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Computer simulations of the collapse of a granular column

Roberto Zenit[a)]
Instituto de Investigaciones en Materiales, Universidad Nacional Autónoma de México, Apdo. Postal 70-360, Ciudad Universitaria, Coyoacán D.F. 04510, México

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Recently, two independent groups reported experimental results on the process of collapse of a cylindrical granular column. It was found that the shape of the final deposit depended mostly on column aspect ratio; surprisingly, the frictional properties of the material appeared not to influence the results significantly. In this investigation, making use of discrete element code, simulations of an equivalent two-dimensional system were carried out. The numerical results qualitatively reproduce the behavior observed in experiments. Performing an energy balance of the system, the different deposit regimes can be discerned. © *2005 American Institute of Physics*. [DOI: 10.1063/1.1862240]

Interest in granular materials arises from the many practical applications and by their ubiquitous presence in natural phenomena. In recent years there has been a significant increase of the number of studies in this subject; however, a general agreement of the governing mechanics of granular flows has not been reached. Most investigations have focused on the steady flow behavior of granular matter; far less is understood about the unsteady behavior of these flows. In particular, the geophysics community has been interested in the behavior of large granular masses that slide and/or collapse under the action of gravity. Events such as landslides and debris flows, are most often studied *a posteriori*, since direct measurements are often impossible and dangerous. Researchers are forced to make claims from the analysis of the final deposits; hence, the dynamics of such flows are poorly understood. Studies on the behavior of granular matter, performed in controlled environments, could contribute to a better understanding of these complex geophysical events.

Recently, a new setup was proposed to study the unsteady behavior of a granular material. Two groups,[1,2] in a simultaneous manner, conducted experiments on a nearly identical configuration: a granular column, initially contained in a cylinder, was released from rest to collapse freely under the action of gravity. Both groups found that the shape of the final deposit changed with the ratio of height to radius, $a = H_o/R_o$, where H_o and R_o are the initial height and column radius, respectively. For short columns, or small a, the edges of the granular mass spread. A deposit with a central undisturbed area connected to a slope is formed for very small a; for slightly larger a values, the entire column erodes, leaving a conical pile. The inclination of the slope at the edge of the deposit is approximately the static angle of repose. For much larger values of a the formation of the final deposit is more complex: the upper part of the column descends centrally, while the foot of the pile is pushed radially outward. The final deposit shows a small central cone and a tapering frontal region. Most importantly, both investigations showed that the shape of the final deposit did not depend strongly on the frictional properties of the particles or the substrate onto which the column spread. This behavior is unexpected since, for such dense granular flows, the particles exchange momentum mainly through direct and enduring contacts; hence, the frictional forces are expected to play a major role in the process. Recently, Goujon *et al.*[3] showed that the runout length and thickness of a granular mass released over an inclined plane, have a strong dependence on the roughness of the surface. Other researchers have also shown the importance of frictional properties on the flow of dense granular materials.[4,5]

Inspired by the interesting nature of the experimental observations, a computational study of an equivalent two-dimensional (2D) configuration (circular disks moving in a plane) was conducted. We are interested in finding out the reasons for the different geometries of the final deposits and the influence of the grain properties. Thus, a computer simulation of the collapse of a granular column using a discrete element (DE) code was implemented. A similar code was used by Campbell *et al.*[6] to simulate landslides. They were able to successfully capture several of the characteristics observed in large landslides.

The DE computer code used in this study was developed by Wassgren,[7] and was adapted to simulate the collapse of a granular column; this technique was first proposed by Cundall and Strack[8] to study granular flows. The details of the implementation of the code are left out for brevity, but can be found in the references. In this case, only gravity and contact forces are considered and both the linear and angular momentum conservation equations are solved at every time step. The contact forces are modelled for both the normal and tangential directions. For the normal direction the linear hysteretic spring model proposed by Walton and Brown[9] was used, which accounts for the collision energy loss using a spring with two different stiffnesses. The loading stiffness is chosen to match Hertzian contact parameters; the unloading stiffness is calculated from the loading stiffness and the coefficient of restitution. For the tangential direction, a linear spring in series with Coulomb sliding friction element model was used, as proposed by Cundall and Strack.[8]

The validity of these type of models has been discussed

[a)]Electronic mail: zenit@servidor.unam.mx

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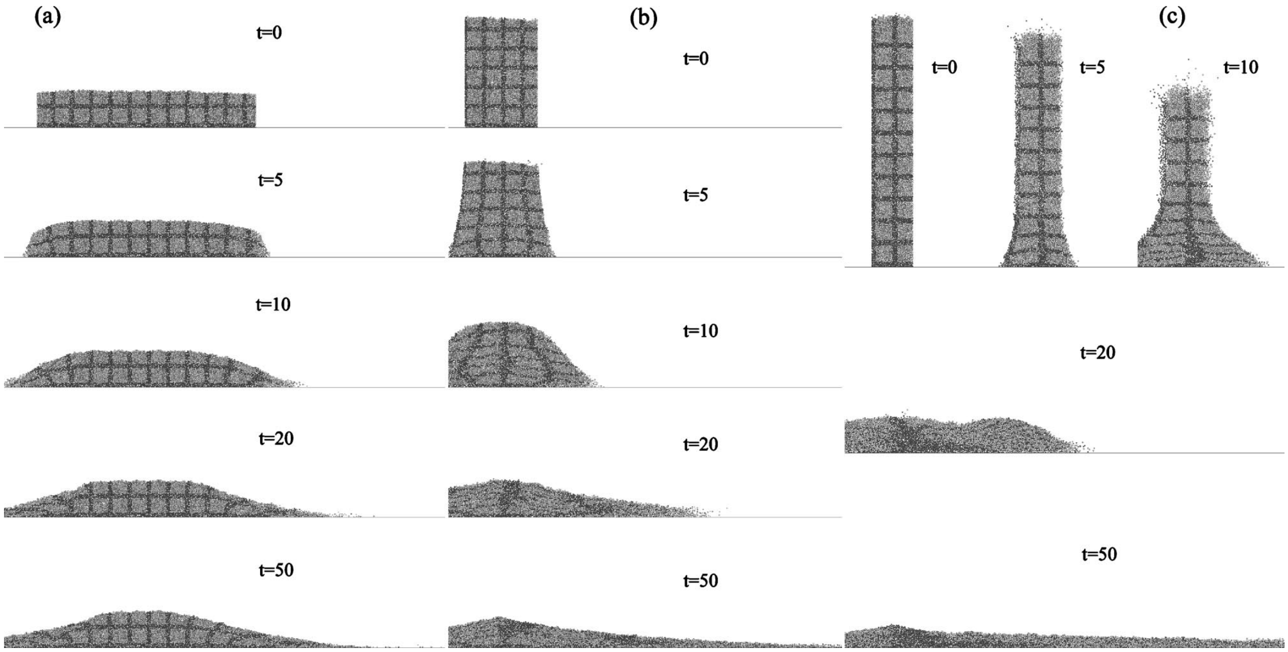

FIG. 1. Snapshots of the column shape for different times. Three cases are shown: (a) $a=0.33$; (b) $a=2.98$; (c) $a=12.38$. The times shown on the figure are dimensionless, $t^*=t/\sqrt{\overline{d_p}g}$. For these three cases $N=5000$, $\epsilon=0.75$, and $\mu_f=0.57$. The stripes in the initial configurations are colored tracer particles. Note that all images are truncated on the left side.

extensively in the granular flow literature (see, for example, Ref. 10). The proper selection of the contact model and properties is critical to obtain physically reasonable results. For dense flows, in which the particle contacts are enduring, the hysteretic spring contact model is the most appropriate (see Schafer *et al.*,[11] for details). In particular, for this model two stiffnesses are considered to account for the energy loss during a contact, loading and unloading stiffnesses, κ_L and κ_U, respectively. The coefficient of restitution, therefore, is

$$\epsilon = \sqrt{\frac{\kappa_L}{\kappa_U}}. \tag{1}$$

The loading stiffness is calculated according to kinematic constraints:[9]

$$\kappa_L = \frac{\overline{m}g}{\overline{r}} \frac{\dot{\alpha}_o}{\alpha_{max}} \exp\left(\frac{-\arctan(\beta)}{\beta}\right), \tag{2}$$

where $\overline{m}$ is the mass of the disks, $\overline{r}$ is mean radius, $\beta = \pi/\ln \epsilon$. α_{max} and $\dot{\alpha}_o$ are the maximum overlap and a characteristic collision speed. The details of the calculation of these parameters can be found in the references.

The use of a soft-particle 2D code was chosen for the following reasons: the contact models are well established and accepted; the visualization of the 2D computer results is straightforward; and, since the number of particles is not very large, many calculations can be performed in reasonable times and without using significant computational resources. One of the main advantages of DE simulations to study granular flows is that practically all the flow information is available at every time step. Quantities which are difficult or impossible to measure can be obtained directly.

The columns are prepared by randomly placing a given number of disks in a region confined by a horizontal and two vertical walls; once placed, a preparation simulation is run and the disks are allowed to settle, arrange and pack themselves under the action of gravity. Once the material comes to rest the preparation simulation is stopped. The main collapse simulation uses the final state of the preparation simulation as an initial condition; the vertical walls, originally confining the disks, are removed. As the simulation starts, the column begins to collapse and the material flows freely on the horizontal wall. The simulation parameters used in the results presented here are those corresponding to 0.35 mm glass particles, with a 10% size distribution. A constant value of the coefficient of restitution, $\epsilon=0.75$, was chosen. For this case the contact model loading stiffness coefficient is $\kappa_L^* = \kappa_L(\overline{d}/(\overline{m}g)) = 6.91\times10^5$, and the unloading one is $\kappa_U^* = \kappa_U(\overline{d}/(\overline{m}g)) = 7.99\times10^5$. Two different values of the friction coefficient $\mu_f=0.3$ and 0.57 were considered. These two values represent the frictional properties of the different materials used in the previous experimental investigations. Also, the same friction coefficient for particle–particle and particle–wall contacts is considered in all cases. The number of particles (disks), N, for each simulation varied from 100 to 10 000 to achieve different values of a. The results shown in Figs. 1 and 2 correspond to the case of $N=5000$. The simulations were stopped once the shape of the deposit ceased to change; a few particles may separate from the deposit and continue to move.

Three typical simulations, for different values of a, are shown in Fig. 1. As reported in the previous experimental investigations, the shape of the final deposit depends on the initial aspect ratio of the column. For small values of a, Fig.

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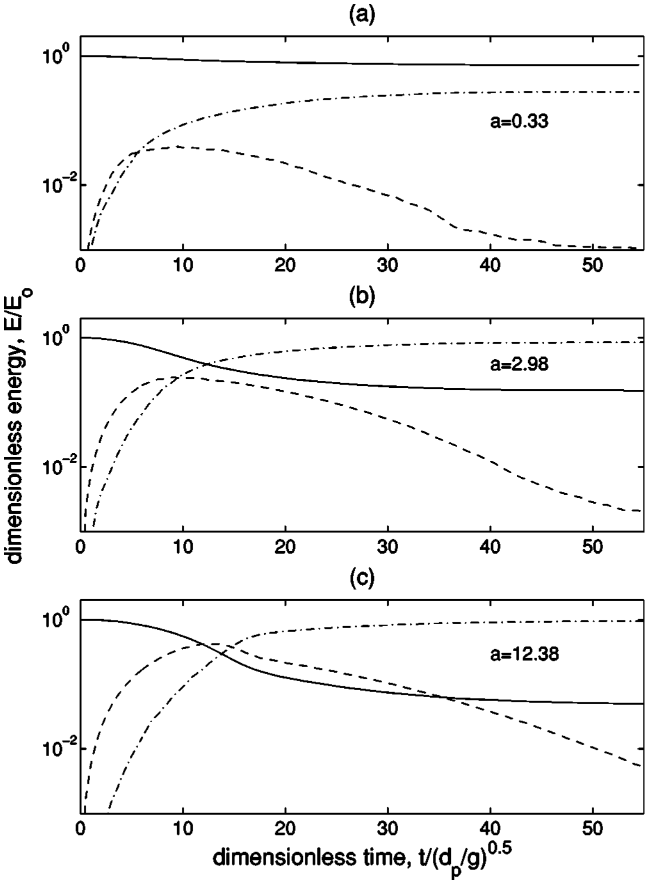

FIG. 2. Dimensionless energy as a function of time for the three values of the initial aspect ratio shown in Fig. 1. The solid line shows the potential energy, E_p^*; the dashed line shows kinetic energy, E_k^*; and the dashed–dotted line shows the cumulative energy dissipation E_{diss}^*.

1(a), the shape of the final deposit is a truncated cone; only the edges of the column avalanche, leaving the central region unchanged. For intermediate values of a, Fig. 1(b), the final deposit takes the shape of a cone. Since the column is approximately as tall as it is wide, the avalanche of the edges meet in the center only leaving a central triangular region unchanged. For larger values of a, Fig. 1(c), the formation process of the deposit is different. The tall column accelerates and splashes down; the particles moving down collide with the stagnant material in the center and are pushed radially out. The radial flow spreads and thins out. For all cases, the details of the collapse process (shapes and times) observed experimentally are captured by the simulations.

From the simulation results it is straight forward to calculate an energy balance of the collapse process. The dimensionless potential energy of the column, E_p^* is calculated as

$$E_p^* = \frac{\sum_{i=1}^{N} m_i g z_i}{E_o}, \tag{3}$$

where m_i and z_i are the mass and z-position (elevation) of particle i, respectively. E_o is the initial energy, which is taken to be the potential energy at $t=0$; hence, the elastic energy stored in particles in the compacted state is neglected. The total kinetic energy, also in dimensionless form, is calculated using

$$E_k^* = \frac{\sum_{i=1}^{N} m_i |V_i|^2}{2E_o}, \tag{4}$$

where $|V_i| = \sqrt{V_x^2 + V_y^2}$ is the magnitude of the velocity of particle i. Note that at the end of the simulation time, the kinetic energy may not be zero. In some cases, a few particles continue to move at long distances from the column center.

Clearly, as the column collapses the potential energy will either be converted into kinetic energy or be dissipated due to the frictional and inelastic nature of the particle–particle interactions. The energy dissipation can also be calculated directly from the numerical simulations;[12] however, for brevity, here we calculate the accumulated dissipated energy using a simple energy balance:

$$E_{diss}^* = 1 - E_p^* - E_k^*. \tag{5}$$

Figure 2 shows the dimensionless energies for the collapse of columns corresponding to the cases shown in Fig. 1. For all cases, the potential energy decreases monotonically as the column collapses. The total decrease of the potential energy increases with a, in accordance with Lajeunesse *et al.*[2] Also for all cases, the kinetic energy increases from zero, reaches a maximum value, and then decreases again for longer times. For short columns, Fig. 2(a), the material cannot gain much kinetic energy since the potential energy does not decrease significantly. Also, the amount of energy dissipation is comparable to the kinetic energy, which contributes to rapidly damp the motion of the particles. On the other extreme, for large a, Fig. 2(c), the amount of kinetic energy rapidly increases, reaching values larger than those of the potential energy during parts of the collapse. In contrast to what is observed for short columns, the energy dissipation does not play a major role during the initial part of the collapse, which also contributes to the vigorous collapse process. For intermediate values of a, Fig. 2(b), the energy balance has characteristics which are in between the two extreme cases discussed above.

Figures 3 and 4 show the dimensionless final height and spread of the columns, as functions of the initial aspect ratio, a, obtained from many simulations. The final height is measured at the center of the column, where the motion has subsided. The final spread is measured from the center to the edge of the deposit, in which several particles remain in contact disregarding individual loose particles. Furthermore, simulations for two different values of the friction coefficient were run. Along with the numerical results, the experimental results from both previous studies[1,2] are included. It must be noted that the comparison between the present numerical and previous experimental results is merely qualitative. There is not an obvious way to account for the different spatial dimensionality.

As in the experiments, our investigation shows that the dimensionless height of the final deposit changes its functional dependence with respect to a when a critical value is reached: $a_{crit} \approx 0.39$ for $\mu_f = 0.3$ and $a_{crit} \approx 0.49$ for $\mu_f = 0.57$; hence, we do observe a dependence, however small, on the friction coefficient. Clearly, the critical value found in the experiments is larger ($a_{crit} = 0.7$ in Lajeunesse *et al.*[2] and

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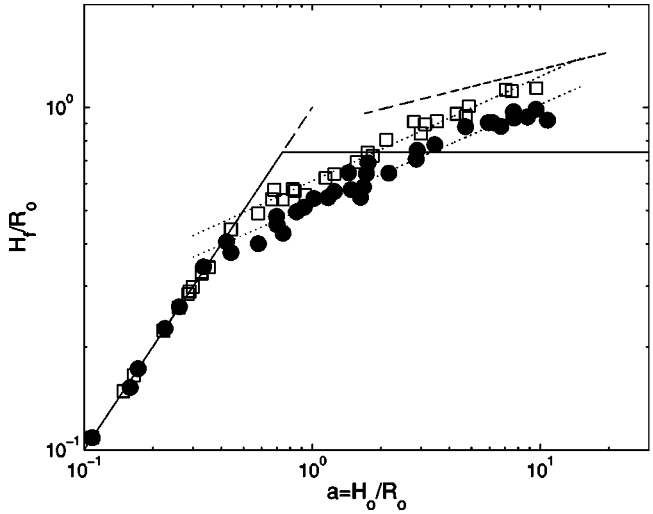

FIG. 3. Dimensionless height as a function of initial aspect ratio. The solid circles show the simulation results for $\mu_f=0.3$ and the empty squares show the results for $\mu_f=0.57$. The dashed lines show the results from Lube *et al.* (Ref. 1), the solid lines show the results from Lajeunesse *et al.* (Ref. 2). The dotted lines show the best fit of the simulation results for $a > a_{crit}$.

$a_{crit}=1.7$ in Lube *et al.*[1]). Again, the differences can be attributed to the geometrical differences between disks and spheres. It can be argued that the angle of repose of a 2D array is smaller than that of an equivalent 3D configuration. Also, a constant value of the dimensionless final height for values of a larger than a_{crit}, as reported by Lajeunesse *et al.*,[2] was not found; for columns with initial aspect ratio larger than a_{crit}, the dimensionless height of the deposit increases slightly, at a rate of approximately $H_f/R_o \sim a^{0.3}$, for both values of μ_f. This is almost twice the rate reported by Lube *et al.*[1] Since the height of the final deposit was smaller than that in the experiments, it is expected that the numerical radial spread will also be larger than the experimental one. This is clearly shown in Fig. 4. Despite this difference, the general behavior is monotonically increasing, which is in good qualitative agreement with the experiments. It is interesting to note that a small difference between the results for the two values of the friction coefficient was observed: the final deposits from columns with higher coefficient of friction were slightly higher and spread for shorter distances. An additional set of simulations (not shown) was conducted to corroborate that the changes in the coefficient of restitution did not affect the results significantly.

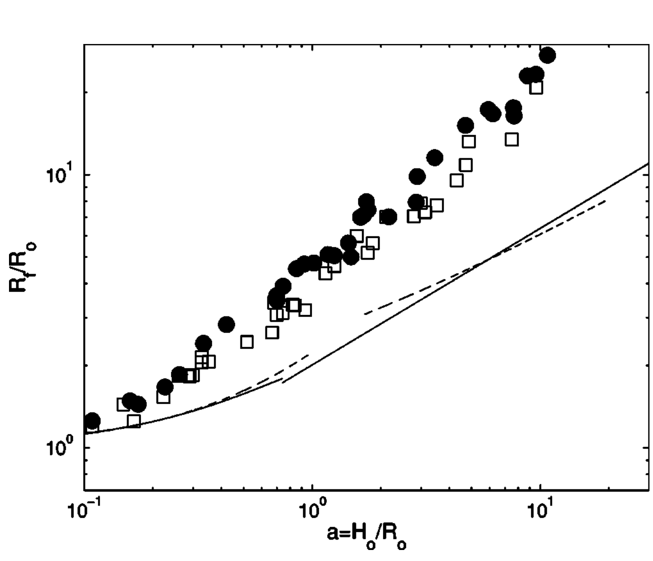

FIG. 4. Dimensionless radial spread as a function of initial aspect ratio. Same symbols as in Fig. 3.

In conclusion, the behavior of 2D collapsing columns was found to be qualitatively the same as in axisymmetric experiments. Despite the obvious differences between the simulated 2D system and the 3D experimental cases, the same characteristics of the phenomena were found: a critical value of a, that determined whether or not the final deposit had a flat top, was found; the radial spread of final deposit increased monotonically with a. Both the dimensionless final height and radial spread compared qualitatively well with the experimental results. Comparisons with all the experimental results reported by both previous studies can also be performed. Finally, the energy balance of the collapsing column can be used as a criterion to determine the regime changes. The characteristic differences that lead to the different shapes of the deposits can be clearly observed from this energy balance. A comprehensive comparison of the simulation and experimental results, as well as a more in depth study of the effect of the contact properties, will be presented in a full-length article currently under preparation.

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