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ROBUST STATISTICAL MECHANISM FOR PHONON-DRIVEN HIGH-T_c SUPERCONDUCTIVITY

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Though commonly unrecognized, a superconducting BCS condensate consists of equal numbers of two-electron (2e) and two-hole (2h) Cooper pairs (CPs). A new *complete* (in the sense that 2h-CPs are not ignored) boson-fermion statistical ternary gas model, however, is able to depart from this perfect 2e-/2h-CP symmetry. It yields robustly higher T_c's without abandoning electron-phonon dynamics, and reduces to all the known statistical theories of superconductors, including the BCS-Bose "crossover" picture—but goes considerably beyond it.

Keywords: Boson-fermion statistical model; BCS-Bose crossover theory; Bose-Einstein condensation.

1. Introduction

Boson-fermion (BF) models of superconductivity (SC) as a Bose-Einstein condensation (BEC) go back to the mid-1950's,[1–4] pre-dating even the BCS-Bogoliubov theory.[5–7] Although BCS theory only contemplates the presence of "Cooper correlations" of single-particle states, BF models[1–4,8–16] posit the existence of actual bosonic CPs. Indeed, CPs appear to be universally accepted as the single most important ingredient of SC, whether conventional or "exotic" and whether of low- or high-transition-temperatures T_c. In spite of their centrality, however, they are poorly understood. The fundamental drawback of early[1–4] BF models, which took 2e bosons as analogous to diatomic molecules in a classical atom-molecule gas mixture, is the notorious absence of an electron energy gap $\Delta(T)$. "Gapless" models cannot describe the superconducting state at all, although they are useful in locating transition temperatures if approached from above, i.e., $T > T_c$. Even so, we are not aware of any calculations with the early BF models attempting to reproduce any empirical T_c values. The gap first began to appear in later BF models.[8−13] With two[11,12] exceptions, moreover, all BF models neglect the effect of *hole* CPs included on an equal footing with electron CPs to give a "complete" BF model (CBFM) consisting of *both* bosonic CP species coexisting with unpaired electrons. Unfortunately, no experiment has yet been done, to our knowledge, that distinguishes between electron and hole CPs.

The "ordinary" CP problem[17] for two distinct interfermion interactions (the δ-well[18,19] or the Cooper/BCS model[5,17] interactions) neglects the effect of two-hole (2h) CPs treated on an equal footing with two-particle (2p) CPs—as Green's functions,[20] on the other hand, can naturally guarantee. A crucial observation[11,12] is that the BCS condensate consists of *equal numbers* of 2p and 2h CPs. This was already evident, though commonly unrecognized, from the perfect symmetry about $\epsilon = \mu$, the electron chemical potential, of the well-known Bogoliubov[22] $v^2(\epsilon)$ and $u^2(\epsilon)$ coefficients, where ϵ is the electron energy. The CBFM "unifies"[13] both BCS and BEC theories as special cases, and predicts substantially higher T_c's than BCS theory without abandoning electron-phonon dynamics.

2. Bethe-Salpeter treatment of Cooper pairing

A Bethe-Salpeter (BS) many-body equation (in the ladder approximation) treating both 2p and 2h pairs on an equal footing reveals that, while the ordinary CP problem [based on an ideal Fermi gas (IFG) ground state (the usual "Fermi sea")] does *not* possess stable energy solutions, it does so when the IFG ground state is replaced by the BCS one. This is equivalent to starting from an unperturbed Hamiltonian that is the BCS ground state instead of the pure-kinetic-energy operator corresponding to the IFG. The remaining Hamiltonian terms are then assumed suitable to a perturbation treatment. Consequently, i) CPs based not on the IFG-sea but on the BCS ground state survive in a *nontrivial* solution as "generalized" or "moving" CPs which are *positive*-energy resonances with an imaginary energy term leading to finite-lifetime effects; ii) as in the "ordinary" CP problem, their dispersion relation in leading order in the total (or center-of-mass) momentum (CMM) $\hbar\mathbf{K} \equiv \hbar(\mathbf{k}_1 + \mathbf{k}_2)$ is also *linear*[21] rather than the quadratic $\hbar^2 K^2/2(2m)$ of a composite boson (e.g., a deuteron) of mass $2m$ moving not in the Fermi sea but in vacuum; and iii) this latter "moving CP" solution, though often confused with it, is physically *distinct* from another more common *trivial* solution sometimes called—even though Bogoliubov[6] was the first to derive it—the Anderson-Bogoliubov-Higgs (ABH)[26,27] (and Ref. 7 p. 44) collective excitation. The ABH mode is also linear in leading order and goes over into the IFG ordinary sound mode in zero coupling. All this occurs in both 2D[28] as well as in the 3D study outlined earlier in Ref. 29. We focus here on 2D because of its interest[30] for quasi-2D high-T_c cuprate superconductors. In general, the results will be crucial for BEC scenarios employing BF models of superconductivity, not only *in exactly 2D* as with the Berezinskii-Kosterlitz-Thouless (BKT)[31,32] transition, but also down to $(1+\epsilon)$D which characterize the quasi-1D organo-metallic (Bechgaard salt) superconductors.[33−35]

If $\mathbf{K} \equiv \mathbf{k}_1+\mathbf{k}_2$ is the CMM and $\mathbf{k} \equiv \frac{1}{2}(\mathbf{k}_1-\mathbf{k}_2)$ the relative wavevectors of the 2e bound state, and $\mathcal{E}_K \equiv E_1+E_2$ is its energy with $E_1$and E_2 the energies of particles 1 and 2, one uses the bare one-fermion Green's functions $G_0\left(\mathbf{K}/2 \pm \mathbf{k}, \mathcal{E}_K/2 \pm E\right)$ for particles 1 and 2, respectively, where $E \equiv \frac{1}{2}(E_1 - E_2)$. The solution of the *complete* BS equation wrt the IFG unperturbed state with *both* 2e- and 2p-CPs included is

$$\mathcal{E}_0 = \pm i2\hbar\omega_D/\sqrt{e^{2/\lambda}-1} \quad \underset{\lambda\to 0}{\longrightarrow} \quad \pm i2\hbar w_D e^{-1/\lambda}. \tag{1}$$

As the CP energy is pure-imaginary there is an obvious instability of the CP problem when both type pairs are included. This was actually reported in Refs. 7 (p. 44) and 36 and contrasts sharply with the familiar solution[17] for 2e-CPs only, $\mathcal{E}_0 = -2\hbar\omega_D/(e^{2/\lambda}-1) \underset{\lambda\to 0}{\longrightarrow} -2\hbar\omega_D e^{-2/\lambda}$ which is exact in 2D and to a very good approximation otherwise if $\hbar\omega_D \ll E_F$, where ω_D is the Debye frequency and $\lambda \equiv VN(E_F)$ with $N(E_F)$ the electronic density of states (DOS) for one spin. This corresponds to a negative-energy, stationary-state (i.e., infinite-lifetime) bound pair. Clearly then, the original CP picture *is meaningless if 2e- and 2h-CPs are treated on an equal footing*, as consistency demands.

However, a BS treatment not about the IFG sea but about the BCS ground state *vindicates the CP concept* as a nontrivial solution. This is equivalent to starting not from the IFG unperturbed Hamiltonian but from the BCS one. Its physical justification lies in recovering three expected items: a) the (trivial) ABH sound mode; b) the BCS $T=0$ gap equation; and c) *finite*-lifetime effects of the "moving CPs" in both 2D[28] or 3D[29]. Thus, e.g., the IFG Green function $G_0\left(\mathbf{K}/2+\mathbf{k}, \mathcal{E}_K/2+E\right) \equiv G_0(\mathbf{k}_1, E_1)$ is replaced by the BCS one $\mathbf{G}_0(\mathbf{k}_1, E_1)$ that contains the energy $E_{\mathbf{k_1}} \equiv \sqrt{{\xi_{k_1}}^2+\Delta^2}$ with $\xi_{k_1} \equiv \hbar^2 k_1^2/2m - E_F$ and Δ the $T=0$ fermionic gap, as well as the Bogoliubov functions[37] $v_{k_1}^2 \equiv \frac{1}{2}(1-\xi_{k_1}/E_{\mathbf{k_1}})$ and $u_{k_1}^2 \equiv 1 - v_{k_1}^2$. There are *two* solutions. 1) The trivial solution is the ABH energy $\mathcal{E}_K$ equation Taylor-expanded about $K=0$ and small λ which in 2D is

$$\mathcal{E}_K = \frac{\hbar v_F}{\sqrt{2}}K + O(K^2) + o(\lambda), \tag{2}$$

where $o(\lambda)$ denote interfermion interaction correction terms that vanish as $\lambda \to 0$. Note that the leading term is just the ordinary sound mode in an IFG with sound speed $v_F/\sqrt{d}$ in d dimensions. 2) The nontrivial *moving CP* solution of the BCS-ground-state-based BS treatment, which is *entirely new*, leads to the pair energy $\mathcal{E}_K$ which in 2D is

$$\pm\mathcal{E}_K = 2\Delta + \frac{\lambda}{2\pi}\hbar v_F K + \frac{1}{9}\frac{\hbar v_F}{k_D}e^{1/\lambda}K^2 - i\left[\frac{\lambda}{\pi}\hbar v_F K + \frac{1}{12}\frac{\hbar v_F}{k_D}e^{1/\lambda}K^2\right] + O(K^3) \tag{3}$$

where the upper and lower signs refer to 2p- and 2h-CPs, respectively, and $k_D \equiv \omega_D/v_F$ with ω_D the Debye frequency. A linear dispersion in leading order again appears, but now associated with the bosonic moving CP. From (3) the *positive*-energy 2p-CP resonance has a width Γ_K and a lifetime $\tau_K \equiv \hbar/2\Gamma_K = \hbar/2\left[(\lambda/\pi)\hbar v_F K + (\hbar v_F/12k_D)e^{1/\lambda}K^2\right]$ that diverges at $K = 0$, falling to zero as K increases. Thus, "faster" moving CPs are shorter-lived and eventually break up, while "non-moving" $K = 0$ ones are in infinite-lifetime stationary states.

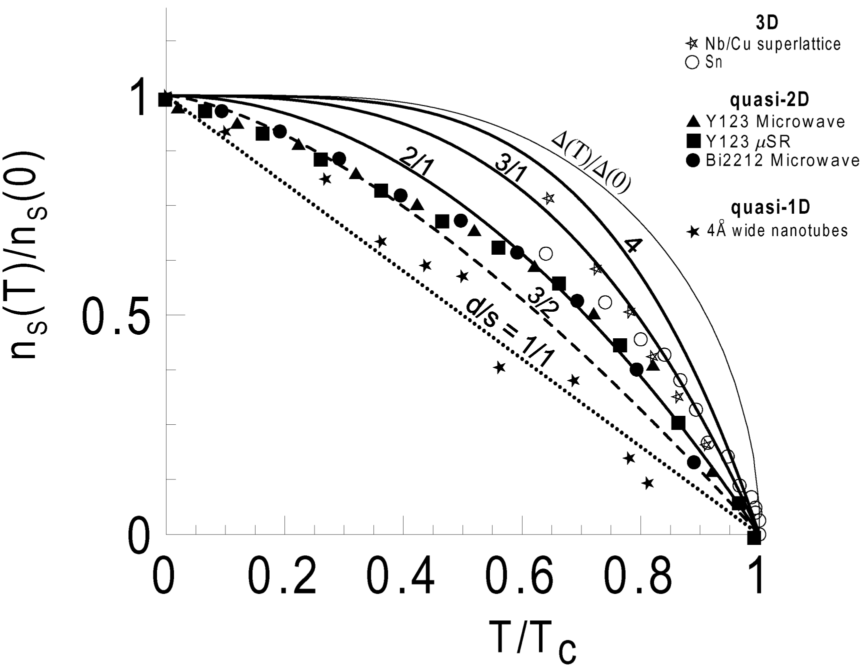

Fig. 1. BE condensate-fraction curves $1-(T/T_c)^{d/s}$ for bosons in $d = 3$, 2, or 1 with dispersion relation $\eta \simeq C_s K^s$ with $s = 2$ or 1, for a *pure* phase of either 2e- or 2h-CPs as discussed in text, compared to empirical data for 3D SCs (Nb/Cu and Sn), two quasi-2D SCs (Y123 and Bi2212) and a quasi-1D SC (4 A-wide nanotubes). Data for the latter are for $\Delta(T)/\Delta(0)$ but are plotted as $[\Delta(T)/\Delta(0)]^2$ so as to reflect 2h-CP condensate fraction $m_0(T)/m_0(0)$ according to (14). The curve marked 1/1 strictly corresponds to $T_c \equiv 0$; however, it serves as a lower bound to all curves with $d/s = (1+\epsilon)/1$ for small but nonzero ϵ for which $T_c > 0$. The ordinate axis is labeled with the superelectron number density $n_s(T)$ in units of the normal electron density n. Also shown, for reference, are the two-fluid model[46] curve $1-(T/T_c)^4$ and the BCS gap $\Delta(T)/\Delta(0)$ order parameter.[47]

Empirical evidence for the *linearly-dispersive* nature of Cooper pairs in cuprates has been argued by Wilson[38] to be suggested by the scanning tunneling microscope conductance scattering data in BSCCO. More direct evidence is shown[39] in Fig. 1 via experimental data (mostly from penetration-depth measurements) for two 3D

SCs,[40,41] two quasi-2D cuprates,[42−44] and a quasi-1D SC.[45] The data are seen to agree quite well, at least for $T \gtrsim 0.5T_c$, with the *pure-phase* (only 2e- *or* 2h-CP) BE condensate fraction formula $1-(T/T_c)^{d/s}$ for $d=3$, 2 and 1, respectively, *provided one assumes* $s=1$. For lower T's, one can argue[12] that since a *mixed* BEC phase containing both 2e- and 2h-CPs becomes more stable (i.e., has lower Helmholtz free energy), the simple pure-phase formula $1-(T/T_c)^{d/s}$ is no longer strictly valid. These remarks apply only to the boson component of the BF mixture while the unpaired-electron background plays a *passive* role as in Cooper's[17] original treatment of CPs. Thus, BF models assuming this CP linearity for the boson component [instead of the quadratic $\hbar^2K^2/2(2m)$ assumed in Refs. 1, 9-12 among many others] can give BEC for all $d>1$, including exactly 2D without the need to invoke the BKT transition. Such BF models can then in principle address not only quasi-2D cuprate but also quasi-1D organo-metallic superconductors.[33−35]

3. Complete Boson-Fermion Model (CBFM)

The CBFM[11,12] is described in d dimensions by the Hamiltonian $H=H_0+H_{int}$. The unperturbed Hamiltonian H_0 corresponds to a non-Fermi-liquid "normal" state which is an *ideal* (i.e., noninteracting) ternary gas mixture of unpaired fermions and both types of CPs, two-electron (2e) and two-hole (2h), and is

$$H_0=\sum_{\mathbf{k}_1,s_1}\epsilon_{\mathbf{k_1}}a^+_{\mathbf{k}_1,s_1}a_{\mathbf{k}_1,s_1}+\sum_{\mathbf{K}}E_+(K)b^+_{\mathbf{K}}b_{\mathbf{K}}-\sum_{\mathbf{K}}E_-(K)c^+_{\mathbf{K}}c_{\mathbf{K}} \tag{4}$$

where as before $\mathbf{K}\equiv\mathbf{k_1}+\mathbf{k_2}$ is the CP CMM wavevector while $\epsilon_{\mathbf{k_1}}\equiv\hbar^2k_1^2/2m$ are the single-electron, and $E_\pm(K)$ the 2e-/2h-CP *phenomenological,* energies. Here $a^+_{\mathbf{k}_1,s_1}$ ($a_{\mathbf{k}_1,s_1}$) are creation (annihilation) operators for fermions and similarly $b^+_{\mathbf{K}}$ ($b_{\mathbf{K}}$) and $c^+_{\mathbf{K}}$ ($c_{\mathbf{K}}$) for 2e- and 2h-CP bosons, respectively. Two-hole CPs are postulated to be *distinct* and *kinematically independent* from 2e-CPs, all of which provides a *ternary* BF gas mixture. In a sense, our H_0 "explains" magnetic-flux quantization experiments[48] that establish CP charge carriers along with unpaired electron ones.

The interaction Hamiltonian H_{int} consists of four distinct BF interaction vertices each with two-fermion/one-boson creation or annihilation operators, depicting how unpaired electrons (subindex $+$) [or holes (subindex $-$)] combine to form the 2e- (and 2h-) CPs assumed in the d-dimensional system of size L, namely

$$H_{int}=L^{-d/2}\sum_{\mathbf{k},\mathbf{K}}f_+(k)\{a^+_{\mathbf{k}+\frac{1}{2}\mathbf{K},\uparrow}a^+_{-\mathbf{k}+\frac{1}{2}\mathbf{K},\downarrow}b_{\mathbf{K}}+a_{-\mathbf{k}+\frac{1}{2}\mathbf{K},\downarrow}a_{\mathbf{k}+\frac{1}{2}\mathbf{K},\uparrow}b^+_{\mathbf{K}}\}$$

$$+L^{-d/2}\sum_{\mathbf{k},\mathbf{K}}f_-(k)\{a^+_{\mathbf{k}+\frac{1}{2}\mathbf{K},\uparrow}a^+_{-\mathbf{k}+\frac{1}{2}\mathbf{K},\downarrow}c^+_{\mathbf{K}}+a_{-\mathbf{k}+\frac{1}{2}\mathbf{K},\downarrow}a_{\mathbf{k}+\frac{1}{2}\mathbf{K},\uparrow}c_{\mathbf{K}}\} \tag{5}$$

where $\mathbf{k}\equiv\frac{1}{2}(\mathbf{k_1}-\mathbf{k_2})$ is the relative wavevector of a CP. The energy form factors $f_\pm(k)$ in (5) are essentially the Fourier transforms of the 2e- and 2h-CP intrinsic wavefunctions, respectively, in the relative coordinate of the two fermions. In Refs. 11 and 12 they are taken as

$$f_\pm(\epsilon) = \begin{cases} f & \text{if} \quad \frac{1}{2}[E_\pm(0) - \delta\varepsilon] < \epsilon < \frac{1}{2}[E_\pm(0) + \delta\varepsilon] \\ 0 & \text{otherwise.} \end{cases} \tag{6}$$

We now introduce the quantities E_f and $\delta\varepsilon$ as *new* phenomenological dynamical energy parameters (in addition to the positive BF vertex coupling parameter f) that replace the previous such $E_\pm(0)$, through the definitions

$$E_f \equiv \tfrac{1}{4}[E_+(0) + E_-(0)] \quad \text{and} \quad \delta\varepsilon \equiv \tfrac{1}{2}[E_+(0) - E_-(0)], \tag{7}$$

where $E_\pm(0)$ are the (empirically *un*known) zero-CMM energies of the 2e- and 2h-CPs, respectively. Clearly

$$E_\pm(0) = 2E_f \pm \delta\varepsilon. \tag{8}$$

The quantity E_f serves as a convenient energy scale; it is not to be confused with the Fermi energy $E_F = \frac{1}{2}mv_F^2 \equiv k_B T_F$ where T_F is the Fermi temperature. The Fermi energy E_F equals $\pi\hbar^2 n/m$ in 2D and $(\hbar^2/2m)(3\pi^2 n)^{2/3}$ in 3D, with n the total number-density of charge-carrier electrons, while E_f is the same with n replaced by, say, n_f. The quantities E_f and E_F coincide *only* when perfect 2e/2h-CP symmetry holds, i.e. when $n = n_f$; see below (14).

The interaction Hamiltonian (5) can be further simplified by dropping all $\mathbf{K} \neq 0$ terms, as is done in BCS theory. Constructing the grand potential

$$\Omega(T, L^d, \mu, N_0, M_0) \; = -k_B T \ln\left[\mathrm{Tr} e^{-\beta(H - \mu\hat{N})}\right] \tag{9}$$

where "Tr" stands for "trace," minimizing with respect to N_0 (the number of zero-CMM 2e-CPs) and M_0 (the same of 2h-CPs), and simultaneously fixing the total number N of electrons by introducing the electron chemical potential μ, one can specify an *equilibrium state* of the system with volume L^d and temperature T by requiring that

$$\frac{\partial\Omega}{\partial N_0} \; = \; 0, \qquad \frac{\partial\Omega}{\partial M_0} \; = \; 0, \quad \text{and} \quad \frac{\partial\Omega}{\partial\mu} \; = \; -N. \tag{10}$$

Here N evidently includes both paired and unpaired CP electrons. Some algebra then leads to the three coupled integral Eqs. (6)–(8) of Ref. 11. The relation between the fermion spectrum $E(\epsilon)$ and fermion energy gap $\Delta(\epsilon)$ turns out to be BCS-like, i.e.,

$$E(\epsilon) = \sqrt{(\epsilon - \mu)^2 + \Delta^2(\epsilon)} \quad \text{but where} \quad \Delta(\epsilon) \equiv \sqrt{n_0} f_+(\epsilon) + \sqrt{m_0} f_-(\epsilon). \tag{11}$$

This last expression for the gap $\Delta(\epsilon)$ implies a simple T-dependence rooted in the 2e-CP $n_0(T) \equiv N_0(T)/L^d$ and 2h-CP $m_0(T) \equiv M_0(T)/L^d$ number densities of BE-condensed bosons, i.e.,

$$\Delta(T) = \sqrt{n_0(T)} f_+(\epsilon) + \sqrt{m_0(T)} f_-(\epsilon). \tag{12}$$

Self-consistent (at worst, numerical) solution of the aforementioned *three coupled equations* then yields the three thermodynamic variables of the CBFM

$$n_0(T, n, \mu), \quad m_0(T, n, \mu), \quad \text{and} \quad \mu(T, n). \tag{13}$$

Most significantly, the three CBFM equations contain the key equations of *five* different statistical theories as special cases. Perfect 2e/2h CP symmetry signifies equal number of 2e- and 2h-CPs, i.e., $n_B(T) = m_B(T)$ *as well as* $n_0(T) = m_0(T)$. With (8) this implies that E_f coincides with μ, and the CBFM then reduces to:

i) the gap and number equations of the *BCS-Bose crossover theory* for the BCS model interaction—if the BCS parameters V and Debye energy $\hbar\omega_D$ are identified with the BF interaction Hamiltonian H_{int} parameters $f^2/2\delta\varepsilon$ and $\delta\varepsilon$, respectively. The crossover picture for unknowns $\Delta(T)$ and $\mu(T)$ is now supplemented by the key relation

$$\Delta(T) = f\sqrt{n_0(T)} = f\sqrt{m_0(T)}. \tag{14}$$

The crossover picture is associated with many authors beginning in 1967 with Friedel and coauthors[49]; for a review see Ref. 50. However, it is widely unrecognized to be a very modest improvement, at least for the Cooper/BCS model interaction, over BCS theory *per se* since an unphysically large λ of about 8 is required to bring $\mu(T_c)/E_F$ down from 1.00 to 0.998; indeed, T_c-values differ very slightly[51] all the way up to $\lambda \sim 50$ when the Fermi surface pinned at μ disappears so that the model interaction breaks down. If one imposes that $\mu(T_c) = E_F$ exactly, as follows from the number equation for weak BF coupling f, the crossover picture is well-known to reduce to:

ii) *ordinary BCS theory* which is characterized by a *single* equation, the gap equation for any T. Thus, *the BCS condensate is precisely a BE condensate* whenever both $n_B(T) = m_B(T)$ and $n_0(T) = m_0(T)$ *and* the BF coupling f is small. For small coupling λ the CBFM $T = 0$ superconducting state has the *same* condensation energy in lowest order in λ as the BCS state, and in fact *lower* in next-to-lowest order.[12,52]

On the other hand, for no 2h-CPs present the CBFM reduces[11] also to:

iii) the *BEC BF model* in 3D of Friedberg and Lee[9,10] characterized by the relation $\Delta(T) = f\sqrt{n_0(T)}$, first reported in Ref. 8; but lacking 2h-CPs this model cannot be fully related to BCS theory. When $f = 0$ it reduces to:

iv) the *ideal BF model* (IBFM) of Refs. 15 and 16 that predicts nonzero 2e-CP BEC T_c's even in 2D. The "gapless" IBFM cannot describe the superconducting phase. But considered as a model for the *normal state* it should provide feasible T_c's as singularities within a BE scenario that are approached from *above* T_c, and this seems to be indeed[16] the case. Finally, in 3D this model in the limit of no unpaired electrons reduces to:

v) the familiar T_c-formula of ordinary BEC in 3D.

The very general CBFM has been applied in both 2D and 3D and gives sizeable enhancements in T_c's over BCS theory that emerge for moderate departures from perfect 2e/2h-pair symmetry. This is attained for the *same* Cooper/BCS interaction model (coupling strength λ and cutoff $\hbar\omega_D$) parameters often used in conventional superconductors. The three coupled equations of the CBFM that determine the d-dimensional BE-condensate number-densities $n_0(T)$ and $m_0(T)$ of 2e- and 2h-CPs, respectively, as well as the electron chemical potential $\mu(T)$, were solved numerically. At $n/n_f = 1$ one has perfect 2e/2h-CP symmetry; n_f can be seen[12] to be the number $n_f(T)$ of unpaired electrons per unit area when $\Delta = 0$ and $T = 0$. The third, or "complete" number, equation then explicitly reads

$$2n_0(T) + 2n_{B+}(T) - 2m_0(T) - 2m_{B+}(T) + n_f(T) = n \tag{15}$$

with $m_{B+}(T)$, e.g., being precisely the number of "preformed" $K > 0$ 2h-CPs. Besides the *normal* phase (n) consisting of the ideal BF ternary gas described by H_0, three different stable BEC phases emerge, see Fig. 2.

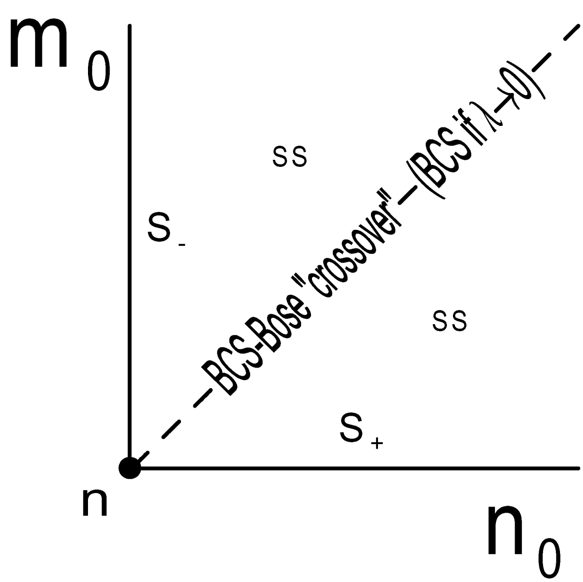

Fig. 2. Illustration on the n_0-m_0 plane of three CBFM condensed phases (the pure 2e-CP and 2h-CP BE condensate phases $s+$ and $s-$ existing only along the horizontal ($m_0 = 0$) and vertical (n_0) axes, respectively, and the mixed phase ss) along with the normal ternary BF non-Fermi-liquid phase n which is a single point at the origen.

We next focus on $s = 1$ which occurs in the leading term for "ordinary" CPs in a Fermi sea as well as for "generalized" CPs in a BCS state. For the latter, the boson excitation energy η to be used has a leading term in the many-body Bethe-Salpeter (BS) CP dispersion relation given by $\eta \simeq (\lambda/2\pi)\hbar v_F K$ in 2D.[28] As before, $\lambda \equiv VN(E_F)$ where $N(E_F)$ is the electron DOS (for one spin) at the Fermi surface. Note that η is *not* the quadratic $\hbar^2K^2/2(2m)$ appropriate for a composite boson of

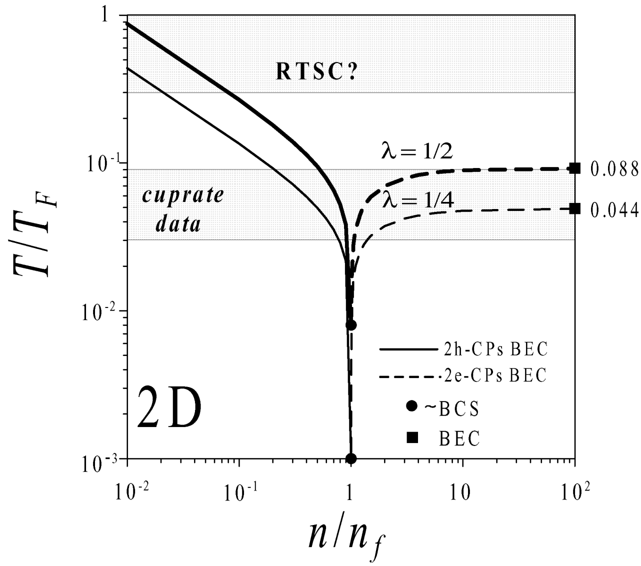

Fig. 3. Phase diagram in 2D temperature T (in units of T_F) and electron density n (in units of n_f as defined in text) showing the phase boundaries of T_c's for pure 2e-CP BEC phases (dashed curves) determined by $\Delta(T_c) = f\sqrt{n_0(T_c)} \equiv 0$ and pure 2h-CP BEC phases (full curves) given by $\Delta(T_c) = f\sqrt{m_0(T_c)} \equiv 0$ for $\lambda = 1/4$ and $1/2$ with $\hbar\omega_D/E_F = 0.05$. Intersections corresponding to $n_0(T) = m_0(T)$ approximately reproduce the BCS T_c as given by (14) and are marked by black dots. Black squares mark the exact BEC limit where all electrons are imagined paired into 2e-CP bosons.

mass $2m$ moving not in the Fermi sea but in vacuum.[1−4,9−12] In 2D the electronic DOS per unit area L^2 is constant, namely $N(\varepsilon) = m/2\pi\hbar^2$. Using $\eta \simeq (\lambda/2\pi)\hbar v_F K$ we obtain for the bosonic DOS

$$M(\eta) \equiv (1/2\pi)K(dK/d\eta) \simeq (2\pi/\lambda^2\hbar^2 v_F^2)\eta \tag{16}$$

instead of the constant that follows in 2D from the quadratic dispersion $\eta = \hbar^2K^2/2(2m)$. One uses $E_f \equiv \pi\hbar^2 n_f/m = k_BT_f$ as energy/density/temperature scaling factors, and the relation $n/n_f = (E_F/E_f)^{d/2}$, to convert quantities such as T_c/T_f to T_c/T_F, where $E_F \equiv k_BT_F$. We took the two values of $\lambda = 1/4$ (lower set of curves in Fig. 3) and $= 1/2$ (upper set of curves), and $\hbar\omega_D/E_F = 0.05$ (a typical value for cuprates). If $\lambda > 1/2$ the ionic lattice in 3D becomes unstable,[53] and Ref. 1 (p. 204). Solving two coupled equations at a time leads to the T_c/T_F *vs.* n/n_f phase diagram of Fig. 3 for both 2e-CP (dashed curve) and 2h-CP (full curve) pure, stable BEC-like phases. The value $n/n_f = 1$ corresponds to perfect 2e/2h-CP symmetry for the lower-T_c mixed phase (not shown as it occurs *below* the BCS T_c's), while $n/n_f > 1$ (and < 1) signifies more (less) 2e-CPs than 2h-CPs in the mixed phase. The T_c value where both phase-boundary curves $n_0(T_c) = m_0(T_c)$ $= 0$ intersect is marked by the large dots in the figure. These values are consistent with those gotten from the BCS formula $T_c/T_F \simeq 1.134(\hbar\omega_D/E_F)\exp(-1/\lambda)$ which gives $\simeq 0.001$ for $\lambda = 1/4$ and $\simeq 0.008$ for $\lambda = 1/2$, for $\hbar\omega_D/E_F = 0.05$. The black squares in Fig. 3 (upper for $\lambda = 1/2$ and lower for $\lambda = 1/4$) mark the exact BEC analytic values[13] if *all* electrons in our 2D or 3D many-electron system were imagined paired into noninteracting bosons formed with the Cooper/BCS model interelectron interaction. Cuprate data empirically[54] show T_c's and T_F's falling within the range $T_c/T_F \simeq 0.03 - 0.09$. Thus, moderate departures from perfect 2e/2h-CP symmetry enable the CBFM to predict quasi-2D cuprate empirical T_c values, *without abandoning electron-phonon dynamics*—contrary to popular belief. Finally, from Fig. 3 we note that *room temperature superconductivity* (RTSC in figure) is possible but only via 2h-CP BE condensates.

Lastly, we address the unique but mysterious role played by *hole* charge carriers in the normal state of superconductors in general. Chapnik[23] has hypothesized that hole (electron) charge carriers are associated with superconductors (non-superconductors). Indeed, a) of the cuprates, those that are hole-doped have transition temperatures T_c about *six* times higher than electron-doped ones; and b) even in conventional superconductors[24] over 80% of all superconducting elements have positive Hall coefficients (meaning hole charge carriers); while c) over 90% of non-superconducting metallic, non-magnetic elements have electron charge carriers. This greater "efficiency" of individual, unpaired hole carriers in producing higher T_c's is clearly reflected in the "skewed" right-left appearance in Fig. 3 for 2D superconductors—at least insofar as pure 2h-CP BE condensates exhibiting higher T_c's than those associated with pure 2e-CP BE condensates.

4. Conclusions

In conclusion, the new "complete boson-fermion model" (CBFM) includes as limiting cases the following theories: i) BCS and ii) BCS-Bose "crossover" when the BE condensate consists of equal numbers of electron- and hole-pairs, as well (when *no* hole pairs are present) as iii) the Friedberg-Lee BEC model, iv) the "ideal boson-fermion model" (IBFM) and v) BEC theory, when there are no unpaired electrons. The BCS condensate is precisely a BE condensate of equal numbers of 2e/2h-pairs and weak coupling. Cooper pairs are meaningless if defined wrt the ideal Fermi gas "sea," but when defined wrt the BCS ground-state sea survive as positive-energy, finite-lifetime plasmon-like objects with a linear (instead of quadratic) rise in total, or center-of-mass, momentum K. Without abandoning electron-phonon dynamics the CBFM leads to 2-to-3 order-of-magnitude higher T_c's—including room-temperature superconductivity but only via hole-pairs. The CBFM also suggests an "explanation" for the empirical skewed role of holes in producing higher T_c's. A specific recipe for high T_c's would be to engineer superconductors to have many more 2h-CPs than 2e-CPs at T_c^+.

All this rests on four essential ingredients: 1) 2h-CPs cannot and must not be neglected in a fully self-consistent treatment in any many-*fermion* system, otherwise a spurious value of T_c may[55] result; 2) CPs are *bosons*, even though BCS pairs not; 3) CPs are *linearly-dispersive* for small K; 4) to achieve higher T_c's one must depart from the perfect 2e-/2h-CP symmetry of the BE condensate of BCS. Left out thus far, however, are: a) $K > 0$ terms in the boson-fermion vertex interactions; b) boson-boson interactions (as is BCS); c) a $T > 0$ Bethe-Salpeter CP treatment; d) different hole and electron effective masses; and e) ionic-lattice crystallinity effects which might initially be dealt with via Van Hove singularities introduced[56] in the electronic DOS or, ultimately, via "bipolarons"[57] replacing CPs.

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