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A note on the modelling of the bouncing of spherical drops or solid spheres on a wall in viscous fluid

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Abstract

A generalized description of the rebound of spherical drops or solid spheres over a wall is proposed using two parameters: a coefficient of restitution that compares the velocity of restitution to the velocity before impact and the contact time with the wall. During the bouncing, the incident kinetic energy is transferred into deformation energy (stored on the surface for the case of liquid drops or in the bulk for the case of solid particles) and then restored into kinetic energy allowing the particle to leave or not the wall. The corresponding criteria is given by the Stokes number that compares the inertia of the particle (added mass included) and the viscous force exerted on the particle during the drainage of the film formed between the particle and the wall. The general behavior of the coefficient of restitution observed in many experiments can be modelled for solid spheres as well as spherical drops by the use of a unique simple correlation depending on this Stokes number. For solid particles, the contact time with the wall in viscous flows is found to be of the same order as that predicted by the Hertzian theory; hence, the contact with the wall can be described as a discontinuity in the particle motion. On the other hand, for liquid drops, the contact time is significant and of the same order as other characteristic time scales of the particle motion. Therefore, to properly describe the rebound process, both a restitution coefficient and a contact time must be considered. Finally, a simple model is proposed and its predictions are compared with experiments performed for millimetric toluene drops in water.

Keywords: Bouncing; Drop; Fluid mechanics; Hydrodynamics; Modelling; Particle

1. Introduction

In many practical situations where multiphase flows are involved, particle–wall interactions play a major role in the overall dynamics of the flow. As a consequence, the motion of the dispersed phase cannot be described solely by its behavior in unbounded flows and specific knowledge has to be developed concerning these interactions. For example, two phase contactors and reactors involved in chemical and biochemical industries are often equipped with internal walls in order to control the residence time of the dispersed phase as well as its agitation and the mass or heat transfer rate between the two phases. In such equipment, wall interactions have a significant effect on the performance via their influence on the behavior of the dispersed phase within the vessel. Among these interactions, the aim of this work is to consider the collision and bouncing processes.

Many studies have reported experimental results concerning the bouncing of a solid sphere on a wall in air as well as in various fluids. The most recent experiments for this system (Joseph et al., 2001; Gondret et al., 2002) and the analytical derivations using the lubrication theory (Davis et al., 1986) have clearly demonstrated that the restitution coefficient ε (ratio of the velocity after the rebound to the approach velocity) can be scaled by a particle Stokes number St which compares particle inertia to viscous effects. A no-rebound situation ($\varepsilon = 0$) is observed below a critical Stokes number St_c ($\mathrm{St}_c \approx$ 10–15). The restitution coefficient quickly increases after

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the transition at St_c and monotonically reaches an asymptotic value close to the value of ε obtained in air where viscous effects are supposed to be negligible during the interaction with the wall ($St > 10^4$). This asymptotic value is thus close to 1 for elastic materials. The Stokes number dependence is due to the drainage of the liquid film formed between the particle and the wall (Davis et al., 1986).

For drops, the bouncing on a wall is characterized by significant deformation and contact time with the wall, both increasing with the particle diameter and the impact velocity (Richard and Quéré, 2000; Klaseboer et al., 2000; Legendre et al., 2005). The bouncing is also controlled by the drainage of a film formed between the particle and the wall. Legendre et al. (2005) proposed a simple dynamic model (dissipative mass spring system) to describe the rebound of millimetric toluene drop in water. This model is able to predict the deformation of the drop during the bounce, the contact time with the wall and the velocity of restitution.

The aim of this note is to propose a generalized global model for the bounce of particles (solid or liquid) on a wall using two parameters namely the coefficient of restitution and the time of contact with the wall.

2. Coefficient of restitution

Bouncing of solid bodies is usually described using coefficients to model tangential and normal velocities at the end of the bounce. The normal coefficient of restitution $e = -V_{res}/V_{imp}$ is defined as the ratio of the normal component of the velocity when leaving the wall V_{res} to the normal component of the velocity at impact V_{imp}. As shown by Joseph et al. (2001) for solid spheres in glycerol–water mixtures and by Legendre et al. (2005) for drops in water, the approach velocity begins to decrease before the contact with the wall so that the velocity at impact V_{imp} has significantly decreased compared to the approach velocity at some radius from the wall (typically the velocity starts to decrease at one radius for millimetric toluene drops in water). From a global point of view, this decrease can be attributed to the bouncing mechanism and a coefficient of restitution can be defined as the ratio of the velocity V_{res}, to the velocity V_∞ before its decrease due to the interaction with the wall:

$$\varepsilon = -\frac{V_{res}}{V_\infty}. \tag{1}$$

This restitution coefficient ε provides a global description of the bounce including the effect of the wall without describing the detailed action of all the physical mechanisms involved: deceleration due to hydrodynamic interaction, deformation, film drainage and restitution of the initial shape. Note that for large Stokes numbers (for example, drops or solid particles in air), the impact occurs at $V_{imp} \sim V_\infty$ and therefore $e \sim \varepsilon$.

Using the analogy with a dissipative mass-spring system, Legendre et al. (2005) have shown that for drops the coefficient of restitution evolves as

$$\varepsilon = \varepsilon_{max} \exp\left[-\frac{\beta}{St}\right], \tag{2}$$

where St is defined here as the Stokes number of the particle away from the wall:

$$St = \frac{(\rho_p + C_{M\infty}\rho)V_\infty d}{9\mu},$$

where ρ_p is the particle density, ρ is the fluid density, μ is the fluid viscosity and d is the particle diameter. In this case, in order to provide a global description of the wall interactions the Stokes number, St is based on V_∞ and on $C_{M\infty} = \frac{1}{2}$. In relation (2), β is a parameter that includes the viscous effects of the film drainage and ε_{max} is the maximum coefficient of restitution that can be reached by the particle i.e., for $St \to \infty$. In practical situation, ε_{max} is the value measured in air. For solid spheres, it depends on the materials constituting the particle and the wall (Sondergaard et al., 1990). For millimetric water and glycerol drops and for centimetric rubber balloons filled with a mixture of water and glycerol, Richard and Quéré (2000) obtained a nearly constant value $\varepsilon_{max} = 0.91$, for collisions in air.

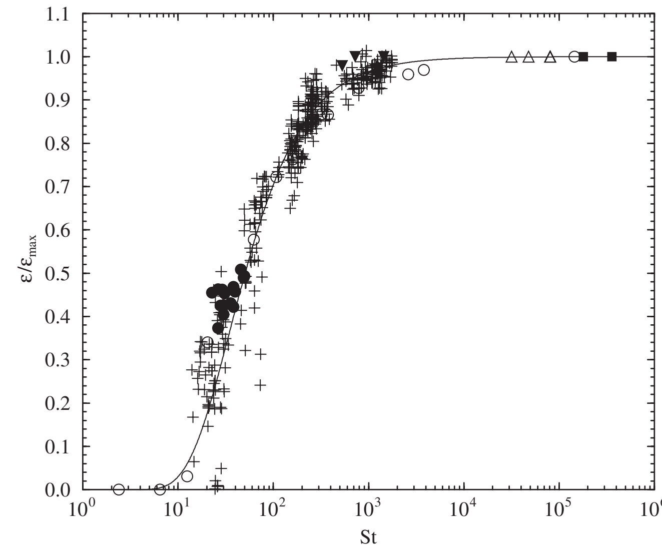

Fig. 1. Restitution coefficient $\varepsilon/\varepsilon_{max}$ for spherical drops and solid spheres versus the Stokes number St. •, toluene drops in water (Legendre et al., 2005); ▼, liquid drop in air (Richard and Quéré, 2000); ■, spherical balloon filled with a mixture of water and glycerol (Richard and Quéré, 2000); +, solid sphere (Joseph et al., 2001); ○, solid sphere (Gondret et al., 2002); △, solid sphere (Foerster et al., 1994); —, correlation given by relation (2) with $\beta = 35$.

Fig. 1 shows experimental results from as many available sources in the literature for spherical drops and also solid spheres colliding both in air and in different liquids. This figure clearly shows that relation (2) with $\beta = 35$ is able to fit the general trend followed by all the experiments in a good manner.

3. Contact time

The collision between a solid sphere and a wall is usually described as a discontinuity in the motion of the sphere assuming that the time of contact is smaller than all the other characteristic time scales describing the particle motion and the surrounding fluid flow. Recent experiments showed that for drops in air (Richard and Quéré, 2000) as well as in water (Legendre et al., 2005), the bouncing with a wall is characterized by a significant contact time. Consequently, the interaction with the wall cannot be described as a discontinuity in the motion. For both types of particles (solid and liquid) the contact time is controlled by the elasticity of the particle. For a drop the elasticity is located on the surface and for a solid particle the elasticity is located in the bulk of the particle.

3.1. Solid spheres

The contact time for a solid sphere in a gas (where the effects of the external fluid can be neglected) can be determined using the classical Hertzian theory. Its applicability is expected to be valid as long as the resulting deformation does not exceed the elastic limit (Goldsmith, 1960; Johnson, 1985). The contact time between a spherical particle and a wall is

$$t_{\mathrm{Hertz}} = 2.54\rho_p^{2/5}\left[\frac{1-v_p^2}{E_p}+\frac{1-v_w^2}{E_w}\right]^{2/5} d V_{\mathrm{imp}}^{-1/5}, \tag{3}$$

where E, ρ and v are the modulus of elasticity, the density and Poisson's ratio, and the sub-indices w and p refer to the wall and the particle, respectively. Zenit (1997) and Falcon et al. (1998) measured the contact time during the bouncing of spherical particle in air and confirmed the $V_{\mathrm{imp}}^{-1/5}$ dependence given by Hertz theory. For a $d = 8\,\mathrm{mm}$ tungsten carbide bead impacting at $V_{\mathrm{imp}} = 19.8\,\mathrm{cm/s}$ in air over a steel wall, Falcon et al. (1998) measured a typical $t_c = 52.4\,\mu\mathrm{s}$. This measurement is close to the contact time $t_{\mathrm{Hertz}} = 43\,\mu\mathrm{s}$ given by (3), considering all the appropriate elastic constants.

For the case in which the surrounding fluid cannot be neglected, two main effects, not included in relation (3), can be considered:

(i) *The effect of the fluid inertia*: The added mass effect increases the kinetic energy that is transformed into deformation during the bounce. This effect can be simply taken into account by replacing ρ_p in (3) by $\rho_p + C_{M\mathrm{wall}}\rho$ with $C_{M\mathrm{wall}} \sim 0.73$, which is the limit value of the added mass coefficient of a spherical body moving toward a wall at contact (Kok, 1995).

(ii) *The effect of the fluid viscosity*: This effect has been studied by Davis et al. (1986) on the particle deformation but no specific information is given for the contact time. The deformation of the sphere is found to be dependent on the fluid viscosity resulting from the drainage of the film formed between the wall and the particle which is found to significantly modify the velocity of restitution. Surprisingly, experimental studies of the effect of the viscosity on the contact time for solid particles are scarce. Since the contact time is very small, it is very difficult to measure it with a satisfactory accuracy using trajectory image processing. It can be argued that the contact time must increase with the fluid viscosity; it can be noted that for St < 10 there is no rebound, which would correspond to an infinite contact time.

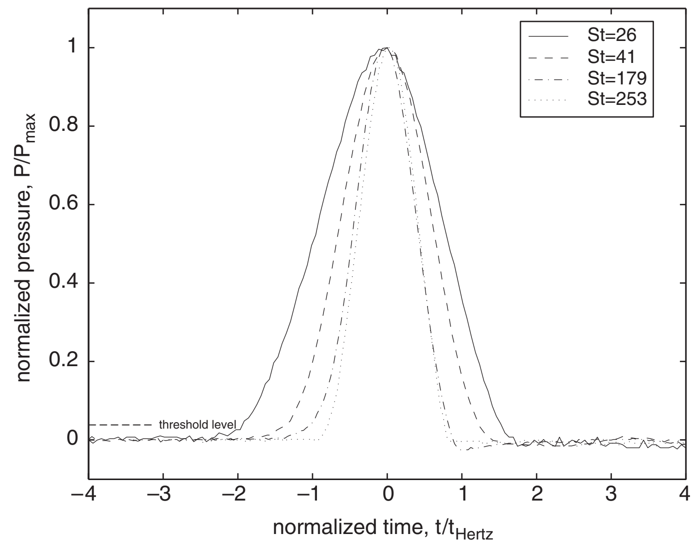

Fig. 2. Typical measured pressure trace resulting from a collision of a solid sphere against a fast pressure transducer. Several typical signals, corresponding to a range of St, are shown. The pressure is normalized by the maximum value achieved for a given experiment. The time is normalized by the collision duration predicted from Hertzian theory for each case.

The data for the contact time of solid particles colliding against a wall in a liquid can be obtained from the experiments of Zenit (1997) and Zenit and Hunt (1999). In these investigations, a fast-time dynamic pressure transducer was used to study the effect of the surrounding liquid in the impulse strength of particle collisions. A wide range of Stokes numbers was studied using a pendulum setup, to vary the impact velocity, and particles of several sizes and densities. Details of the experimental arrangement can be found in the references. Typical measured pressure traces resulting from collisions are shown in Fig. 2. Clearly, the pressure quickly raises from zero to a maximum value, presumably as an indication of solid–solid contact, and then drops to zero again in a nearly for-aft symmetric fashion. These authors found that the liquid immersed collisions had a strength similar to those predicted by a purely elastic contact, e.g. the Hertzian prediction. The deviation from a Hertzian contact was found to increase as the particle Stokes number decreased.

From the original data, a precise measure of the particle contact time can be obtained. As for the case of the measurement of the collision impulse obtained in the original investigation, a threshold level equal to 5% of the collision maximum pressure was considered to obtain a measure of the contact time. The threshold level is shown schematically in Fig. 2. The collision time is defined as the duration for which the pressure signal is above the threshold level. For these experiments, the sampling rate was 1 MHz; hence, the contact time could be determined with an accuracy of $\pm 2\,\mu\mathrm{s}$. The measured contact time did not show a strong dependence of the chosen value of the threshold

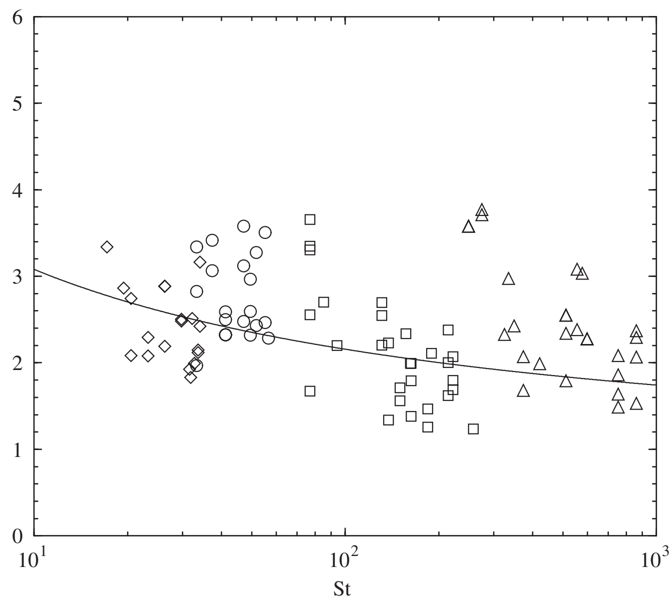

Fig. 3. Ratio between the measured contact time to the Hertz relation (3) where ρ_p is replaced by $\rho_p + C_{M\text{wall}}\rho$. ∘, $d = 3$ mm glass sphere; △, $d = 4.5$ mm steel sphere; □, $d = 6$ mm glass sphere; ⋄, $d = 6.5$ mm nylon sphere. —, correlation (4).

level. Typically, for our experiments, the duration of collision varied approximately 5% for a change of the threshold level of 100%. The only requirement to choose an appropriate threshold level is than its value is larger than the base-line noise, to prevent an erroneous detection of the initiation of the pressure pulse.

Fig. 3 shows the ratio of the measured contact time to the Hertz relation (3), where ρ_p is replaced by $\rho_p + C_{M\text{wall}}\rho$ to include the effect of the fluid inertia. This figure clearly shows that the contact time increases as the Stokes number decreases (i.e., when viscous effects increased compared to the inertia) but remains of the same order of magnitude as the time given by the modified Hertz prediction, at least for the range of Stokes numbers covered by the experiments ($17 < \text{St} < 10^3$). It is not possible to strictly conclude if the contact time tends to a finite value or increases continuously to infinity as St approaches its no-bounce critical value $\text{St}_c \approx 10$–15. From Fig. 3 it can be argued that if the contact time must reach an infinite value (enduring contact), it would increase very fast as $\text{St} \rightarrow \text{St}_c$. Indeed the minimum Stokes number where bouncing occurs for the data reported here is $\text{St} = 17$ and the ratio with the Hertz contact time is 3.3. Note that there is a noticeable scatter in the data presented in Fig. 3. Experiments performed under identical nominal conditions resulted in measurements which varied significantly. Joseph et al. (2001) argued that the measurements depended on the microscopic details of the contact area. Since, for every collision the contact area is slightly different in each case, some *characteristic* variability is expected.

A simple correlation can be proposed to account for the effects of the fluid inertia and viscosity on contact time measured in the experiments. It is found that the following empirical correction in St

$$t_c = t_{\text{Hertz}} \left(\frac{\rho_p + C_{M\text{wall}}\rho}{\rho_p} \right)^{2/5} \frac{1}{1 - 0.85\,\text{St}^{-0.1}} \tag{4}$$

fits the averaged tendency given by the experimental measurements. Eq. (4) is shown in Fig. 3.

3.2. Liquid drops

For drops, the contact time is controlled by the drop's surface tension during the bouncing as shown by Klaseboer et al. (2000), Richard and Quéré (2000), Okurama et al. (2003) and Legendre et al. (2005). It is the time necessary to deform the surface (kinetic energy transferred into surface energy) and to restore the spherical shape (surface energy transferred into kinetic energy). Typically, the order of magnitude of the contact time for a millimetric toluene drop in water is 20 ms which is three orders of magnitude longer than the contact time of a solid sphere with equivalent conditions.

Legendre et al. (2005) showed that the time of contact can be estimated by considering an equivalent mass-spring system where the mass of the system $m^* = (\rho_p + C_{M\text{wall}}\rho)\pi d^3/6$ includes the added mass involved in the motion (with $C_{M\text{wall}} \sim 0.73$) and the stiffness of the system $K = 16\pi\sigma/5$ is deduced from the surface energy due to the deformation. The time of contact with the wall is the half period of this oscillator:

$$t_c = \pi\sqrt{\frac{m^*}{K}} = \pi\sqrt{\frac{5}{96}}\sqrt{\frac{(\rho_p + C_{M\text{wall}}\rho)\,d^3}{\sigma}} \approx 0.717\sqrt{\frac{(\rho_p + C_{M\text{wall}}\rho)\,d^3}{\sigma}}. \tag{5}$$

Fig. 4 shows the comparison between the predictions of relation (5) and the experimental results of Okurama et al. (2003) for drops in air, Klaseboer et al. (2000) and Legendre et al. (2005) for toluene drops in water, Klaseboer et al. (2000) for cyclohexane drops and silicone oil drops in water. In the latter case, the contact time is deduced from the trajectories of the drops obtained using a high-speed camera. The experiments of Klaseboer et al. (2000) were performed using a rate of 50 images per second which is not enough to deduce the contact time with a satisfactory precision. The values reported in Fig. 4 are deduced from the fit of their experimental points (figure 7 in their paper) obtained using a bouncing model proposed by these authors and based on the calculation of the drops trajectory coupled with the calculation of the lubrication film between the drop and the wall. The prediction (5) is found to give a good prediction for bouncing of drops of different fluids in water as well as in air and shows that the simple model proposed for toluene drops in water can be also used to determine the contact time for drops bouncing in a liquid or a gas. This figure also reveals that there is a second-order dependency of the data: the contact time decreases with the Stokes number. Further experiments on a wider range of Stokes numbers are obviously necessary to clarify this point.

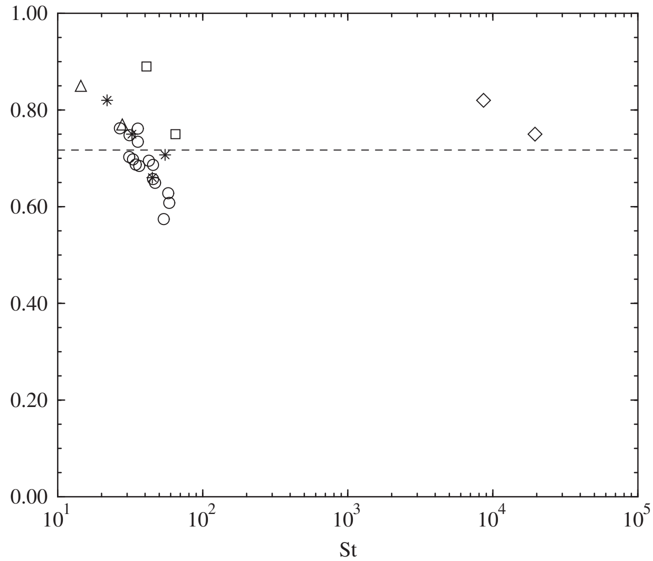

Fig. 4. Normalized contact time $t_c/\sqrt{(\rho_p + C_{M\text{wall}}\rho)\, d^3/\sigma}$ for liquid drops evolution versus the particle Stokes number St. Toluene drops in water ($\rho = 860\,\text{kg/m}^3$, $\sigma = 0,026\,\text{N/m}$): ∘, experiments of Legendre et al. (2005); ⋆, experiments of Klaseboer et al. (2000). Cyclohexane drops in water ($\rho = 780\,\text{kg/m}^3$, $\sigma = 0,047\,\text{N/m}$): □, experiments of Klaseboer et al. (2000). Silicone oil drops in water ($\rho = 960\,\text{kg/m}^3$, $\sigma = 0,030\,\text{N/m}$): △, experiments of Klaseboer et al. (2000). Water drops in air: ⋄, experiments of Okurama et al. (2003). − − −, correlation (5).

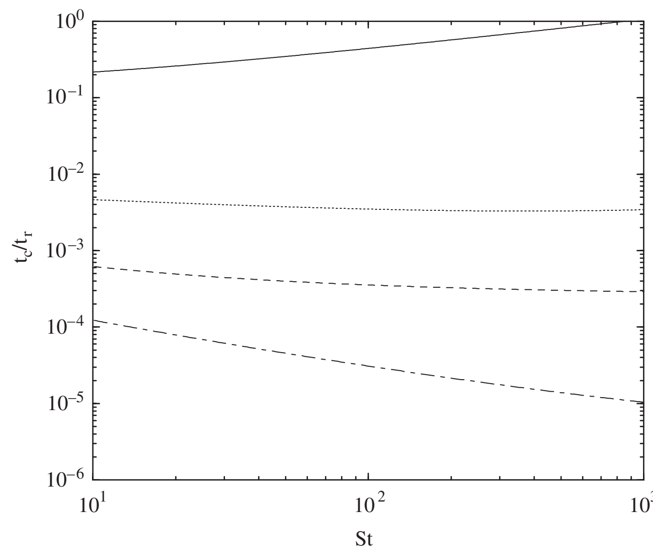

Fig. 5. Ratio between the contact time t_c and the viscous relaxation time t_r versus the Stokes number St. —, toluene drops in water ($\rho_p = 860\,\text{kg/m}^3$, $\sigma = 0.026\,\text{N/m}$); · · · · ·, water drops in air; − − −, glass sphere ($\rho_p = 2540\,\text{kg/m}^3$) in water; − · − · −, glass sphere in air. Note that the plot starts at St > 10 which is the limit for bouncing.

3.3. Comparison with the relaxation time

For practical applications, it is interesting to compare the contact time with the viscous relaxation time of the drop or solid particle. The relaxation time characterizes the time necessary for the particle to adjust its velocity to any unsteady situation (for example, it is the characteristic time required to reach the terminal rising velocity after being released from rest). As observed in many experiments, the drag of a drop usually follows the drag of a solid sphere due to the presence of contaminants and can be estimated using the Shiller and Nauman's (1933) correlation which is used here to estimate the relaxation time of the solid or fluid particle:

$$t_r = \frac{(\rho_p + C_{M\infty}\rho)d^2}{18\mu(1 + 0.15\,Re^{0.687})}. \quad (6)$$

The ratio between the contact time (given by (5) for drops and (3) for solid spheres) and the relaxation time is plotted in Fig. 5 versus the particle Stokes number for a particle impacting on a wall at its terminal velocity. The terminal velocity is obtained by the use of Shiller and Nauman's drag correlation. Four cases are presented, toluene drops moving in water, water drops moving in air, glass spheres moving in water and glass spheres moving in air. For toluene drops in water, the contact time is found to be always of the same order of magnitude as the viscous relaxation time. For example, for a $d = 2.5$ mm drop (St = 32) the relaxation time given by (6) is $t_r = 76$ ms while the contact time is $t_c = 21$ ms. This result has a significant implication for the Lagrangian description of the drop motion. Let us consider, for example, a numerical calculation of the drop motion by solving its trajectory. In order to give an accurate description of the trajectory, the time step must be lower than the relaxation time of the drop and can thus be lower than the contact time. This condition would cause a drop to appear immobilized on the wall for several time steps. For water drops in air and for the diameters considered here ($0.01\,\text{mm} \leqslant d \leqslant 10\,\text{mm}$) the contact time is found to be always at least two orders of magnitude less than the viscous relaxation time. In such a situation, the motion of the drop can be modelled as a discontinuity in the drop's motion.

For solid spheres, the comparison of the contact time (4) and the relaxation time clearly shows that for all the situations (in air or water) the contact time is several order smaller than the relaxation time of the particle. Considering, for example, a $d = 2.5$ mm glass sphere impacting the wall at its terminal velocity $V_\infty = 33$ cm/s in water (St = 296), the relaxation time given by (6) is $t_r = 0.7$ s, several orders of magnitude larger than the contact time $t_c = 52.4\,\mu$s which can be described as a discontinuity ($t_c \sim 0$) for the particle motion.

4. Model for bouncing

The results presented above clearly show that for a solid particle the bouncing can be described as a discontinuity for the particle trajectory. The velocity after the impact can be simply deduced from the coefficient of restitution and the velocity before the impact. On the other hand, a liquid drop can remain in contact with the wall a significant amount of time. This particular interaction can be modelled considering the contact time results presented here.

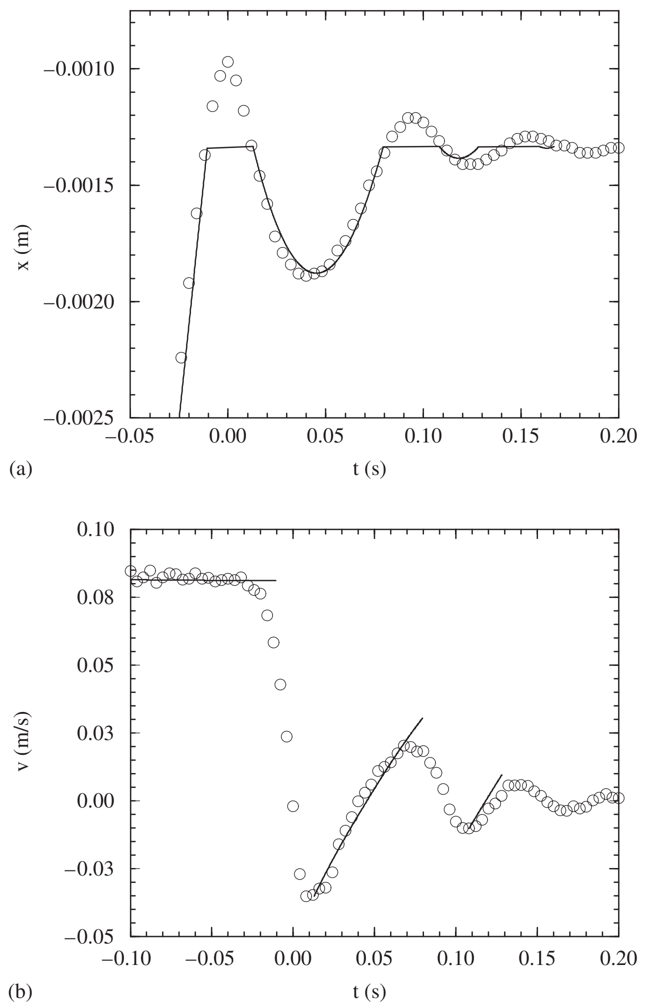

Fig. 6. (a) Trajectory and (b) velocity evolution of a $d = 2.75$ mm toluene drop (Legendre et al., 2005). ∘, experimental results; —-, model.

The simple bouncing model which is proposed here is based on the two parameters presented above: the coefficient of restitution ε or e (see Section 2) and the contact time with the wall t_c (see Section 3). When the collision between the wall and the drop is detected, the particle is immobilized at the wall during t_c. The particle is then released to the surrounding fluid with a normal velocity $V_{rest} = -\varepsilon V_\infty$ or $V_{rest} = -e\, V_{imp}$ where V_∞ is the velocity of the drop before "feeling the wall" and V_{imp} is the velocity of the particle when it touches the wall. The choice between e and ε depends on how the particle motion is described. If the force balance includes all the effects of the wall, then the use of e is required. If no wall effects are included in the force balance, the use of ε is more appropriate. In this section, we test the last situation by comparing with the experiments reported by Legendre et al. (2005), the motion of a millimetric spherical toluene drop (position and velocity) moving in water before and after the impact with a wall. A typical result from this investigation is shown in Fig. 6. These results were obtained using a high-speed camera operating at 250 images per second.

To model the trajectory of the drop, a force balance equation is proposed. The force balance for a drop rising involves only buoyancy, steady drag and added mass:

$$\left(\rho_p + \frac{1}{2}\rho\right)\frac{\pi d^3}{6}\frac{d\,\mathbf{v}}{\mathrm{d}t} = -3\pi\mu d(1 + 0.15\,Re^{0.687})\mathbf{v} + \frac{\pi d^3}{6}(\rho_p - \rho)\mathbf{g}, \tag{7}$$

where the added mass coefficient without wall interaction is $\frac{1}{2}$ for a spherical drop. In the experiments the drag force is found to follow the Shiller and Nauman's correlation where $Re = \rho d|\mathbf{v}|/\mu$ is the instantaneous Reynolds number of the drop based on its instantaneous velocity $\mathbf{v}$. The trajectory equation of the drop (7) is integrated using a second-order Runge–Kutta method, considering initial conditions amenable for direct comparison with the experimental results. The initial position of the droplet is greater than 10 drop diameters from the wall; its initial velocity is zero. Figs. 6a and b present, respectively, the trajectory and the velocity evolution of a $d = 2.75$ mm diameter drop, which corresponds to a Stokes number, based on the terminal velocity, equal to $\mathrm{St} = (\rho_p + 0.5\rho)\,Re/9\rho = 38$. Clearly, the drop position and velocity are in good agreement with the experiments up to the second collision with the wall, even the second impact being well reproduced by this basic modelling. It must be noted that no history force expressions are available for drops. Since the drop's interface is contaminated, it may be possible to add the long time history force for solid sphere (Lawrence and Mei, 1995) as done by Gondret et al. (2002) who showed that the history force cannot be neglected for bouncing trajectories of solid spheres in liquid. Gondret et al. (2002) obtained a satisfactory fit with their experiments by adjusting the history force with a fitting coefficient which had to be adjusted for each case.

Finally, one can observe that this simple model gives a good description of the bouncing between a drop and a wall: the contact time is modelled and the conditions of rebound from the wall are reproduced. Having in mind that approximately 80–90% of the kinetic energy is dissipated during the first impact, in this example, the simple model presented here accounts for the major part of momentum and energy exchanges during drop–wall interactions.

5. Conclusion

It was found that to properly characterize the process of collision and rebound of a spherical particle, solid or liquid, on a wall immersed in a fluid two parameters must be considered: the coefficient of restitution and the contact time with the wall. The coefficient of restitution follows the same evolution for solid or fluid particles as shown by many experiments. This behavior can be modelled by the simple expression $\varepsilon = \varepsilon_{max} \exp[-35/\mathrm{St}]$ where St is the Stokes number that includes the added mass of the particle and is based on the velocity before its change by the hydrodynamic effects due to the wall. ε_{max} is the maximum coefficient of restitution that can be reached for the particle under consideration and is the value measured in air. By further analysis of available data, it was found that the contact time for solid particles in viscous fluids is of the same order of magnitude as that predicted by Hertz's theory. Also, it was found that the contact time is always much smaller than the viscous relaxation time; hence, for the case of solid spheres, the bouncing can be described in a simplified manner as a discontinuity in the particle motion. A correlation depending on the Stokes number is proposed to consider the effect of the viscosity while the effect of the fluid inertia is considered by the introduction of the added mass in the inertia involved in the contact time expression. For millimetric drops in water the contact time, modelled using an analogy with a mass-spring system, can be of the same order than the viscous relaxation time and the drops appear to stick on the surface during the time of the rebound. These two parameters combined with a simple trajectory equation including no wall effects are able to give a satisfactory description of the bouncing of drops on a wall.

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