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Light transmission in quasiperiodic multilayers of porous silicon

R. Nava [a], V. Agarwal [b,c], J.A. del Río [b], C. Wang [a,*]

[a] *Instituto de Investigaciones en Materiales, UNAM, A.P. 70-360, 04510 México D.F., Mexico*
[b] *Centro de Investigación en Energía, UNAM, A.P. 34, 62580 Temixco, Mor., Mexico*
[c] *CIICAp, UAEM, Av. Universidad 1001, Col Chamilpa, Cuernavaca, Mor., Mexico*

Abstract

Porous silicon is an efficient photo- and electro-luminescent material and represents a promising candidate for opto-electronic applications. On the other hand, quasiperiodic structures have been shown to be effective media for light localization and third harmonic generation. In this work, we present a photonic model for quasiperiodic multilayer structures, which are built experimentally by alternating porous silicon layers with high and low refractive indices. The analysis of the light propagation through these structures is based on the transfer matrix theory. The theoretical reflectance spectrum is compared with experimental data, observing a good agreement.

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1. Introduction

Porous silicon is an interesting opto-electronic material due to its luminescence and dielectric properties. It has applications as light-emitting diodes, optical filters, Bragg reflector microcavities, etc. [1]. These optical applications can be enormously enhanced by constructing periodic multilayers of such material, with high enough refractive index contrast layers, which induce a band structure for photon propagation [2], analogous to the electronic band structure in a semiconductor. Recently, photonic quasiperiodic multilayers have been built [3] and they present peculiar transmission spectra due to the non-periodic long-range order [4,5]. One of the most studied quasiperiodic structures is the Fibonacci sequence (F_j), which can be constructed by defining $F_1 = A$, $F_2 = BA$, and the addition rule, $F_j = F_{j-2} \oplus F_{j-1}$, understood as the joining of sequences. Therefore, we have $F_3 = ABA$, $F_4 = BAABA$, and so on.

In this paper, we present a theoretical model and experimental data of the light propagation in a Fibonacci multilayer structure of porous silicon, paying special attention to the importance of the optical path length, the refractive index and its frequency dependence. The exprimental data and the numerical results are found in good agreement.

* Corresponding author. Tel.: +52-55 5622 4634; fax: +52-55 5616 1251.
E-mail address: chumin@servidor.unam.mx (C. Wang).

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2. The model

Based on the transfer matrix theory [6], the light propagation through the entire multilayer can be described by a transfer matrix (M), which is the product of the matrices $T_{n-1|n}$ and T_n, representing respectively the transmission across the interfaces of layers $n-1$ and n,

$$T_{n-1|n} \equiv \begin{pmatrix} 1 & 0 \\ 0 & \frac{\eta_n \cos\theta_n}{\eta_{n-1}\cos\theta_{n-1}} \end{pmatrix}, \quad (1)$$

and that inside each layer n

$$T_n \equiv \begin{pmatrix} \cos\delta_n & -\sin\delta_n \\ \sin\delta_n & \cos\delta_n \end{pmatrix}, \quad (2)$$

where $\delta_n = k\eta_n d_n/\cos\theta_n$, being θ_n the incidence angle, k the wave vector in vacuum, d_n and η_n the thickness and the refractive index of layer n, respectively. The general expression of the transmittance (T) and reflectance (R) can be written as

$$T = \frac{4}{(m_{11}^2 + m_{12}^2 + m_{21}^2 + m_{22}^2) + 2(m_{11}m_{22} - m_{12}m_{21})} \quad (3)$$

and

$$R = \frac{(m_{11}^2 + m_{12}^2 + m_{21}^2 + m_{22}^2) - 2(m_{11}m_{22} - m_{12}m_{21})}{(m_{11}^2 + m_{12}^2 + m_{21}^2 + m_{22}^2) + 2(m_{11}m_{22} - m_{12}m_{21})}, \quad (4)$$

where $m_{\mu,\nu}$ are elements of the transfer matrix M.

In particular, if the multilayer is built by two types of dielectrics A and B, arranged following the Fibonacci sequence, it is convenient to use the renormalized transfer matrices [7], i.e., $Q_A \equiv T_{A|B}T_B T_{B|A}T_A$ for block BA and $Q_B \equiv T_{A|A}T_A$ for block A. In this way, the transfer matrix M for a Fibonacci multilayer can be written as $M = \cdots Q_B Q_A Q_A Q_B Q_A$. Notice that Q_A and Q_B are unimodular, thus the product of them is also unimodular, i.e., $\det(M) = 1$ and $T + R = 1$.

3. Experiment

The porous silicon multilayers are obtained by an electrochemical dissolution, in which a p-type crystalline silicon (c-Si) substrate with a resistivity of 0.001–0.005 Ω cm is etched in an aqueous HF/ethanol/glycerol electrolyte with a ratio of 3:7:1 [8]. Constant currents between the wafer and the electrolyte are applied to produce the multilayers, i.e., 37 mA/cm^2 for the low refractive index layers and 3.7 mA/cm^2 for the high refractive index ones. The layer thickness is controlled by the etching time, so in this case $t_A = 8$ s and $t_B = 28$ s. Fibonacci multilayers were fabricated and thermally oxidized in an oxygen atmosphere at 300 °C during 10 min. As the multilayer structure is formed on the c-Si substrate we measured reflectance instead of transmittance.

4. Results

The transmittance (T) of a Fibonacci multilayer of generation six as a function of the inverse of wavelength (λ^{-1}) is shown in Fig. 1(a) for $\eta_A d_A = \eta_B d_B = \lambda_0/4$, in Fig. 1(b) for $\eta_A d_A = \lambda_0/4.5$ and $\eta_B d_B = \lambda_0/4$, and Fig. 1(c) for $\eta_B d_B = \lambda_0/4$,

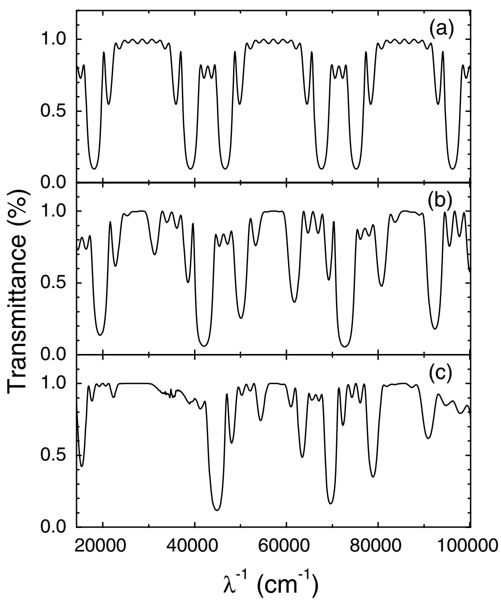

Fig. 1. Transmittance versus the inverse of wavelength (λ^{-1}) for a Fibonacci multilayer containing 13 dielectric layers with optical path length: (a) $\eta_A d_A = \eta_B d_B = \lambda_0/4$, (b) $\eta_A d_A = \lambda_0/4.5$ and $\eta_B d_B = \lambda_0/4$, and (c) $\eta_B d_B = \lambda_0/4$, $d_A = \lambda_0/5.6$ and $\eta_A(\lambda)$.

$d_A = \lambda_0/5.6$ and $\eta_A(\lambda)$ taken from a sample of porous silicon with 70% porosity [9]. In these systems $\lambda_0 = 700$ nm, $\eta_A = 1.4$, and $\eta_B = 2.3$ have been considered. Notice that the spectrum of Fig. 1(a) is periodic, since the layers have the same optical path length, and it reproduces that obtained in Ref. [4]. A small variation in the optical path length of the layers breaks the translational optical-path symmetry and then the spectrum looses the periodicity. Furthermore, if the wavelength dependence of the refractive index $[\eta_A(\lambda)]$ is considered, the transmission spectrum is strongly distorted. Certainly, the constant optical path length is an ideal situation and difficult to be fulfilled in real multilayers. Additionally, the refractive index has an important variation of the wavelength.

Finally, the numerical results (dashed line in Fig. 2), obtained from the same Fibonacci multilayer as in Fig. 1(a) except for the boundary conditions, are compared with the experimental data of a porous silicon multilayer (solid line in Fig. 2). Although, we have neglected the possible experimental variation of the optical path length and the wavelength dependence of the refractive index, we observe a good agreement between the numerical calculations and experimental results. It would be worth mentioning that the numerical results of reflectance shown in Fig. 2 are obtained by considering $\lambda_0 = 770$ nm and a substrate of c-Si with $\eta_{Si} = 3.4$, i.e., a boundary condition different from that used in Fig. 1.

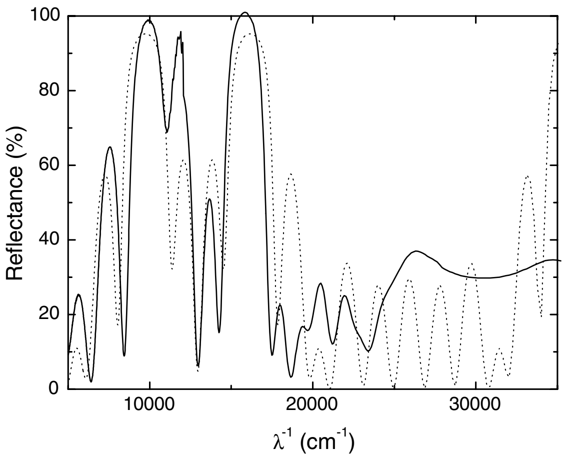

Fig. 2. Measured (solid line) and calculated (dashed line) reflectance spectrum of a porous-silicon 13-layer Fibonacci sample on a c-Si substrate.

5. Conclusions

We have studied the optical properties of quasiperiodic multilayers and found a periodic structure in the transmittance spectrum, when the optical path length of the layers is considered constant. This periodic pattern is sensitive to small variations in the optical path length of the layers or in the refractive index. The numerical results are compared with those obtained from samples of porous-silicon quasiperiodic multilayers, and a good agreement is observed, since for the visible region the quality of the interfaces is not crucial. Finally, this work will be extended to analyze the non-linear optics of these quasiperiodic multilayers, since efficient emissions of second and third harmonics are observed in ferroelectric Fibonacci multilayers [10]. Moreover, the luminescence of quasiperiodic multilayers is an interesting and not widely studied subject. Porous silicon multilayers could be a good candidate and this study is currently in progress.

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