Velocity fluctuations resulting from the interaction of a bubble with a vertical wall

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The interaction and motion of a spherical bubble with a vertical wall moving in an ideal fluid are analyzed. This case is representative of a bubble that moves at a high Reynolds number and has a small Weber number. Using potential flow theory, the velocity potential is obtained using a double expansion in spherical harmonics. The presence of the wall is approximated by considering the motion of a bubble and its virtual image. The motion equations, obtained considering an energy conservation argument, are solved numerically for a range of initial conditions. The mean and variance of the bubble velocity are calculated. In particular, the bubble velocity variance is obtained as a measure of the bubble velocity fluctuations resulting from the interaction with the wall. The predictions of this model are in good qualitative agreement with some recent experimental results. © *2005 American Institute of Physics*. [DOI: 10.1063/1.2074707]

When a gas bubble moves in a clean liquid, free of surfactants, of relatively small viscosity, its motion and the motion of the surrounding fluid can be modeled using potential theory. In particular, if the Reynolds number is high and the Weber number is small, the agreement between theoretical predictions and experiments is very good.[1] The Reynolds number is defined as $\mathrm{Re}=U_b d_b/\nu$, where ν is the fluid kinematic viscosity, and U_b and d_b are the velocity and diameter of the bubble, respectively. The Weber number is defined as $\mathrm{We}=\rho U_b^2 d_b/\sigma$, where σ is the surface tension of the liquid-gas interface and ρ is the density of the fluid. Air bubbles of 1–2 mm in diameter moving in clean water satisfy this dual limit.

Based on this fact, many authors have proposed theories to describe the flow properties of bubbly liquids (see, for example, Ref. 2 and references therein). Although these theoretical attempts have been proved to qualitatively describe the behavior of monodispersed noncoalescing bubbly liquids,[3] improvements should be considered to achieve proper quantitative predictions. One of the limitations of these models is that the influence of the containing walls is not considered. Zenit *et al.*[3] identified this effect as one of the possible mechanisms responsible for the limited agreement between experimental and theoretical results. In particular, these authors identified that the mean rise velocity of bubbles ascending in a vertical column was somehow smaller than that observed for individual bubbles. They observed that the bubbles in the column interacted with each other but also significantly with the containing walls. There have been recent studies that analyze the behavior of a bubble moving near a wall, for the dual limit of high Re and low We numbers.[4,5] Both studies observed that, for this flow regime, the bubbles were attracted to the wall and collisions and rebounds occurred repeatedly. Figure 1 shows the motion of a small nitrogen bubble moving in clean water, in the vicinity of a vertical wall (obtained from Ref. 4). It can be clearly observed that the bubble is attracted towards the wall and repeatedly collides with it. During the collision, the mean velocity of the bubble is reduced. After a short time, the bubble continues to move away from the wall, but it is attracted to it again. The hydrodynamic interactions between pairs of bubbles were recently reported by Legendre *et al.*[6] They found that the direction of the force between two bubbles changes direction from repulsion to attraction as the bubble Reynolds number increases.

In this paper the effect of a wall on the motion of an individual bubble using potential flow theory is explored. A system of two identical bubbles moving side by side at the same velocity is considered to simulate the presence of a vertical wall. The movement of a pair of bubbles in potential flow has been investigated by several authors.[7–10] In particular, we followed closely the work of Kok[9] and Kumaran[10] to implement the problem of our interest. In this manner, the wall-induced fluctuations are calculated for an idealized system. Clearly the predictions obtained from potential flow theories are of limited *use*; however, no other reliable models that include viscous and wake effects are yet available.

Following the framework first proposed by Biesheuvel and van Wijngaarden,[8] Kok[9] and Kumaran[10] modeled the case of different sized bubbles moving in arbitrary directions in a perfect irrotational fluid. The velocity potential for the bubble pair can be obtained using a double expansion in spherical harmonics considering the size and velocity of each bubble and the separation between them. The coefficients of the spherical harmonics are obtained considering the boundary conditions over the bubbles' surfaces. Once the potential function is obtained, the equation of motion of the bubble pair can be obtained in two different ways. The first one, used by Kok,[9] consists in calculating the kinetic energy of the system and obtain the motion equation from Lagrange equations.[7] The second formulation, used by Kumaran,[10] calculates the pressure around the spheres, using Bernoulli's

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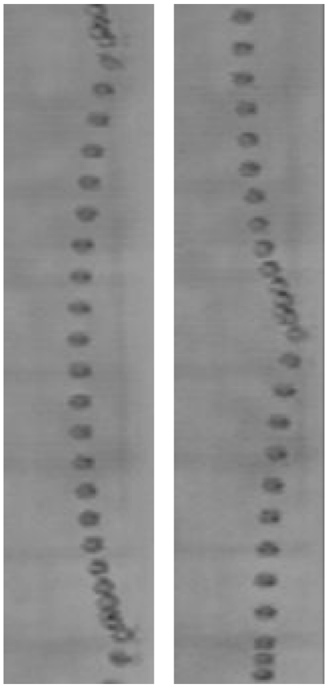

FIG. 1. Bubble moving near a wall. d_b=1.5 mm, Re=210, We=0.51, S/R=4.2. Two different experiments for the same nominal conditions are shown. The bubble image was captured in the same plate for different time instants. Taken from Lima (Ref. 4).

equation. Once the forces are known, a momentum balance equation can be written for each bubble considering the added mass for each bubble. We have verified that these two approaches give identical results.[11] The results presented here were obtained considering the conservation of energy approach. The kinetic energy of the fluid[7] is calculated, integrating the fluid motion around the bubble pair. Once the kinetic energy is known, Lagrange's equations[7] are used to obtain the equation of motion for each bubble.

As mentioned above, these equations were obtained for a general case both by Kok[9] and Kumaran.[10] Our interest is to use these equations to study the motion of a single bubble in the vicinity of a solid wall. The motion of two equal bubbles aligned horizontally, moving at the same vertical velocity, is considered. One bubble will be considered the *image* of the other one; the half distance between their centers will therefore be the distance to the wall. The case under consideration is shown in Fig. 2.

The simplified equations of motion are

$$\rho_l V_b \frac{d}{dt}\left[a_{11}\frac{dx}{dt}\right] = F_1, \tag{1}$$

$$\rho_l V_b \frac{d}{dt}\left[a_{33}\frac{ds}{dt}\right] = \frac{1}{2}\rho_l V_b\left[\dot{x}^2\frac{\partial}{\partial s}a_{11} + \dot{s}^2\frac{\partial}{\partial s}a_{33}\right] + F_3, \tag{2}$$

where V_b is the bubble volume, and $\dot{x}$ is the bubble velocity in the vertical direction. The half distance between the bubble and wall is s and $\dot{s}$ is velocity at which the bubble approaches the wall. F_1 and F_3 are the external forces on the bubble (drag, buoyancy, etc.). The coefficients a_{11} and a_{33} are calculated from the velocity potential of the bubble pair.

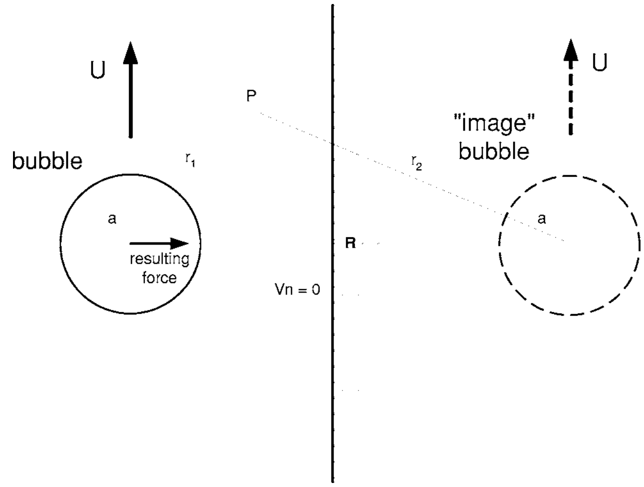

FIG. 2. A bubble moving parallel to a wall.

The following functional form for the coefficients, in powers of R/s, can be deduced:

$$a_{11} = -1 + \frac{3}{2}\left(\frac{R}{s}\right)^3 - \frac{3}{4}\left(\frac{R}{s}\right)^6 + \cdots, \tag{3}$$

$$a_{33} = \frac{1}{4} + \frac{3}{4}\left(\frac{R}{s}\right)^3 + \frac{3}{4}\left(\frac{R}{s}\right)^6 + \cdots, \tag{4}$$

where R is the bubble radius. The system of equations (1) and (2) can be solved for given initial conditions $[x(0),\dot{x}(0),s(0),\dot{s}(0)]$. External forces, F_1 and F_3, could be considered; however, for this analysis we are interested on the free motion of spheres in an ideal environment to only evaluate the inertial effects. A program was written in MATLAB© to solve this system. The program was written such that any number of terms could be used to calculate a_{11} and a_{33}.

The bubble is placed at $t=0$ at a certain distance away from the wall, $s(0)$, at $x(0)=0$. An arbitrary initial vertical velocity is also imposed $\dot{x}(0)=U_\infty$, with zero horizontal velocity, $\dot{s}(0)=0$. The system evolves with time; the position, velocity, and acceleration are calculated using a Runge-Kutta scheme for each time step. The time step is refined according to an optimization algorithm to ensure the accuracy of the integration.

The coefficients a_{11} and a_{33} could be calculated up any order of the power (R/s). The numerical accuracy of the coefficients was evaluated by calculating their values using an increasing number of terms. For instance, using 100 terms in the expansions results in a 0.65% difference compared to the calculations of Kok who used only five terms. To ensure a consistent accuracy, $N=20$ was chosen for all the results shown in this paper. A complete analysis of the dependence of the results on the number of terms, and an analysis of the convergence of the series as $s/R \rightarrow 1$, can be found in Ref. 11.

Typical results of the trajectory are shown in Fig. 3. Several initial conditions were tested, keeping the initial dimensionless distance fixed. The bubble moves initially upwards, but it is attracted to the wall. For the first case the

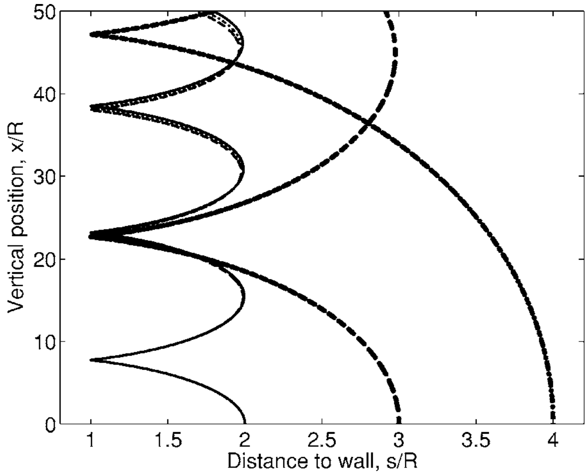

FIG. 3. Trajectories of a bubble interacting with a wall. First case (thin lines), different bubble diameters and initial velocities, initial distance to wall $s(0)/R=4$: solid line, $R=0.5$ mm, $U_\infty=0.25$ m/s; dashed line, $R=0.25$ mm, $U_\infty=0.25$ m/s; dash-dotted line, $R=0.7$ mm, $U_\infty=0.25$ m/s; dotted line, $R=0.7$ mm, $U_\infty=0.5$ m/s. Second case (thick lines), $R=1$ mm, $U_\infty=0.25$ m/s; dashed line, $s(0)/R=3$; dash-dotted line, $s(0)/R=4$.

initial separation is only $s/R=4$ but the first collision with the wall occurs at height of $x/R=7$, approximately. This attraction results from the hydrodynamics force that the fluid exerts on the bubble. It can also be observed that the bubble *collides* against the wall (as its center reaches a distance of $s/R=1$). To observe the evolution of the bubble trajectory for longer times, and repeated collisions, an artificial collision condition was imposed. When s/R reached 1.0001, an instantaneous elastic rebound was imposed: the direction of the horizontal velocity, $\dot{s}$, was inverted. After the *rebound* the simulation was allowed to continue. Since no dissipation mechanisms are considered, the bubble returns to its original horizontal position and recovers its initial velocity.

Clearly, the results presented in this dimensionless form are nearly the same despite the different initial conditions. For the case of the vertical and horizontal velocities, shown in Fig. 4, the results for the different cases are again collapsed onto approximately the same curve. As the bubble moves closer to the wall, the horizontal velocity increases; as a result, the vertical velocity decreases. At the moment when the bubble *touches* the wall, the collision condition is applied and the direction of motion of the bubble is reversed. The velocity reversal models the collision-rebound process in the simplest manner. Note also that at the moment of collision, the vertical bubble velocity reaches its minimum value of approximately 80% of the unperturbed value. A similar velocity reduction is observed in the experiments of single bubbles interacting with a vertical wall.[5] However, one must keep in mind that the experiments show a much more complex process during the collision.

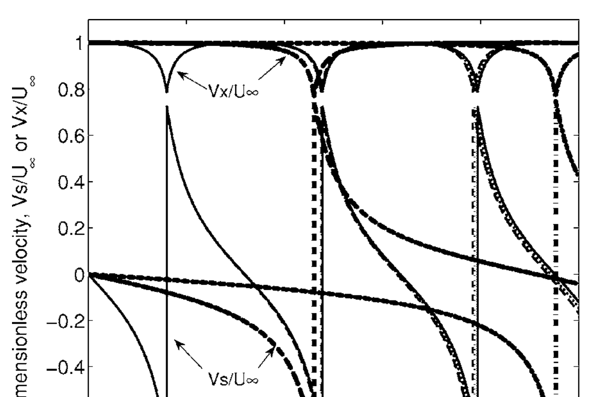

FIG. 4. Horizontal and vertical velocities as a function of time. The results for the same cases are shown in Fig. 3.

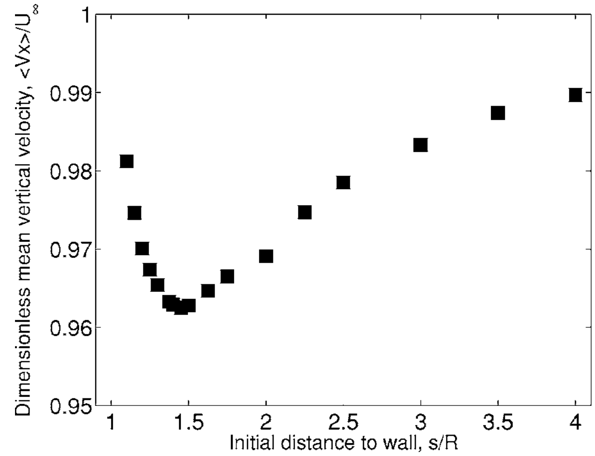

FIG. 5. Dimensionless mean vertical bubble velocity as a function of the normalized wall distance.

The results for different initial distances to the wall are also shown in Figs. 3 and 4 (thick lines). For all cases the same general behavior is observed: the bubble is attracted to the wall and its horizontal velocity increases as it moves towards the wall. The vertical velocity decreases reaching a minimum at the moment of collision, of the same magnitude for all initial distances. Clearly, as the distance from the wall increases the strength of the hydrodynamic force decreases; hence, the vertical distance at which the bubble collides with the wall increases with the initial distance from the wall. In other words, the bubble collision frequency decreases as the initial distance to the wall increases. For all cases, the initial position and velocity are recovered after the collisions with the wall.

To obtain a measure of the wall-induced velocity fluctuations the mean and variance of the bubble velocity were calculated. The mean vertical velocity, $\langle V_x \rangle$, and the standard deviation of the vertical and horizontal velocity components, $\sqrt{\langle V_x'^2 \rangle}$ and $\sqrt{\langle V_s'^2 \rangle}$, were obtained for a range of initial wall distances. Figure 5 shows the mean dimensionless vertical velocity as a function of the normalized initial bubble position. The mean vertical velocity shows an interesting nonmonotonic behavior. The curve shows a minimum value at a certain distance from the wall, approximately $s/R=1.45$. For this case, the maximum amount of velocity reduction is approximately 4%. For large distances the mean value approaches the unperturbed velocity, as expected. However, for

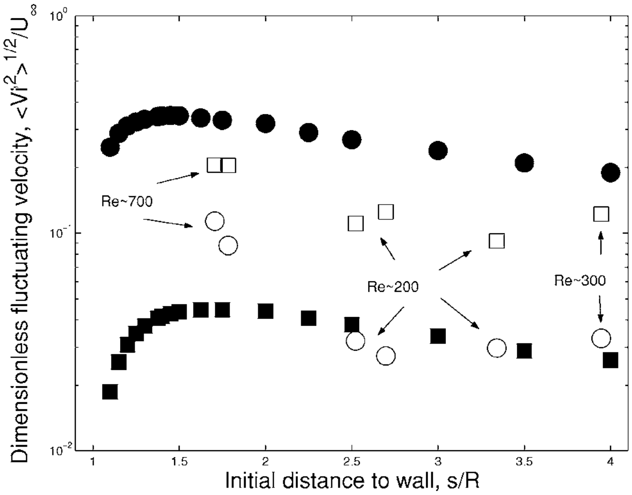

FIG. 6. Dimensionless fluctuating velocity as a function of the normalized wall distance: circles, horizontal component; squares, vertical component. The filled symbols show the model predictions. The empty symbols show the experimental measurements calculated from Ref. 5.

very small distances the velocity of the bubble appears to increase. As it can be seen in Figs. 3 and 4, the trajectory of the bubble changes with the initial distance. If the distance is large, the collision frequency is small. On the other hand, when the bubble is very near the wall, although the collision frequency is high, the deceleration caused by each collision is small; therefore, the mean velocity does not decrease. This effect can be clearly observed in Fig. 6, which presents a measure of the fluctuating components of the bubble velocity. The velocity fluctuations reach a maximum value at approximately the same distance where the mean vertical velocity shows a minimum value. Actually, experiments have shown that for such small velocities sliding on the wall, rather than repeated bouncing, is observed. Again, for such small initial distances the results of the model are not expected to give accurate results. Note also that the horizontal component is significantly larger than the vertical one.

A comparison with experimental results is possible. Lima[4] and DeVries[5] performed experiments to investigate the motion of small air bubbles near a vertical wall in water; however, the experiments by DeVries were performed with clean bubbles which behavior is expected to be closer to that predicted by potential flow theory. In Fig. 6, along with the model predictions, experimental measurements of the two components of the velocity fluctuations are also shown for several of the experimentally obtained bubble trajectories obtained by DeVries,[5] for different bubbles sizes, velocities, and initial distances to the wall. Only the experiments that showed repeated bouncing were considered. The experimental results have been grouped in three different ranges of Re number.

For all cases the vertical velocity fluctuations calculated with the model are under predicted by approximately a factor of 3. Clearly, the bubble velocity does not decrease as much as what is observed experimentally. However, the predicted fluctuations are of the same order of magnitude as the experimental measurements. On the other hand, the predicted horizontal fluctuations are approximately four times larger than those observed experimentally.

A comparison of the predicted reduction of the mean velocity with other experimental measurements can also be obtained. The experiments performed by DeVries[5] showed that the velocity of individual bubbles is reduced significantly (up to 50% of the terminal bubble velocity) during one collision event. The reduction calculated in this investigation was only of the order of 20%. This is a clear indication that the calculation ceases to be valid when the bubble is very near the wall.

In summary, this investigation presents the predictions of a model to calculate the motion of a clean spherical bubble near a wall at a high Re number. The results show that the inertial hydrodynamic interaction of a bubble with a wall results in an attractive force that makes the bubble to migrate towards the wall and collide with it repeatedly. In particular, these results are in relative agreement with experiments. With the model near-analytic predictions can be obtained for the bubble velocity fluctuation resulting from the wall interaction. Moreover, the comparison with experimental measurements clearly shows the limitations of the idealized model and demonstrates that viscous effects must be accounted for to properly predict wall-induced fluctuations. We hope that these calculations can be used to guide future more complete models. In order to fully calculate the bubble-wall interaction process, a complete Navier-Stokes solver would have to be implemented for the problem studied here. We intend to pursue this objective in the future.

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