_T$,

$N_2 = -2\delta_6\zeta(\delta_3 - \delta_7)/\rho_o\nu k_T$, $N_3 = -2\delta_6\zeta\delta_7 d^2/Ik_T^2$, $N_4 = D_1/k_T$, $N_5 = D_2/k_T$, $N_6 = D_3/(-2\delta_6\zeta)$.

Note that $D_1 = \delta_2\delta_3 - (\delta_4 + \delta_5)/2 - \delta_6\delta_7$ and $D_2 = \delta_2\delta_3 - (\delta_4 - \delta_5)/2 + \delta_6\delta_7$ contain diffusivity constants of the rotational of polarization (transverse mode) and of the polarization charge density (longitudinal mode), respectively [2]. $D_3 = \delta_2\mu - \delta_6\zeta$, represents products of vortex and fluid viscosities with coupling terms of the symmetric and anti-symmetric stress tensors. Equation (10), obtained in a natural way by means of extended irreversible thermodynamics [3], is a generalized version of the one proposed by Stiles and Hubbard [4].

A combination of the equations resulting from the second rotational of Eqs. (9) and (11), the first rotational of Eq. (10) and use of Eqs. (12) and (13) lead to one equation for the z-component of the velocity. That is

$$\mathcal{G}\left\{\left(\frac{\partial}{\partial t} - \nabla^2\right)\left[\frac{2N_3}{\Pr}\frac{\partial}{\partial t}\nabla^2 w + \left(\frac{N_1}{2\tau} - 2N_3(1+M)\right)\nabla^4 w\right] - 2N_3 R\nabla_\perp^2 w\right] - \mathcal{H}\nabla^4 w = 0, \tag{14}$$

with

$$\mathcal{G} = \left\{\left(\frac{\partial}{\partial t} + \frac{1}{\tau}\right)^2 - \left[N_4\left(\frac{\partial}{\partial t} + \frac{1}{\tau}\right) - 2N_3\right]\nabla^2\right\}, \tag{15}$$

$$\mathcal{H} = \left(\frac{\partial}{\partial t} - \nabla^2\right)\left[N_1\left(\frac{\partial}{\partial t} + \frac{1}{\tau}\right) - 4MN_3\right]\left[\left(\frac{\partial}{\partial t} + \frac{1}{\tau}\right)\frac{1}{2\tau} + \left(2N_3N_6 - \frac{1}{2\tau}N_4\right)\nabla^2\right], \tag{16}$$

where $\mathcal{G}$ and $\mathcal{H}$ are differential operators and $\nabla_\perp^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the two-dimensional laplacian. Note that it was supposed that $\nabla\cdot\vec{\omega} = 0$ (however, this term relaxes to zero as shown from the divergence of the linear spin equation).

The boundary conditions are the following. For stress free boundaries $w = D^2 w = 0$ at $z = 0, 1$ and $\theta = 0$ for a fixed temperature at $z = 0, 1$. Besides, it is supposed that at $z = 0, 1$, $DP_{p1} = DP_{p2} = P_{p3} = 0$ and that $\omega_1 = \omega_2 = D\omega_3 = 0$. This boundary conditions are satisfied consistently if it is supposed that all the components of the perturbation vector fields have the form $f = AF(z)\exp i(\alpha_x x + \alpha_y y + \omega t)$ and that they have the next dependence on z

$$\vec{u} \sim [\cos(n\pi z), \cos(n\pi z), \sin(n\pi z)],$$
$$\vec{P}_p \sim [\cos(n\pi z), \cos(n\pi z), \sin(n\pi z)],$$
$$\vec{\omega} \sim [\sin(n\pi z), \sin(n\pi z), \cos(n\pi z)].$$

Here, A is a constant amplitude, α_x and α_y are the x- and y-components of the wavenumber, ω is the frequency and n is an integral number.

The solvability condition of Eq. (14) for the stationary case will be given first. In this case the solution for the marginal Rayleigh number is

$$\mathrm{R} = \frac{1}{\alpha^2}(\pi^2 + \alpha^2)^3 \times\left[1 - \frac{\tau(N_1 - 4\tau MN_3)(2N_6 - 1)(\pi^2 + \alpha^2)}{2[1 + \tau(N_4 - 2\tau N_3)(\pi^2 + \alpha^2)]}\right]. \tag{17}$$

The magnitude of the wavenumber vector is defined as $\alpha^2 = \alpha_x^2 + \alpha_y^2$. It can be shown that $n = 1$ gives the smaller marginal Rayleigh number. A detailed analysis of the constant parameters and their products shows that they can be simplified and that it is possible to find a better expression for them in Eq. (17). Therefore, it is found that only a few parameters are needed to describe the instability. The new expression of the Rayleigh number (see Appendix) is

$$\mathrm{R} = \frac{1}{\alpha^2}(\pi^2 + \alpha^2)^3\left[1 + \frac{\tau_2(\pi^2 + \alpha^2)}{1 + (\tau_1 + \tau_2)(\pi^2 + \alpha^2)}\right]. \tag{18}$$

The new parameters are

$$\tau_1 = \tau_D\delta_2\delta_3/d^2,$$
$$\tau_2 = -\tau_D\left(\delta_4 + \frac{\delta_5}{2d^2}\right),$$

which compare diffusivity times of polarization with the Debye relaxation time. It might seem that the spin effects already disappeared from the stationary Rayleigh number mainly because in the absence of polarization the effects of the spin are not present in stationary convection. However, if calculations are made for the polarization coupled alone with the velocity field a similar equation for R is obtained but with more coefficients. Therefore, it is concluded that if it were not for the contribution of the spin the resulting equation would not be as simple as Eq. (18). In this way, it is concluded that the full system of coupled equations is needed to describe, in a simple manner, convection in polar suspensions. This result was not expected due to the large number of parameters involved.

The same analysis can be made in the complete equation for the marginal Rayleigh number of oscillatory convection. From the solvability condition of Eq. (16) the equation is

$$\mathrm{R} = \frac{1}{\alpha^2}(i\omega + \pi^2 + \alpha^2)(\pi^2 + \alpha^2)\left\{\frac{i\omega}{\Pr} + (\pi^2 + \alpha^2)\left[\left(M + 1 - \frac{N_1}{4N_3\tau}\right) + \frac{R_1}{R_2}\right]\right\}, \tag{19}$$

where

$$R_1 = \left[\frac{N_1}{4N_3\tau}(i\omega\tau + 1) - M\right] \times \left[i\omega\tau + 1 - \tau(4\tau N_3 N_6 - N_4)(\pi^2 + \alpha^2)\right], \quad (20)$$

$$R_2 = (i\omega\tau + 1)^2 + \tau[N_4(i\omega\tau + 1) - 2\tau N_3](\pi^2 + \alpha^2). \quad (21)$$

Here, the coefficients may also be transformed as in the stationary case (see Appendix). The result is

$$\mathrm{R} = \frac{1}{\alpha^2}(i\omega + \pi^2 + \alpha^2)(\pi^2 + \alpha^2) \times \left[\frac{i\omega}{\mathrm{Pr}} + (\pi^2 + \alpha^2)\frac{R_A}{R_B}\right], \quad (22)$$

where

$$R_A = \tau_2(\pi^2 + \alpha^2) + (i\omega\tau + 1)^2 + i\omega\tau M(i\omega\tau + 1) + i\omega\tau(\pi^2 + \alpha^2) \times \left[\tau_2 + \tau_3 + M\left(\tau_1 - \tau_1\delta + \tau_2 - \frac{\tau_3}{\delta M}\right)\right], \quad (23)$$

$$R_B = (i\omega\tau + 1)^2 + (\pi^2 + \alpha^2) \times \left[\tau_1 + \tau_2 + i\omega\tau(\tau_1 + \tau_2 + \tau_3)\right]. \quad (24)$$

Note that, besides Pr the parameters τ and M are still present in the Rayleigh number for oscillatory convection. The new parameters are $\tau_3 = -\tau_D\delta_6\delta_7/d^2$ and $\delta = -\delta_6/\delta_2$. When $\omega = 0$ it is easily seen that Eq. (22) reduces to Eq. (18). These two equations are analyzed numerically in detail in the following section.

4. Numerical results

4.1. Stationary convection

The numerical results are presented in graphs of the critical Rayleigh number against each of the two parameters which appear in Eq. (18). The critical Rayleigh number of stationary convection is defined as the minimum of the marginal curves with respect to the wavenumber. The wavenumber corresponding to the minimum is the critical one. It has been shown [5] that in the absence of spin and polarization the Newtonian critical Rayleigh number for stationary convection is $\mathrm{R_c} = 27\pi^4/4 = 657.51$ under the conditions of stress free and very good thermal conducting boundaries. From Eq. (18) it is clear that it is enough to make $\tau_1 = 0$ to recover the newtonian marginal Rayleigh number. This will be reflected in the numerical results. Figure 1, for stationary convection, presents graphs of the critical Rayleigh numbers and corresponding wavenumbers against τ_1 and τ_2 for two values of the fixed parameter. As reference, the values of the newtonian $\mathrm{R_c} = 657.5$ and $\alpha_c = 2.221$ are plotted with a horizontal starred line. In Fig. 1a the continuous curves correspond to $\mathrm{R_c}$ against τ_1 for two fixed values of τ_2. It is shown that $\mathrm{R_c}$ has a fast decrease with τ_1 and that it attains its newtonian magnitude as τ_1 tends to zero. Therefore, τ_1 has a strong destabilizing effect on the suspension convection. The diffusivities represented by τ_1 are the contributions coming from the shear of polarization in the symmetric stress tensor and the constitutive equation for polarization. The dashed curves in Fig. 1a correspond to $\mathrm{R_c}$ against τ_2 for two fixed values of τ_1. In this case, $\mathrm{R_c}$ tends to a small but finite value when τ_2 tends to zero, depending on the contribution of τ_1. As τ_2 increases the critical Rayleigh number approaches the newtonian one. The diffusivities in τ_2 are contributions of the symmetric and anti-symmetric polarization shears appearing in the constitutive equation of polarization. The continuous curves of Fig. 1b show the dependence of the critical

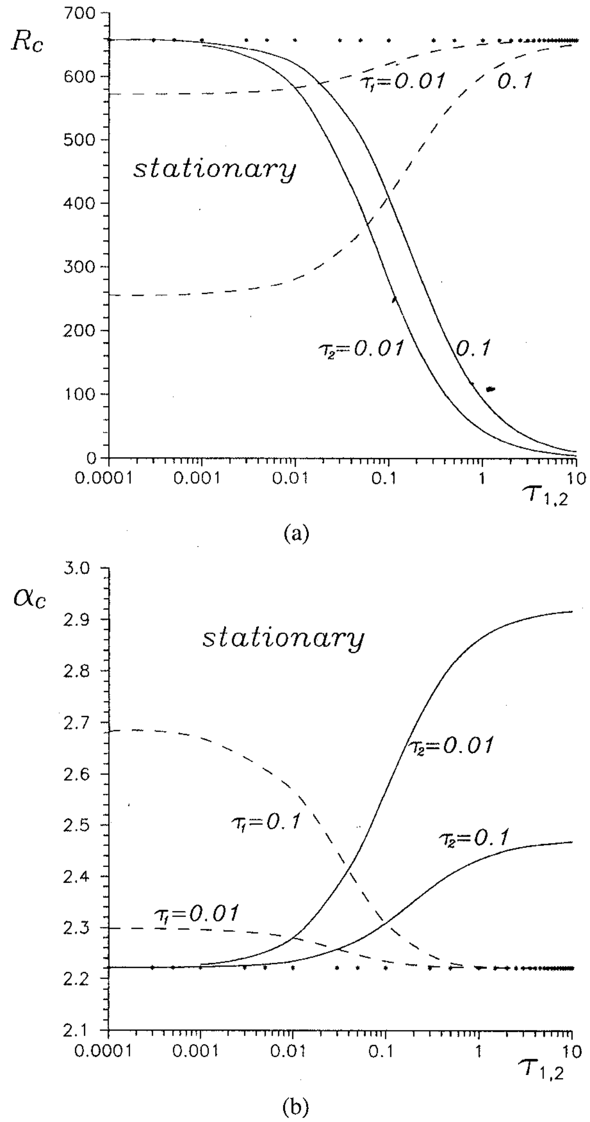

FIGURE 1. Stationary convection. a) Critical Rayleigh number against τ_1 and τ_2, and b) critical wavenumber against τ_1 and τ_2. Where (——) correspond to critical values against τ_1 and two values of $\tau_2 = 0.01, 0.1$ and (- - -) correspond to critical values against τ_2 and two values of $\tau_1 = 0.01, 0.1$.

wavenumber against τ_1 for two values of τ_2. It can be seen that α_c increases with τ_1 separating far from the newtonian value. The contrary occurs in the dashed curves corresponding to α_c against τ_2 for two different values of τ_1. In this case they tend to the horizontal newtonian line.

4.2. Oscillatory convection

The Rayleigh number of Eq. (22) is a complex quantity. Therefore, the marginal Rayleigh number of oscillatory convection is obtained looking for a real frequency which makes zero the imaginary part of the Rayleigh number for a given wavenumber and fixed parameters. By changing the wavenumber the marginal curve of the Rayleigh number is obtained. The minimum of this curve is the critical Rayleigh number with its corresponding critical wavenumber for the given parameters values.

It is well known [5] that it is not possible to have linear oscillatory convection in a newtonian fluid. Besides, note that in the absence of polarization but with the presence of spin, it is easy to obtain a quadratic equation for the square of the frequency which makes zero the imaginary part of the Rayleigh number. It is found that the three coefficients of the quadratic equation are positive and that both roots of the square of the frequency should be negative. Thus, it is not possible to have a real frequency which satisfies the condition for a marginal Rayleigh number. In other words, with spin coupling alone it is not possible to have linear oscillatory convection. As will be shown below, the coupling of the spin and velocity fields with polarization makes it possible to have oscillatory convection before the stationary one.

The numerical results are plotted in Fig. 2 for Pr $= 7$, $M = 0.5$ and $\delta = 1$. This Figure presents three different horizontal lines. The horizontal continuous and dashed lines represent the magnitudes of the stationary critical Rayleigh and wave numbers corresponding to the parameter values of the curves plotted in the Figure. The horizontal starred line represents the newtonian critical Rayleigh and wave numbers for stationary convection. In Fig. 2, two curves of critical quantities against τ_3 are given corresponding to two different sets of parameter values. The continuous curve corresponds to $\tau = 0.01$, $\tau_1 = 0.01$, $\tau_2 = 0.01$ and the dashed curve to $\tau = 0.1$, $\tau_1 = 0.1$, $\tau_2 = 0.1$. The magnitude of $M = 0.5$ was selected because it is supposed that the vortex viscosity is smaller than that of the fluid but of almost the same order of magnitude. The selection of $\delta = 1$ means that the coupling terms for the symmetric and anti-symmetric stress tensors in the constitutive polarization equation make the same contribution. The diffusivities represented by τ_3 come from the anti-symmetric coupling in the polarization equation and from the anti-symmetric polarization shear in the $\mathbf{Q}^a$ stress tensor. Therefore, as will be seen, the anti-symmetric contributions are important to let oscillatory convection to appear before stationary convection for the same fixed parameters.

In Fig. 2a the curves of the critical Rayleigh number are plotted. As it can be seen, R_c decreases with τ_3 even far below its corresponding stationary value (see Fig. 1a) which already was below the newtonian one given by the starred line. Besides, it is of interest to see that both critical wavenumbers in Fig. 2b are larger than their corresponding stationary ones and that they present maxima with respect τ_3. The curves for the critical frequency of oscillation also present maxima with respect to τ_3 as shown in Fig. 2c. Note that increasing

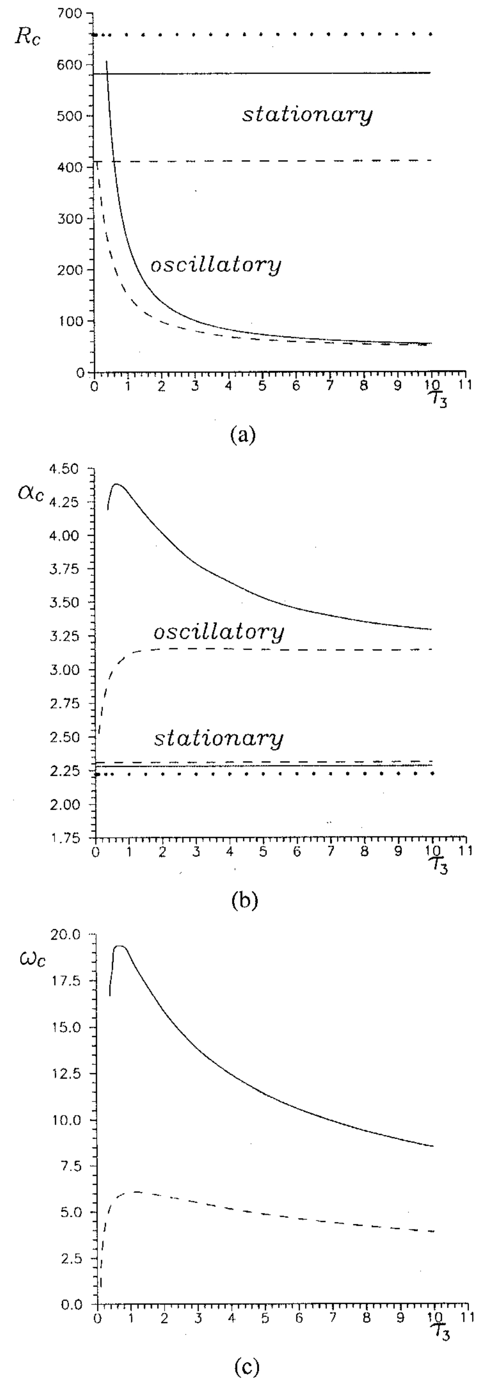

FIGURE 2. Oscillatory convection for Pr $= 7$, $M = 0.5$ and $\delta = 1$. a) Critical Rayleigh number against τ_3, b) critical wavenumber against τ_3, and c) critical frequency against τ_3. Where (——) and (- - -) correspond to critical values against τ_3 for $\tau = \tau_1 = \tau_2 = 0.01$ and $\tau = \tau_1 = \tau_2 = 0.1$, respectively.

the values of the τ's decreases the magnitude of the critical Rayleigh number making the suspension more susceptible to oscillatory convection even though the corresponding line for stationary R_c is reduced simultaneously.

5. Conclusions

It has been found that the coupling of spin and polarization with the suspension flow velocity field can give surprising results not expected even with the coupling of spin and velocity field alone. If only the spin and velocity fields were coupled the effects of spin disappear in stationary convection. If the spin is absent when polarization and velocity are coupled Eq. (18) should contain more parameters. In this way, it has been shown that the triple coupling of the equations of motion, spin and polarization is essential to obtain a simple expression for the stationary Rayleigh number in Eq. (18). Curves of criticality were obtained by means of numerical analysis of this Equation. It is found that the parameter τ_1 plays an essential role in the stationary instability. The instability becomes that of a newtonian fluid when it tends to zero and the critical Rayleigh number decreases rapidly with τ_1 making the suspension layer very unstable. Therefore, the suspension layer is more susceptible to stationary convection when the symmetric shear of polarization is more important in the polarization equation ($\tau_1 \backsim \delta_2\delta_3$). Note that for very large τ_2 the R_c tends to its newtonian value.

In the formula of the Rayleigh number for oscillatory convection it is more clear that the coupling of the full system of equations is important for the instability. It has been shown, that from the spin contribution alone it is not possible to obtain marginal oscillatory Rayleigh numbers because no real frequency of oscillation exists which can make zero the imaginary part of the Rayleigh number. However, the coupling with spin and polarization makes it possible to find a real frequency and, consequently, a real marginal Rayleigh number.

An example was investigated in which all the parameters are fixed except τ_3. It has been shown that τ_3 is important in the temporal instability of the suspension layer. It was found that, in spite of the decrease in the critical stationary Rayleigh number with respect to the newtonian one, R_c of oscillatory convection becomes smaller with τ_3 very fast. This brings about the possibility of having oscillatory convection before the stationary one when the anti-symmetric shear (rotational) of polarization is important in the polarization equation ($\tau_3 \backsim \delta_6\delta_7$).

The results presented in this paper show how polarization and spin contribute simultaneously to the instability of a convecting suspension layer. Besides, they demonstrate that spin and polarization are not able alone to bring a simple description of the flow as they do coupled with the velocity field. The example of natural convective instability given in this paper shows the relevance the set of constitutive equations obtained by Dávalos-Orozco and del Castillo [2], [3] may have in the description of suspensions flows.

Appendix

In this appendix some steps are presented to obtain the expressions of the marginal Rayleigh numbers Eq. (18) and (22) which include the new parameters. First, the steps to obtain the terms of the stationary case are given.

Using the definitions of the original parameters it is found that the terms of the numerator are equal to

$$-(N_1 - 4\tau M N_3) = \frac{2\delta_6\zeta\delta_3}{\mu k_T}, \tag{25}$$

$$2N_6 - 1 = -\frac{\mu\delta_2}{\zeta\delta_6}, \tag{26}$$

therefore

$$-\tau(N_1 - 4\tau M N_3)(2N_6 - 1) = 2\tau_1. \tag{27}$$

The term of the denominator is

$$\tau(N_4 - 2\tau N_3) = \frac{\tau}{k_T}\left(\delta_2\delta_3 - \frac{\delta_4 + \delta_5}{2}\right) = \tau_1 + \tau_2. \tag{28}$$

These complete the terms needed for the expression of R in Eq. (18). The calculations for R in the oscillatory case take a little longer. After some simple algebraic transformations it follows that

$$\frac{N_1}{4\tau N_3} = M\left(\frac{\delta_3}{\delta_7} + 1\right) = M\left(\frac{\delta\tau_1}{\tau_3} + 1\right). \tag{29}$$

This expression is used in different terms as in

$$\frac{N_1}{4\tau N_3} - M - 1 = M\frac{\delta\tau_1}{\tau_3} - 1, \tag{30}$$

$$M\left[\frac{N_1}{4\tau N_3 M}(i\omega\tau + 1) - 1\right] = M\left[\left(\frac{\delta\tau_1}{\tau_3} + 1\right)(i\omega\tau + 1) - 1\right]. \tag{31}$$

Next, the other factor of the numerator is

$$-\tau(4\tau N_3 N_6 - N_4) = \tau_1 + \tau_2 - \frac{\tau_3}{\delta M}, \tag{32}$$

and finally, the term in the denominator is transformed into

$$\tau[N_4(i\omega\tau + 1) - 2\tau N_3] = (\tau_1 + \tau_2 + \tau_3)(i\omega\tau + 1) + \tau_1 + \tau_2. \tag{33}$$

The expression of Eq. (22) is obtained after addition and subsequent simplification of the algebraic fraction inside the square brackets of Eq. (19).

Acknowledgments

The author would like to thank DGAPA-UNAM for support to project IN119200. Besides, thanks are due to Mr. Raúl Reyes for technical support.

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