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Transverse Stark effect of electrons in a hollow semiconducting quantum wire

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Abstract

We investigate the effect of an electric field applied transverse to the axis of cylindrical symmetry of a hollow cylindrical quantum wire on the energy ground state of the electrons in this wire. We investigate how the shift in the energy with field depends upon the ratio of the outer and inner radii of the hollow wire.

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1. Introduction

There has been much work on the transverse Stark effect in quantum wells where an electric field is applied along the direction of confinement in the well [1–7].

The effect of the electric field on the energy levels of electron, excitons and hydrogenic impurities have been investigated both theoretically and experimentally. Also, the Stark effect has also been investigated in quantum dots [8]. We have previously theoretically investigated the Stark effect when the electric field is applied transverse to the axis of cylindrical symmetry (transverse Stark effect) in a solid cylindrical quantum wire [9]. Dupertuis et al. [10] investigated the effect of an electric field on the energies of electrons in V shaped quantum wires but did not show what they used for the confining potential. The electric field in their case had components parallel and perpendicular to the V shaped wire. They found excited states with many nodes which get split with the electric field. Huynh Thanh et al. [11] investigated the confined Stark effect in quantum wires with parabolic confinement and found a quadratic Stark effect at all electric fields. Benner and Haug [12] considered the Stark effect in quantum wires also assuming parabolic confinement. There has also been experimental work on the photoluminescence in quantum wires in an electric field [13,14]. Arakawa et al. [13] found a blue shift in the photoluminescence due to excitons in quantum wires in the presence of an electric field. In their case, the quantum wires were V shaped. Rinaldi et al. [14] also observed a small blue shift which they believed was due to the piezoelectric field caused by strain in the quantum wires. Here, we wish to investigate the Stark effect when the electric field is applied transverse to the axis of cylindrical symmetry, but in hollow cylindrical quantum wires as a function of the ratio of the inner and outer radius of the hollow cylinder. In Section 2, a variational calculation is performed to obtain the energy shift due to the transverse electric field using an infinite confining potential for the carriers, and in Section 3 we discuss our results.

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2. Calculation

The Hamiltonian of a carrier in a cylindrical quantum wire in the presence of an electric field applied transverse to the axis of cylindrical symmetry, and parallel to the x-direction, is

$$H = \frac{p^2}{2m^*} - qF\rho\cos\theta + V_c(\rho), \tag{1}$$

where F is electric field, m^* and q are the carrier effective mass and charge, ρ is the distance of the carrier from the axis of cylindrical symmetry, θ is the angle between the position vector of the carrier and the electric field direction and $V_c(\rho)$ is the confining potential using the infinite well model which vanishes for $r<\rho<R$ and becomes infinite $\rho>R$ and $\rho<r$. Here, r is the inner radius of the hollow cylindrical wire and R is its outer radius. Our assumption of a constant electric field transverse to the wire axis is based on a negligible difference between the dielectric constants in the wire and its surroundings. This would be the case for a GaAs wire embedded in $Ga_{1-x}Al_xAs$ surroundings.

Following an approach similar to that used by Brown and Spector [15,16] in calculating the energies of excitons and hydrogenic impurities in a cylindrical quantum wire, we chose as our variational wave function

$$\Psi(\rho,\theta,z) = N\Phi(\rho)\exp(\beta\cos\theta)\exp(ikz), \tag{2}$$

where k is the wave vector of the carrier along the axis of cylindrical symmetry, β is a real variational parameter which depends upon the electric field, N is the normalization constant of the wave function and

$$\Phi(\rho) = J_0(k_1\rho) - \left[\frac{J_0(k_1 r)}{Y_0(k_1 r)}\right] Y_0(k_1\rho). \tag{3}$$

Here $J_0(k_1\rho)$ is the ordinary cylindrical Bessel function and $Y_0(k_1\rho)$ is the cylindrical Neumann function. The quantity k_1 is determined by the boundary conditions at $\rho = r$ and $\rho = R$ which yields the transcendental equation

$$J_0(k_1 R)Y_0(k_1 r) - J_0(k_1 r)Y_0(k_1 R) = 0. \tag{4}$$

The first exponential factor in Eq. (2) is chosen in analogy to the trial wave functions used for carriers in an electric filed in bulk semiconductors and semiconducting quantum well which seem to give good results for the ground state when compared to the exact wave functions involving Airy functions [2,17]. A simple calculation shows that the normalization is given by

$$N^{-2} = 2\pi M_0(\beta), \tag{5}$$

where

$$M_0(\beta) = \int_0^R d\rho\rho\Phi(\rho)^2(k_1\rho)I_0(2\beta\rho). \tag{6}$$

Here $I_0(x)$ is a modified Bessel function of the first kind.

A straight forward calculation yields the following expression for the expectation value of the Hamiltonian using the variational wave function given by Eq. (2)

$$E(\beta) = \frac{\hbar^2}{2m^*}(k^2 + k_1^2 + \beta^2) - \frac{qF}{2}\frac{d}{d\beta}\ln M_0(\beta). \tag{7}$$

The change in the carriers energy due to the transverse electric field is

$$\Delta E = E(\beta) - E(0) = \frac{qF}{2}\frac{d}{d\beta}\ln M_0(\beta) - \frac{\hbar^2}{2m^*}\beta^2.$$

3. Results

The expression for ΔE as a function of β is minimized to obtain a lower limit of ΔE as a function of the inner and outer radii of the wire and the electric field. In Figs. 1–3, $-\Delta E$ is shown as a function of the inner wire radius r for various electric fields F and for various outer wire radii. In these figures the shift in the energy is given in electron Rydberg units where $Ry = (e^2/2\kappa a)$, the wire radii is given in electron Bohr radii where $a = (\hbar^2\kappa/m^*e^2)$, the electric field is given in atomic units $F_0 = (e/2\kappa a^2)$ and κ is the dielectric constant of the semiconductor. The figures show that except for the inner wire radius close to the outer wire radius, the results are similar to those for a solid cylindrical wire [9]. For fixed inner and outer radii, $-\Delta E$ increases with electric field. Interestingly, even though the Stark field rises the potential at one side of the origin the same amount it lowers the potential at the other side of the origin, the total effect in a given charged particle is not canceled. The reason is that the particles tend to move to the region of lower energy. All these facts yield the result that the quantum system lowers its energy as the Stark field increases. For fixed electric field and outer radius, $-\Delta E$ increases when the inner wire radius increases. This effect is due to the facts that carriers are more confined and that the electric potential is large at ρ near R (which is far from the center of the cylinders) when the inner wire radius increases

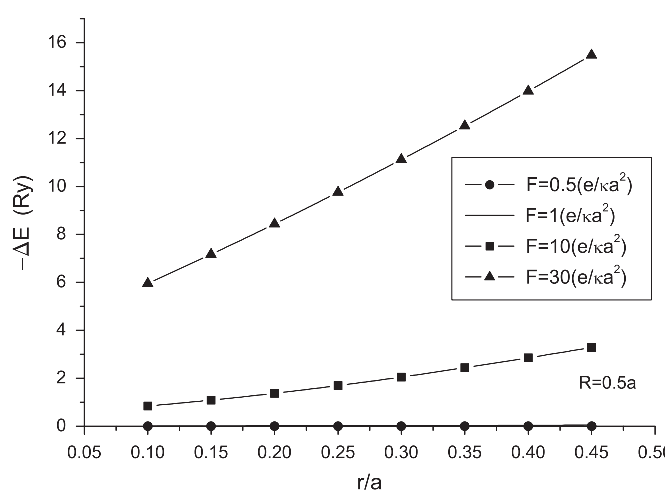

Fig. 1. The Stark shift of the carrier's energy $-\Delta E$ is shown as a function of the inner wire radius r for various electric fields and for an outer wire radius $R = 0.5a$. In this figure and the following figures, the quantities are given in atomic units appropriate for the electron in the semiconductor.

for fixed electric field and outer radius. In Figs. 4–6, we show the dependence of the variational parameter β on r for various values of R and F. In these figures β is given in units of inverse Bohr radii. This variational parameter increases with electric field for fixed inner and outer wire radii and increases with the outer wire radius for fixed inner wire radii and electric field. For small outer radii, the variational parameter increases slightly with increasing inner wire radius while for larger outer radii, it remains almost constant until it drops when the difference between the inner and outer radii becomes small.

In Fig. 7, $-\Delta E$ is shown as a function of the electric field for various outer radii of the wire and for an inner radius of $r = 0.5a$. $-\Delta E$ increases with the electric field for all values of the outer radius with the increase being greater, the larger the outer radius. Also, the greater the difference between the outer and inner radii, the larger $-\Delta E$ is. In

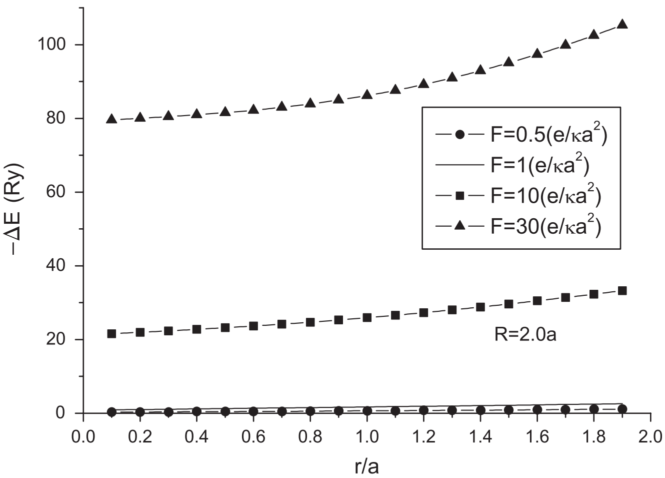

Fig. 2. $-\Delta E$ as a function of the inner wire radius r for various electric fields and for an outer wire radius $R = 2a$.

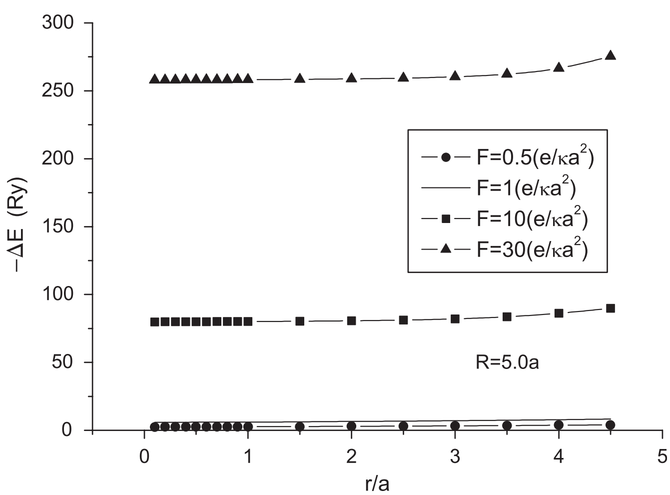

Fig. 3. $-\Delta E$ as a function of the inner wire radius r for various electric fields and for an outer wire radius $R = 5a$.

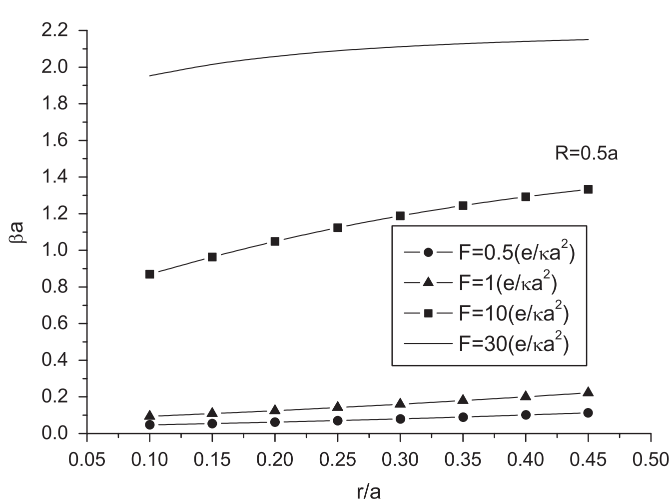

Fig. 4. The variational parameter β is shown as a function of the inner wire radius r for various electric fields and for an outer wire radius $R = 0.5a$.

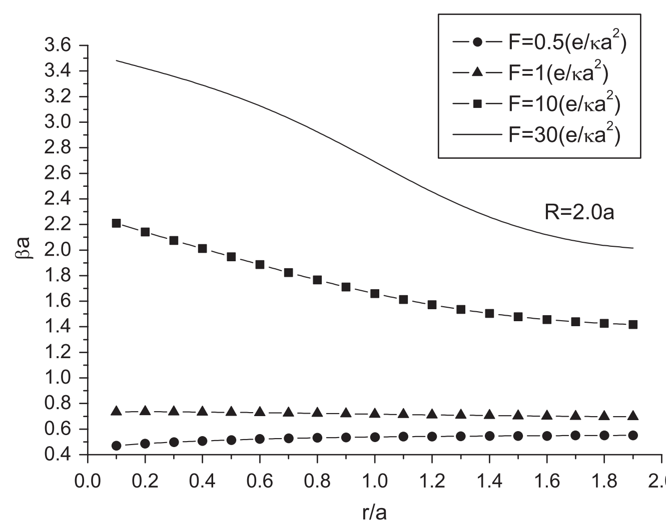

Fig. 5. β as a function of the inner wire radius r for various electric fields and for an outer wire radius $R = 2a$.

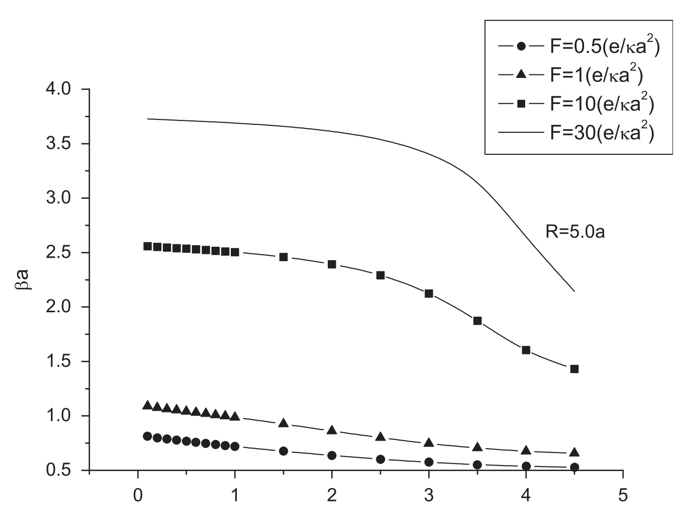

Fig. 6. β as a function of the inner wire radius r for various electric fields and for an outer wire radius $R = 5a$.

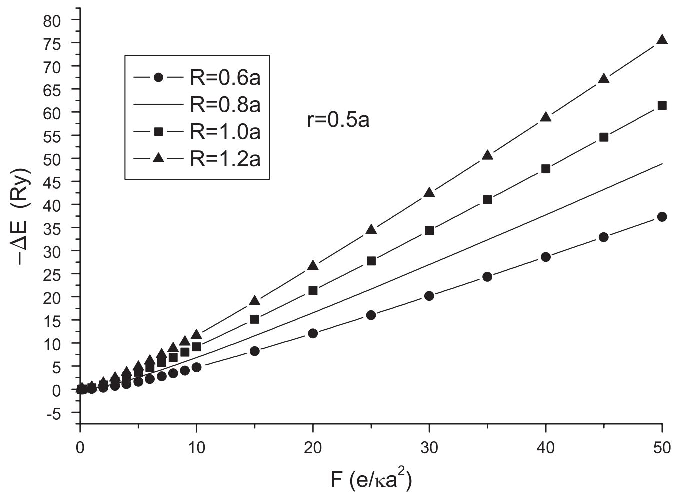

Fig. 7. $-\Delta E$ as a function of the electric field F for $r = 0.5a$ and for various outer radii R.

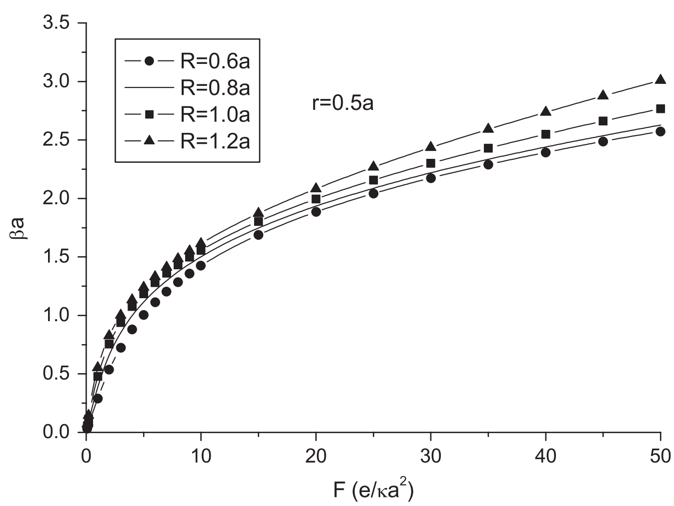

Fig. 8. β as a function of the electric field F for various outer wire radii R and $r = 0.5a$.

Fig. 8, we see that β also increases with the electric field, with the increase being greater, the larger the outer radius of the wire. Again, the greater the difference between the outer and inner radii, the larger β is.

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