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Generalizing BCS for Exotic Superconductors

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A new boson–fermion statistical model with two-hole (h) as well as two-electron (e) Cooper pairs (CP) exhibiting Bose–Einstein condensation (BEC)—which simultaneously reduces to BCS theory in weak coupling for perfect eh symmetry *and* to BEC when no hole CPs are present—yields reasonable transition temperatures for exotic superconductors, whether quasi-2D cuprate or 3D ones, for moderate departures from perfect eh symmetry.

KEY WORDS: Boson–fermion models; BCS theory; Bose–Einstein condensation; exotic superconductors.

Boson–fermion (BF) models of superconductivity go back to the mid-1950s [1–3], pre-dating even the BCS–Bogoliubov theory [4,5]. Although the latter only contemplates the presence of Cooper "correlations," BF models [1–3,6–12] posit the existence of real bosonic Cooper pairs (CPs). Unfortunately, however, there seems to be no experiment yet that distinguishes between electron (e) and hole (h) CPs. With two exceptions [10,11] all BF models neglect 2h-CPs formulated on an equal footing with 2e-CPs and so cannot contain BCS theory where perfect eh symmetry holds. In our complete BF model (CBFM) of unpaired electrons coexisting with *both* CP species, 2h-CPs are *distinct* from (and kinematically independent of) 2e-CPs as their Bose commutation relations exhibit a sign change while electron and hole Fermi anticommutation relations do not. The unperturbed Hamiltonian corresponds to an ideal BF gas, while the interaction one is reminiscent of the Fröhlich (or Dirac QED) interaction Hamiltonians involving two fermion and one boson operators, but with *two* types of CPs instead of phonons (or photons). Those Hamiltonians are the most natural ones to use in a many-electron/phonon (or photon) system; one can conjecture likewise for the BF system under study, as had already been assumed by several authors [6–9].

The CBFM leads to a set of three coupled integral equations (Ref. [10], Eqs. (7)–(9)): two gap-like relations (one each for the 2e-CP and 2h-CP BE-condensed boson number densities) plus a third "number" equation involving *both* number densities. They encompass *four* different theories as special cases. For perfect electron-hole (eh) symmetry in the CPs the CBFM reduces to (i) *ordinary BCS theory* if the CBFM interaction parameters are properly identified with those of BCS. On the other hand, for no 2h-CPs present the CBFM also contains (ii) the *Bose-Einstein condensation (BEC) BF model* in 3D of Friedberg and Lee [7], and for zero coupling (iii) the *ideal BF model* of Ref. [12] that predicts nonzero BEC T_cs even in 2D, as well as (iv) the familiar transition-temperature T_c-formula of *ordinary BEC* in 3D.

Here we sketch how the CBFM yields sizeable enhancements in T_cs over BCS theory for moderate departures from perfect eh symmetry, for the *same* (BCS model) interaction. In Ref. [11] the three coupled equations were solved numerically in 3D for the usual BCS interaction parameters $\lambda = 1/5$ and $\hbar\omega_D/E_F = 0.001$ where it was found that, along with the *normal* phase consisting of an ideal BF gas [12], three different stable (plus unstable, probably metastable, i.e., of higher Helmholtz free energy) BEC phases emerged—all surrounding the BCS T_c value on the T/eh-symmetry plane. They consisted of two

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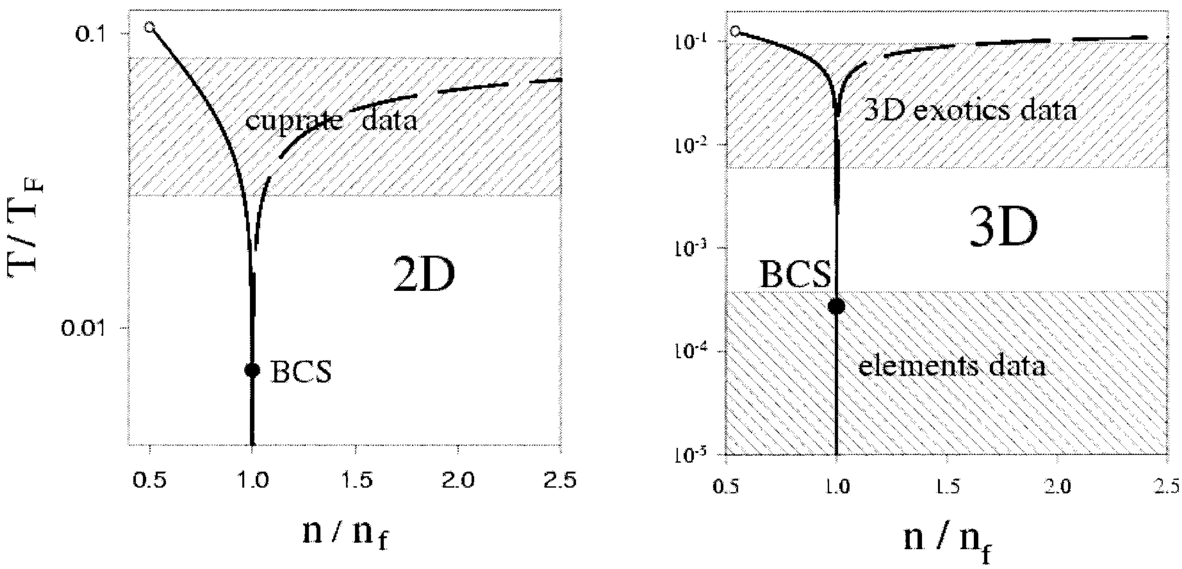

Fig. 1. Phase boundaries in the temperature/eh-symmetry plane according to the CBFM for 2h-CP (full) and of 2e-CP (dashed) BEC T_cs in 2D (left) and 3D (right). Dot is BCS result while open circle is explained in text. Data taken from Refs. [14,15].

pure phases of *either* 2e- *or* 2h-CP BE-condensates, and a lower temperature *mixed* phase with arbitrary proportions of 2e- *and* 2h-CPs. There is far greater physical interest in the two higher-T_c pure phases, so we focus only on them in both 2D and 3D.

For the boson energy vs. center-of-mass-momentum $\hbar K$ dispersion we employ the *linear* leading term $(\lambda/2\pi)\hbar v_F K$, with v_F the Fermi velocity, in the many-body Bethe–Salpeter treatment of CPs (see Ref. [13] for derivation in 3D)—rather than [1–3,6–11] the quadratic $\hbar^2 K^2/2(2m)$ appropriate for a composite boson of mass $2m$ moving not in the Fermi sea but in vacuum, as e.g. an isolated deuteron. In 2D $\lambda = 1/2$ and $\hbar\omega_D/E_F = 0.05$ (typical of cuprates) were used to determine the BEC T_c/T_F vs. n/n_f and are graphed as phase boundaries in Fig. 1 for both 2e- (dashed curve) and 2h-CP (full curve) pure, stable phases. Here n is the total number-density of charge-carrier electrons while n_f is that of *unpaired* electrons at zero-temperature and coupling, and depends on the average value of the zero-momenta 2e- and 2h-CPs (unknown, but phenomenological) energies. Perfect eh symmetry corresponds to $n/n_f = 1$. BCS theory predicts $T_c/T_F \simeq 1.134(\hbar\omega_D/E_F)\exp(-1/\lambda) \simeq 0.008$ and is marked by the dot in figure. The open circle on the 2h-CP BEC (full) curve marks the value of T_c/T_F beyond which a complex solution for n/n_f develops. The 2e-CP BEC (dashed) curve tends to the asymptotic value of 0.088 that follows for $n \gg n_f$ similarly as with Eq. (24) of Ref. [10]. Cuprate data fall [14] in the range $T_c/T_F = 0.03 - 0.09$. Thus, moderate departures from perfect eh symmetry can *reach empirical* T_c *values for the quasi-2D cuprates*.

Results in 3D are also encouraging. Whereas BCS theory can reproduce T_c/T_F values well for the elements [even with smaller values of the coupling λ than our (admittedly large) value of 1/2] it takes only moderate departures from perfect eh symmetry to access empirical [15] T_c/T_F values for 3D exotic superconductors.

To summarize, a very general *complete* (in the sense that 2h-CPs are not neglected) BF model—encompassing *four* different theories as special cases, including the BCS and the BEC theories—can produce in either 2D or 3D, with the BCS electron–phonon model interaction, sizeable enhancements over the BCS predicted T_c values for moderate departures from perfect eh symmetry. The results lie well within empirical ranges in 2D for exotic cuprates, as well as in 3D for other exotic superconductors, and are *higher* for 2h- than for 2e-CP BEC.

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