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Two-dimensional Bose-Einstein condensation in cuprate superconductors

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Abstract

A binary gas of noninteracting, temperature-dependent Cooper pairs in chemical/thermal equilibrium with unpaired fermions is studied in a two-dimensional (2D) boson-fermion statistical model analogous to an atom plus diatomic-molecule system. The model naturally suggests a more convenient definition for the bosonic chemical potential whereby access into the degenerate Fermi region of positive fermion chemical potential is now possible. The linear (as opposed to quadratic) dispersion relation of the pairs yields substantially higher T_cs than with BCS or pure-boson Bose-Einstein condensation theories, and fall within the range of empirical T_cs for quasi-2D copper oxide superconductors. © 2002 Elsevier Science Ltd. All rights reserved.

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1. Introduction

We provide support for a widespread conjecture (or 'paradigm') that superconductivity in general is a Bose-Einstein condensation (BEC) of the charged Cooper pairs [1] (CPs) observed in magnetic flux quantization experiments in classical [2,3] as well as copper-oxide (or cuprate) [4] superconductors. The same general conjecture is also often made regarding the superfluidity of liquid helium-3 in terms of CPs consisting of neutral-atom ^{3}He fermions. As CPs coexist in a superconductor with unpaired electrons (or holes), a *binary gas* model suggests itself. BEC as a *statistical* (as opposed to a *dynamical*) mechanism of superconductivity has been entertained, among others, by Anderson [5], by Lee [6] and by Mott [7] and their co-workers. But BEC normally occurs only for dimensions $d > 2$ while some modern superconductors are quasi-2D or even quasi-1D materials [8–10]. In this paper we recall, however, how CPs *can* undergo BEC for *all* $d > 1$ and obtain reasonable critical temperatures T_c without any adjustable parameters, thus bolstering the earlier mentioned conjecture even before including full many-body self-consistency. More detailed, sophisticated treatments actually link [11–16] BEC (characterized by a nonnegligible *bosonic condensate fraction*) with the BCS theory (characterized by a *fermionic energy gap*), but report no successful attempts to calculate specific T_cs to compare with experiment. A binary gas BEC picture of superconductivity is consistent with the recent discovery of a so-called 'pseudogap' in the electronic density of states [17–23] above T_c in certain cuprates, at least with one of its major interpretations as 'pre-formed CPs' without long-range coherence or condensation, while in BCS theory CP formation and condensation occur simultaneously below the *same* T_c. We submit here that, along with the zero-center-of-mass–momentum (CMM) CPs, a natural candidate for such pre-formed CPs are the nonzero-CMM CPs usually neglected in BCS theory.

In Section 2 we recall the *linear* dispersion relation for Cooper pairs in a 2D gas of electrons at $T = 0$ interacting via a constant pairing interaction, nonzero only in a thin annulus about the Fermi surface—viz. the *BCS model interaction*. The binding energy of a *single* pair near the Fermi surface (CP problem) decreases almost linearly with

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the CMM of the pair for all values of the momentum below a breakup momentum which is typically only about four orders of magnitude smaller than the Fermi momentum. In Section 3 a simple boson-fermion binary gas statistical model is introduced by constructing the Helmholtz free energy for an ideal mixture of pairable but unpaired fermions plus paired fermions (both zero and nonzero CMM pairs), all in chemical and thermal equilibrium, to give the relative number of bosons in the mixture as function of both coupling and temperature. In Section 4 the critical BEC singularity temperature is obtained first by ignoring the unpaired fermions in a pure boson-gas model and then exactly for the boson-fermion binary gas mixture model from a T-dependent dispersion relation derived and calculated numerically, and results are compared with empirical cuprate superconductor T_cs. Finally, Section 5 contains conclusions.

2. Dynamics of Cooper pairs and their dispersion relation

To fix the interfermion dynamics take a 2D system of N fermions of mass m confined in a square of area L^2 interacting pairwise via the BCS model electron–phonon interaction $V_{\mathbf{k},\mathbf{k}'} = -V$, with $V > 0$, whenever $\mu(T) - \hbar\omega_D < \epsilon_{k_1}(\equiv \hbar^2 k_1^2/2m)$, $\epsilon_{k_2} < \mu(T) + \hbar\omega_D$, and zero otherwise, where $\mathbf{k} \equiv 1/2(\mathbf{k}_1 - \mathbf{k}_2)$ is the relative wave-vector of the two particles; $\mu(T)$ the *ideal* Fermi gas (IFG) chemical potential, which at $T = 0$ becomes the Fermi energy $E_F \equiv \hbar^2 k_F^2/2m$ with k_F the Fermi wavenumber; and ω_D the Debye frequency. In most materials $\hbar\omega_D \ll E_F$. After much debate on the precise interfermion dynamical nature of copper-oxide materials, striking direct evidence for significant electron–phonon coupling in high-temperature cuprate superconductors from angle-resolved photoemission spectroscopy (ARPES) experiments has recently been reported [24].

If $\hbar\mathbf{K} \equiv \hbar(\mathbf{k}_1 + \mathbf{k}_2)$ is the CMM of a CP, let E_K be its *total* energy (besides the CP rest-mass energy). The *original* CP [1] eigenvalue equation at $T = 0$ is

$$1 = V \sum_{\mathbf{k}}{}' \frac{\theta(k_1 - k_F)\theta(k_2 - k_F)}{2\epsilon_k + \hbar^2 K^2/4m - E_K}, \tag{1}$$

where $\theta(x)$ is the Heaviside unit step function, and the prime on the summation sign denotes the conditions $k_{1,2} \equiv |\mathbf{K}/2 \pm \mathbf{k}| < (k_F^2 + k_D^2)^{1/2}$ ensuring that the pair of particles *above* the Fermi 'surface' cease interacting beyond the annulus of energy thickness $2\hbar\omega_D \equiv \hbar^2 k_D^2/m$, thereby restricting the summation over $\mathbf{k}$ for a given fixed $\mathbf{K}$. Nonzero CMM pairs have $\mathbf{K}$ values in all directions and are thus unrelated to the current-carrying state having a drift-velocity momentum in a fixed direction. Setting $E_K \equiv 2E_F - \Delta_K$, a pair is *bound* if $\Delta_K > 0$, so that Eq. (1) becomes an equation for the (positive) pair binding energy Δ_K. Our Δ_K and Δ_0 follow Cooper's notation and should *not* be confused with the BCS energy gap $\Delta(T)$ at $T = 0$. Let $g(\epsilon)$ be the electronic density-of-states (for each spin) in the normal (i.e. interactionless) N-fermion state; in 2D it is constant, $g(\epsilon) = L^2 m/2\pi\hbar^2 \equiv g$. For $K = 0$ it becomes a single elementary integral, with the familiar [1] solution $\Delta_0 = 2\hbar\omega_D/(e^{2/\lambda} - 1)$ valid for all coupling $\lambda \equiv gV \geq 0$. The Cooper equation (1) for the unknown quantity Δ_K can be analyzed beyond the usual zero-CMM, $K = 0$, case and for small K and λ gives [25]

$$\Delta_K \underset{K\to 0}{\longrightarrow} \Delta_0 - \frac{2}{\pi}\hbar v_F K + O(K^2), \tag{2}$$

where $v_F \equiv \sqrt{2E_F/m}$ is the Fermi velocity. This *linear dispersion relation* is the 2D analog of the 3D result stated by Schrieffer as far back as 1964 in Ref. [26], p. 33 (see also Ref. [27], p. 336). Although some treatments (Ref. [28]) of CPs are more sophisticated than the original Cooper picture (1) numerically yield *resonant* pairs (i.e. with an imaginary term in the energy) with a leading quadratic dispersion, *linearly-dispersive resonances* appear analytically from a Bethe–Salpeter equation many-body approach [29] to CPs in 3D—provided it is based on the BCS (where holes are treated on an equal footing with particles), not the IFG, ground state. In 2D, see also Refs. [30,31]. These linarly-dispersive CPs are commonly confused with the also linearly-dispersive sound phonons of the collective excitation sometimes referred to as the Anderson–Bogoliubov–Higgs mode (which for zero coupling reduces [32] to the IFG result $\hbar v_F K/\sqrt{d}$). The IFG sound speed $c = v_F/\sqrt{d}$ follows trivially from the zero-temperature IFG pressure $P = n^2[\mathrm{d}(E/N)/\mathrm{d}n] = 2nE_F/(d+2)$ via the familiar thermodynamic relation $\mathrm{d}P/\mathrm{d}n = mc^2$, where E is the ground-state energy and $n \equiv N/L^d = k_F^d/2^{d-2}\pi^{d/2} d\Gamma(d/2)$ is the fermion-number density. But the simple result (2) in fact refers to actual 'moving' (or 'excited') CPs *in the Fermi sea*, which clearly 'break up' for $K > K_0$ as defined by $\Delta_{K_0} \equiv 0$. *Both* kinds of *distinct* soundwave-like solutions—moving CPs and ABH phonons—appear in the many-body, ladder-summation scheme of Ref. [29].

In the opposite limit of *strong* coupling (and/or low density) instead of the linear term in Eq. (2) one gets, at least for an attractive interfermion delta interaction [31], the quadratic $\hbar^2 K^2/2(2m)$ as expected in the vacuum limit of no Fermi sea. But as quite well-known this expression gives a zero BEC T_c, see Eq. (11) below.

These CP integer-spin objects are considered bosons even though their creation and annihilation operators for fixed $\mathbf{k}_1$ and $\mathbf{k}_2$ (or, alternatively, fixed relative $\mathbf{k}$ and total CM $\mathbf{K}$ wavevectors) are known not to obey the usual Bose commutation relations (Ref. [26] p. 38). However, indefinite occupation in a state of given $\mathbf{K}$, needed to ensure the Bose-Einstein *distribution* in turn required for BEC, indeed occurs (see Ref. [33], p. 181 ff.) for the objects whose energy E_K is a solution of Eq. (1) as it depends *only* on $\mathbf{K}$ but not on $\mathbf{k}$. As a result, in the thermodynamic limit, for *any* coupling

(Ref. [25], see Fig. 1) there will be indefinitely many values of the relative momenta **k** for a given **K** so that CPs formed with the BCS model interaction do in fact obey the BE distribution.

3. Boson-fermion binary gas in chemical/thermal equilibrium

The number of bosons in the boson-fermion mixture to be studied turns out to be both coupling- and temperature-dependent. And it is *in conserving the total fermion number* self-consistently that a BEC-like singularity arises. As is the case for the pure boson gas, a linear rather than a quadratic dispersion relation is needed to obtain BEC in 2D. This emerges in a statistical model for the ideal binary *mixture* of bosons (the CPs) and unpaired (both pairable and unpairable) fermions in chemical equilibrium [35,36] for which thermal pair-breaking into unpaired pairable fermions is explicitly allowed [37]. Assuming the BCS model interaction the total number of fermions in 2D at any T is $N = L^2k_F^2/2\pi = N_1 + N_2$, where N_1 is the number of unpairable (i.e. noninteracting) fermions while N_2 is the number of pairable (i.e. active) ones. The unpairable fermions also obey the usual Fermi–Dirac distribution with the IFG chemical potential μ but the N_2 pairable ones are simply those in the interaction shell of energy width $2\hbar\omega_D$ so that, if $\beta \equiv (k_BT)^{-1}$,

$$N_2 = 2\int_{\mu-\hbar\omega_D}^{\mu+\hbar\omega_D} d\epsilon \frac{g(\epsilon)}{e^{\beta(\epsilon-\mu)}+1} = 2g\hbar\omega_D, \tag{3}$$

which is independent of T. At fixed interfermionic coupling λ and temperature T these N_2 fermions form an ideal mixture of pairable but unpaired fermions plus CPs created near the single-fermion energy $\mu(T)$, with binding energy $\Delta_K(T) \geq 0$ and total energy

$$E_K(T) \equiv 2\mu(T) - \Delta_K(T). \tag{4}$$

This generalizes the $T = 0$ equation $E_K \equiv 2E_F - \Delta_K$ introduced below Eq. (1).

The Helmholtz free energy $F = E - TS$, where E is the internal energy and S the entropy, of this binary 'composite boson/pairable-but-unpaired-fermion system' at temperatures $T \leq T_c$ is then readily constructed [37] in terms of: (a) the average number of unpaired but pairable fermions with fixed energy; (b) $N_{B,K}(T)$, the number of CPs with nonzero-CMM, $0 < K \leq K_0$, with the CP-breakup value K_0 defined [25] by $\Delta_{K_0} \equiv 0$; and (c) $N_{B,0}(T)$, the number of CPs with zero CMM at temperature T. The free energy F_2 of just the N_2 *pairable* fermions is to be minimized subject to the constraint that N_2 is conserved, i.e. one seeks the minimum of $F_2 - \mu_2N_2$ with respect to (a)–(c) just mentioned. The total number of pairable but unpaired fermions $N_{20}(T)$ is then

$$N_{20}(T) = 2g\int_{\mu-\hbar\omega_D}^{\mu+\hbar\omega_D} d\epsilon \frac{1}{e^{\beta(\epsilon-\mu_2)}+1} = \frac{2g}{\beta}\ln\left[\frac{1+e^{-\beta(\mu-\mu_2-\hbar\omega_D)}}{1+e^{-\beta(\mu-\mu_2+\hbar\omega_D)}}\right]. \tag{5}$$

Note that if $\mu_2 = \mu$ this becomes the rhs of Eq. (3), as it should. The relevant *number equation* for the pairable fermions is thus

$$N_2 = N_{20}(T) + 2[N_{B,0}(T) + \sum_{K>0}^{K_0} N_{B,K}(T)] \equiv N_{20}(T) + 2N_B(T), \tag{6}$$

where $\sum_{K>0}^{K_0} N_{B,K}(T) = \sum_{K>0}^{K_0}[e^{\beta\{E_K(T)-2\mu_2\}} - 1]^{-1}$ is the *total* number of excited CPs (namely with CMM values $0 < K < K_0$). One can rewrite $E_K(T) - 2\mu_2$ here as $\varepsilon_K(T) - \mu_B(T)$, with $\varepsilon_K(T) \equiv \Delta_0(T) - \Delta_K(T) \geq 0$ a (non-negative) excitation energy as suggested by Eq. (2). Hole–hole and particle–particle CPs can be shown to have the same excitation energy $\varepsilon_K(T)$. The remaining unknown $\mu_B(T)$ is then

$$\mu_B(T) = 2[\mu_2(T) - \mu(T)] + \Delta_0(T) = 0, \tag{7}$$

for $0 \leq T \leq T_c$ since $N_{B,0}(T)$ is negligible for all $T > T_c$. This is precisely the BEC condition for a *pure* boson gas, although one now has a binary boson-fermion *mixture*. Thus, rather than $2\mu_2$ Eq. (7) is a more convenient definition for a binary mixture of the boson chemical potential. Furthermore, Eq. (7) improves upon the definition in Ref. [36] p. 123, which is precisely Eq. (7) but with $2\mu(T)$ instead of our $2[\mu_2(T) - \mu(T)]$; that definition severely handicaps the binary system since $\mu_B \leq 0$ implies $2\mu \leq -\Delta_0$ and so the system 'can never enter the Fermi degeneracy region' [36] where a *positive* $\mu(T)$ guarantees a Fermi surface.

To determine $N_B(T)$ from Eqs. (5) and (6) we use Eq. (7) and see that

$$N_{20}(T) = \frac{2g}{\beta}\ln\left[\frac{1+e^{-\beta\{\Delta_0(T)/2-\hbar\omega_D\}}}{1+e^{-\beta\{\Delta_0(T)/2+\hbar\omega_D\}}}\right], \tag{8}$$

for $0 \leq T \leq T_c$. Thus $2N_B(T)/N_2 \equiv 1 - N_{20}(T)/N_2$ is obtainable for $0 \leq T \leq T_c$ from Eq. (8) if $\Delta_0(T)$ were known.

For this, $\theta(k_1 - k_F) \equiv \theta(\epsilon_{k_1} - E_F)$ in Eq. (1) becomes $1 - n(\xi_{k_1})$ where $n(\xi_{k_1}) \equiv (e^{\beta\xi_{k_1}} + 1)^{-1}$ with $\xi_{k_1} \equiv \epsilon_{k_1} - \mu(T)$, the IFG chemical potential $\mu(T)$ in 2D being given exactly by $\mu(T) = \beta^{-1}\ln(e^{\beta E_F} - 1) \rightarrow E_F$ as $T \rightarrow 0$. Similar arguments hold for $\theta(k_2 - k_F)$. Since for $K = 0$, $k_1 = k_2$ which implies that $\xi_{k_1} = \xi_{k_2}$, Eq. (1) then provides a simple generalization to finite-T of the $K = 0$ CP equation, namely

$$1 = \lambda\int_0^{\hbar\omega_D} d\xi(e^{-\beta\xi} + 1)^{-2}[2\xi + \Delta_0(T)]^{-1}. \tag{9}$$

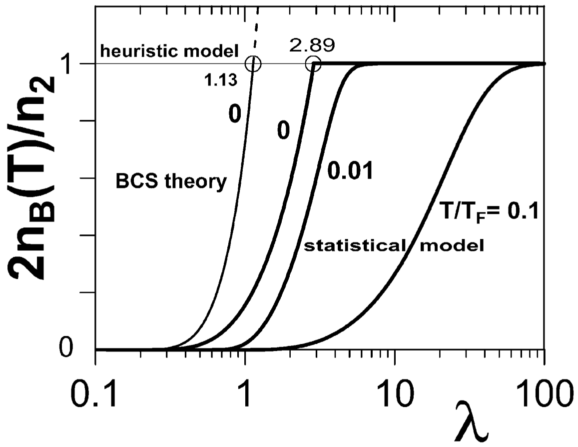

Fig. 1. Fractional number of pairable fermions that are actually paired vs. coupling λ for the present statistical model at three different temperatures (thick curves) and estimated for BCS theory at $T = 0$ as explained below Eq. (10) (thin curve). The number of pairable fermions with the BCS model interaction used is just Eq. (3); all of them are paired at $T = 0$ (unrealistically) in the heuristic BEC model, Ref. [34] Eq. (23).

Its numerical solution shows $\Delta_0(T)$ to decrease monotonically with T for fixed λ and $\hbar\omega_D$, and zero only at infinite T. (This infinite 'de-pairing' temperature is obviously spurious as the BCS model interaction loses meaning when $\mu(T)$ turns negative at large T.) Thus also $2N_B(T)/N_2$ decreases with T; it is plotted in Fig. 1 as $2n_B(T)/n_2$, since $n_B(T) \equiv N_B(T)/L^2$ and $n_2 \equiv N_2/L^2$. Using Eq. (3) for N_2 the *fractional number of pairable fermions that are actually paired* at $T = 0$ becomes simply

$$2N_B(0)/N_2 = \Delta_0/2\hbar\omega_D = (e^{2/\lambda} - 1)^{-1} \underset{\lambda\to 0}{\longrightarrow} e^{-2/\lambda}, \tag{10}$$

for $\lambda \leq 2/\ln 2 \simeq 2.89$, and equals 1 (all pairable fermions paired into bosons) for $\lambda \geq 2.89$. This fraction is also plotted against coupling λ in Fig. 1. It contrasts sharply with the 'heuristic model' of Ref. [34], Eq. (16), where $2N_B(0)/N_2 \equiv 1$ for all coupling. The fractional number is now qualitatively similar to that of BCS theory—which is not [38] a BEC theory—where, in any d, a coupling-dependent fraction is estimated (Ref. [39] p. 128) to be $(\Delta/\hbar\omega_D)^2 \equiv (\sinh 1/\lambda)^{-2} \to 4\,e^{-2/\lambda}$ as $\lambda \to 0$. Here Δ (again, not to be confused with the CP binding energy Δ_0) is the $T = 0$ BCS energy gap for the same BCS model interaction used in this paper. It is graphed as the thin curve in Fig. 1 and seen to be much larger than Eq. (10) for fixed λ.

4. Critical temperatures in BEC and BCS

For N_B bosons of mass m_B and energy $\varepsilon_K = C_s K^s$ with $s > 0$ and C_s a constant, a BEC temperature singularity occurs at $T_c \neq 0$ for any dimension [40,41] $d > s$ in the number equation $N_B = \sum_{\mathbf{K}} [e^{(\varepsilon_K - \mu_B)/k_B T} - 1]^{-1}$ at vanishing bosonic chemical potential $\mu_B \leq 0$ when the number of $\mathbf{K} = 0$ bosons just ceases to be negligible upon cooling. This critical temperature is given [34] by

$$T_c = \frac{C_s}{k_B}\left[\frac{s\Gamma(d/2)(2\pi)^d n_B}{2\pi^{d/2}\Gamma(d/s)g_{d/s}(1)}\right]^{s/d}, \tag{11}$$

with $n_B \equiv N_B/L^d$, and $g_\sigma(z)$ the usual Bose integrals expandable as infinite series which for $\sigma > 1$ become $\zeta(\sigma)$, the Riemann zeta function of order σ, but diverge for $\sigma \leq 1$. Thus $T_c = 0$ for all $d \leq s$. For $s = 2$ and $d = 3$ one has $\zeta(3/2) \simeq 2.612$, and since $C_2 \equiv \hbar^2/2m_B$ Eq. (11) then reduces to the familiar formula $T_c \simeq 3.31\hbar^2 n_B^{2/3}/m_B k_B$ of 'ordinary' BEC. But for bosons with (positive) excitation energy $\varepsilon_K \equiv \Delta_0 - \Delta_K$ given approximately by the linear term in Eq. (2) for all K (meaning that $s = 1$ and $C_1 \equiv a(d)\hbar v_F$ with $a(d) = 2/\pi$ and 1/2 for $d = 2$ and 3, respectively) the critical temperature T_c is *nonzero for all* $d > 1$—*precisely* the dimensionality range of all known superconductors down to the quasi-1D organo-metallic (Bechgaard) salts [8–10].

If the background unpaired fermions are not considered one has a *pure boson gas* of CPs but with T-dependent number density $n_B(T)$. Converting the explicit T_c-formula (1) for $s = 1$ and $d = 2$ into an *implicit* one by allowing n_B to be T-dependent leaves

$$T_c = \frac{4\sqrt{3}}{\pi^{3/2}}\frac{\hbar v_F}{k_B}\sqrt{n_B(T_c)}, \tag{12}$$

since $g_2(1) \equiv \zeta(2) = \pi^2/6$. This requires $n_B(T) \equiv N_B(T)/L^2$ which from Eq. (6) requires Eq. (8), along with $\Delta_0(T)$ from Eq. (9). Solving these *three* coupled equations simultaneously for $\lambda = 1/2$ gives the remarkably constant value $T_c/T_F \simeq 0.004$ over the entire range of $\nu \equiv \hbar\omega_D/E_F$ values 0.03–0.07 typical of cuprate superconductors, see Fig. 2. On the other hand, the BCS theory formula $T_c^{BCS} \simeq 1.13\Theta_D\,e^{-1/\lambda}$ with $\lambda = 1/2$ yields $T_c/T_F = 0.005$–0.011 over the same range of ν values. Unfortunately, both sets of predictions are well below empirical cuprate values of T_c/T_F varying [42] from 0.03 to 0.09. Pure gas model results [34], where all (pairable) fermions are paired, for either breakable or unbreakable bosons are shown in Fig. 2 to overestimate empirical T_c/T_F values by factors ranging from two to more than two orders of magnitude. All these results are wide off the mark.

To obtain the critical temperature *without* neglecting the background unpaired fermions, one needs the exact CP excitation energy dispersion relation $\varepsilon_K(T) \equiv \Delta_0(T) - \Delta_K(T)$ which is neither precisely linear in K nor independent of T. To determine $\Delta_K(T)$ we need a working equation that generalizes Ref. [25] for $T > 0$ via the new CP eigenvalue Eq. (9). For the critical temperature from the finite-temperature dispersion relation, besides solving for $\Delta_K(T)$, one requires Eqs. (3), (6) and (8). At $T = T_c$ both $N_{B,0}(T_c) \simeq 0$ and $\mu_B(T_c) \simeq 0$ so that Eq. (6) leads [37] to the implicit

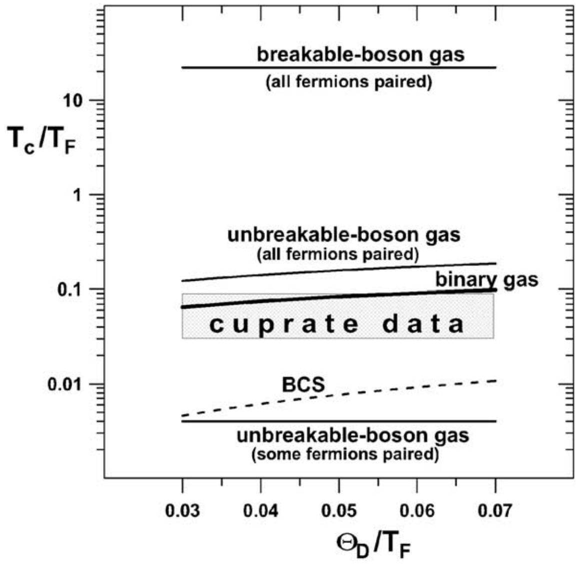

Fig. 2. Critical BEC temperature T_c in units of T_F for the BCS model interaction with $\lambda = 1/2$ for varying $\nu \equiv \hbar\omega_D/\mu(T_c) \simeq \Theta_D/T_F$ for: the pure unbreakable-boson gas with *some* and with all fermions paired, the former being the solution of Eq. (12) and the latter taken from Ref. [34], Eq. (17); for the breakable-boson gas, from Ref. [34], Eq. (18); and for the boson-fermion mixture from Eq. (13) (thick full curve labeled binary gas). Dashed curve is the BCS theory T_c, and cuprate experimental data are taken from Ref. [42].

T_c-equation for the *binary gas*

$$1 = \frac{\tilde{T}_c}{\nu}\ln\left[\frac{1+e^{-\{\tilde{\Delta}_0(\tilde{T}_c)/2-\nu\}/\tilde{T}_c}}{1+e^{-\{\tilde{\Delta}_0(\tilde{T}_c)/2+\nu\}/\tilde{T}_c}}\right] + \frac{8(1+\nu)}{\nu}\int_0^{\kappa_0(\tilde{T}_c)} \frac{\kappa\, d\kappa}{e^{[\tilde{\Delta}_0(\tilde{T}_c)-\tilde{\Delta}_\kappa(\tilde{T}_c)]/\tilde{T}_c}-1}, \quad (13)$$

where quantities with tildes are in units of $\mu(T_c) \simeq E_F$ or T_F, while $\kappa \equiv K/2\sqrt{k_F^2+k_D^2}$ and $\nu \equiv \Theta_D/T_F$. *Four* coupled equations must now be solved self-consistently for the exact T_c for each λ and ν, including Eq. (9) for $\tilde{\Delta}_0(\tilde{T})$, and Eq. (35) of Ref. [37] for both $\tilde{\Delta}_\kappa(\tilde{T})$ and $\kappa_0(\tilde{T}_c)$. Results with $\lambda = 1/2$ labeled 'binary gas' in Fig. 2 show a huge enhancement of T_c, with respect to the self-consistent result from Eq. (12), arising from the equilibrating presence of the unpaired fermions, in spite of the very small number of bosons suggested by Fig. 1 for $\lambda = 1/2$.

For cuprates $d \simeq 2.03$ has been suggested [43] as more realistic since it reflects inter-CuO-layer couplings, but our results in that case would be very close to those for $d = 2$ since, e.g. from Eq. (11) T_c for $s = 1$ (but not for $s = 2$) varies little [44] with d around $d = 2$. Indeed, if $m_{B\perp}$ and m_B are the boson masses *perpendicular* and *parallel*, respectively, to the cuprate planes, an 'anisotropy ratio' $m_B/m_{B\perp}$ varied from 0 to 1 allows 'tuning' d continuously from 2 to 3.

Other boson-fermion models [6,12,16,28,45,46] have been introduced, some even addressing [16,46] d-wave interaction effects as opposed to the pure s-wave considered here, and some also focusing [16,28] on the pseudogap. But successful calculations of cuprate T_c values in quasi-2D without adjustable parameters are not reported–and indeed predict $T_c \equiv 0$ in exactly 2D.

5. Conclusions

A statistical model treating ordinary CPs, of fixed CMM but indefinite relative momenta, as noninteracting bosons in chemical and thermal equilibrium with unpaired fermions yields a boson number that rises very slowly from zero with coupling, and that decreases with temperature. The model naturally suggests a more convenient definition of the boson chemical potential whereby the degenerate Fermi region of positive fermion chemical potential can be accessed unlike previous treatments. When the CP dispersion relation is approximately linear, as it must be because of the Fermi sea, it exhibits a BEC of zero-CMM pairs at a finite temperature at precisely 2D. Such a critical temperature T_c would vanish for a quadratic dispersion appropriate not to the Fermi sea but to vacuum. Transition temperatures for the boson-fermion mixture based on the exact CP dispersion relation for the BCS model electron–phonon interaction are greatly enhanced over both BCS theory as well as over pure-Bose-gas BEC T_cs, and without any adjustable parametersare in rough agreement with empirical cuprate superconductor T_cs.

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References

[1] L.N. Cooper, Phys. Rev. 104 (1956) 1189.
[2] B.S. Deaver Jr., W.M. Fairbank, Phys. Rev. Lett. 7 (1961) 43.
[3] R. Doll, M. Näbauer, Phys. Rev. Lett. 7 (1961) 51.
[4] C.E. Gough, M. Colclough, E.M. Forgan, R.G. Jordan, M. Keene, C.M. Muirhead, I.M. Rae, N. Thomas, J.S. Abell, S. Sutton, Nature 326 (1987) 855.
[5] P.W. Anderson, Science 235 (1987) 1196.
[6] R. Friedberg, T.D. Lee, Phys. Rev. B 40 (1989) 6745.
[7] A.S. Alexandrov, N.F. Mott, Phys. Rev. Lett. 71 (1993) 1075.
[8] D. Jérome, Science 252 (1991) 1509.

[9] J.M. Williams, A.J. Schultz, U. Geiser, K.D. Carlson, A.M. Kini, H.H. Wang, W.K. Kwok, M.H. Whangbo, J.E. Schirber, Science 252 (1991) 1501.
[10] H. Hori, Int. J. Mod. Phys. B 8 (1994) 1.
[11] J. Ranninger, R. Micnas, S. Robaszkiewicz, Ann. Phys. (Paris) 13 (1988) 455.
[12] R. Friedberg, T.D. Lee, H.-C. Ren, Phys. Lett. A 152 (1991) 417 see p. 423.
[13] J. Ranninger, J.M. Robin, Physica C 253 (1995) 279.
[14] J. Ranninger, J.M. Robin, Solid State Commun. 98 (1996) 559.
[15] V.V. Tolmachev, Phys. Lett. A 266 (2000) 400.
[16] Y. Domanski, J. Ranninger, Phys. Rev. B 63 (2001) 134505 see esp. Eq. (53).
[17] D.S. Marshall, D.S. Dessau, A.G. Loeser, C.-H. Park, A.Y. Matsuura, J.N. Eckstein, I. Bozovic, P. Fournier, A. Kapitulnik, W.E. Spicer, Z.-X. Shen, Phys. Rev. Lett. 76 (1996) 4841.
[18] H. Ding, T. Yokoya, J.C. Campuzano, T. Takahashi, M. Randeria, M.R. Norman, T. Mochiku, K. Kadowaki, J. Giapintzakis, Nature 382 (1996) 51.
[19] A.G. Loeser, Z.-X. Shen, D.S. Dessau, D.S. Marshall, C.H. Park, P. Fournier, A. Kapitulnik, Science 273 (1996) 325.
[20] G.V.M. Williams, J.L. Tallon, R. Michalak, R. Dupree, Phys. Rev. B 54 (1996) R6909.
[21] A.V. Puchkov, P. Fournier, D.N. Basov, T. Timusk, A. Kapitulnik, N.N. Kolesnikov, Phys. Rev. Lett. 77 (1996) 3212.
[22] A. Ino, T. Mizokawa, K. Kobayashi, A. Fujimori, Phys. Rev. Lett. 81 (1998) 2124.
[23] M. Kugler, Ø. Fischer, Ch. Renner, S. Ono, Y. Ando, Phys. Rev. Lett. 86 (2001) 4911.
[24] A. Lanzara, P.V. Bogdanov, X.J. Zhou, S.A. Kellar, D.L. Feng, E.D. Lu, T. Yoshida, H. Eisaki, A. Fujimori, K. Kishio, J.-I. Shimoyama, T. Noda, S. Uchida, Z. Hussain, Z.-X. Shen, Nature 412 (2001) 510.
[25] M. Casas, S. Fujita, M. de Llano, A. Puente, A. Rigo, M.A. Solís, Physica C 295 (1998) 93.
[26] J.R. Schrieffer, Theory of Superconductivity, Benjamin, New York, 1964.
[27] A.L. Fetter, J.D. Walecka, Quantum Theory of Many-Particle Systems, McGraw-Hill, New York, 1971.
[28] Q.J. Chen, I. Kosztin, B. Jankó, K. Levin, Phys. Rev. B 59 (1999) 7083 see esp. Eq. (11).
[29] M. Fortes, M.A. Solís, M. de Llano, V.V. Tolmachev, Physica C 364–365 (2001) 95.
[30] S.V. Traven, Phys. Rev. B 51 (1995) 3242.
[31] S.K. Adhikari, M. Casas, A. Puente, A. Rigo, M. Fortes, M.A. Solís, M. de Llano, A.A. Valladares, O. Rojo, Phys. Rev. B 62 (2000) 8671.
[32] L. Belkhir, M. Randeria, Phys. Rev. B 49 (1994) 6829.
[33] S. Fujita, S. Godoy, Quantum Statistical Theory of Superconductivity, Plenum Press, New York, 1996.
[34] M. Casas, A. Rigo, M. de Llano, O. Rojo, M.A. Solís, Phys. Lett. A 245 (1998) 5.
[35] M.R. Schafroth, Phys. Rev. 96 (1954) 1442.
[36] M.R. Schafroth, S.T. Butler, J.M. Blatt, Helv. Phys. Acta 30 (1957) 93.
[37] M. Casas, N.J. Davidson, M. de Llano, T.A. Mamedov, A. Puente, R.M. Quick, A. Rigo, M.A. Solís, Physica A 295 (2001) 425.
[38] J. Bardeen, Phys. Today January (1963) 25.
[39] J.M. Blatt, Theory of Superconductivity, Academic Press, New York, 1964.
[40] J.D. Gunton, M.J. Buckingham, Phys. Rev. 166 (1968) 152.
[41] R.M. Ziff, G.E. Uhlenbeck, M. Kac, Phys. Rep. 32 (1977) 169.
[42] C.P. Poole Jr., H.A. Farach, R.J. Creswick, Superconductivity, Academic Press, New York, 1995, p. 599.
[43] X.-G. Wen, R. Kan, Phys. Rev. B 37 (1988) 595.
[44] F.J. Sevilla, M. Grether, M. Fortes, M. de Llano, O. Rojo, M.A. Solís, A.A. Valladares, J. Low Temp. Phys. 121 (2000) 281 see esp. Fig. 1.
[45] J. Ranninger, S. Robaszkiewicz, Physica B 135 (1985) 468.
[46] V.B. Geshkenbein, L.B. Ioffe, A.I. Larkin, Phys. Rev. B 55 (1997) 3173.