

COOPER PAIRS AS BOSONS

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Although BCS pairs of fermions are known to obey neither Bose–Einstein (BE) commutation relations nor BE statistics, we show how Cooper pairs (CPs), whether the simple original ones or the CPs recently generalized in a many-body Bethe–Salpeter approach, being clearly distinct from BCS pairs at least obey BE statistics. Hence, contrary to widespread popular belief, CPs can undergo BE condensation to account for superconductivity if charged, as well as for neutral-atom fermion superfluidity where CPs, but uncharged, are also expected to form.

Keywords: Cooper pairs; Bose–Einstein condensation; superconductors; superfluids.

1. Introduction

A recent electronic analog of the Hanbury Brown–Twiss photon-effect experiment Samuelsson and Büttiker¹ strongly suggest electron pairs in a normal/superconductor junction to be bosons although further work seems needed to provide compelling empirical proof. On the other hand, assuming Cooper pairs (CPs) to be bosons, it was recently proven^{2,3} within a generalized Bose–Einstein condensation (BEC) model that includes the BCS–Bose crossover theory^{4–20} as a special case, that a BCS condensate is precisely a Bose–Einstein (BE) condensate consisting of equal numbers of particle- and hole-CPs, at least for the Cooper/BCS^{21,22} model interfermion interaction in the limit of weak coupling when the crossover picture reduces to BCS theory. This is significant since BCS argued (Ref. 22, footnote 18) that “*our transition is not analogous to a Bose–Einstein condensation (BEC).*” Somewhat later, Bardeen wrote²³ that “*... the picture by Schafroth (1955) ... of electron pairs ... which at low temperature undergo a BEC, is not valid.*” In contrast, both Feynman²⁴ and Josephson²⁵ referred without proof to CPs as bosons.

On the other hand, a long-standing common objection to attempts^{2,3,26,27} to unify BCS and BEC theories for a description of *superconductivity* has been that

CPs, strictly speaking, cannot be considered bosons. In this paper we establish a clear distinction between BCS pairs and CPs, showing that while the former are not bosons the latter are very definitely, at least insofar as they obey BE statistics. The distinction, though elementary, is hardly if ever mentioned in the superconductivity literature, though it appears not to be as unacceptable among the neutral-fermion *superfluidity* community where it is also applied, albeit with a different interfermion interaction, to the BCS and Cooper pairs as in liquid ^3He (Ref. 28) and in ultra-cold trapped alkali Fermi gases such as ^{40}K (Ref. 29) and ^6Li (Ref. 30). Recent studies^{31–35} have also appeared addressing superfluidity in degenerate Fermi gases. Indeed, a BE-like condensate of dimers formed from neutral ^{40}K (Ref. 36) or ^6Li (Refs. 37–39) fermionic atoms via magnetically-tunable “Feshbach resonances” has only recently finally been observed.

2. Cooper Pairs

The original or ordinary CPs²¹ emerge by solving a Schrödinger-like equation in momentum space for two *particles* above the ideal-Fermi-gas (IFG) sea of the remaining $N - 2$ system fermions. If two-hole pairs are also included, the extended Cooper eigenvalue equation yields^{40,42} purely imaginary values. This precludes defining CPs relative to an IFG sea. However, a more both general and self-consistent treatment starts with an unperturbed Hamiltonian representing not the Fermi sea but BCS-correlated ground state in the Bethe–Salpeter (BS) integral equations *with* hole propagation. When solved^{43–45} in the ladder approximation for bound two-particle and two-hole states, non-purely-imaginary eigenvalues are restored for what has been called the “moving CP” solution of the BS equations. The BS results in 3D (Ref. 43), 2D (Ref. 44), and 1D (Ref. 45) are consistent with each other, as expected.

Unlike the Fermi-atom case, the interfermionic interactions in (especially cuprate) superconductors are a deep, problematic and highly controversial issue.^{46–49} So, we confine ourselves to an electron gas interacting pairwise via the ingeniously simple Cooper/BCS model interfermion interaction^{21,22}

$$V_{\mathbf{k}\mathbf{k}'} = \begin{cases} -V & \text{if } k_F \text{ or } \sqrt{k_F^2 - k_D^2} < \left| \mathbf{k} \pm \frac{1}{2}\mathbf{K} \right|, \left| \mathbf{k}' \pm \frac{1}{2}\mathbf{K} \right| < \sqrt{k_F^2 + k_D^2} \\ 0 & \text{otherwise,} \end{cases} \quad (1)$$

where $V_{\mathbf{k}\mathbf{k}'} \equiv L^{-d} \int d\mathbf{r} \int d\mathbf{r}' e^{-i\mathbf{k}\cdot\mathbf{r}} V(\mathbf{r}, \mathbf{r}') e^{i\mathbf{k}'\cdot\mathbf{r}'}$ is the double Fourier transform of $V(\mathbf{r}, \mathbf{r}')$, the (possibly nonlocal) interaction in d -dimensional coordinate space, with \mathbf{r} the relative coordinate of the two electrons. Here $V > 0$, and $\hbar\omega_D \equiv \hbar^2 k_D^2 / 2m$ is the maximum energy of a phonon associated with the vibrating ionic lattice underlying the electron gas, while $E_F \equiv \hbar^2 k_F^2 / 2m$ is the Fermi energy and m is the effective electron mass. In Ref. 21 there are no hole pairs (*viz.*, below E_F) so the lower limit in Eq. (1) was k_F , while in Ref. 22 it is $\sqrt{k_F^2 - k_D^2}$. In the simpler Cooper case it means that two electrons with momentum wavevectors \mathbf{k}_1

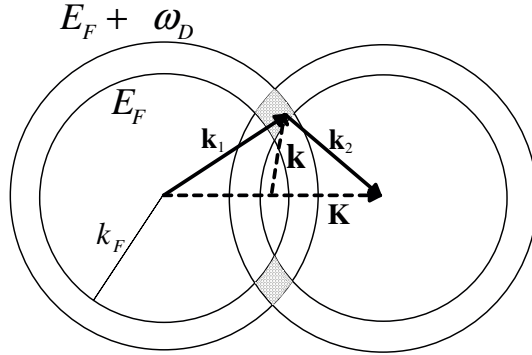


Fig. 1. Cross-section of overlap volume in k -space (shading) where the tip of the relative pair wavevector $\mathbf{k} \equiv \frac{1}{2}(\mathbf{k}_1 - \mathbf{k}_2)$ must point for the attractive Cooper/BCS model interaction (1) to be nonzero for a pair of total (or center-of-mass) momentum $\hbar\mathbf{K} \equiv \hbar(\mathbf{k}_1 + \mathbf{k}_2)$. The tail of \mathbf{k} is at the midpoint of \mathbf{K} .

and \mathbf{k}_2 interact with a constant net attraction $-V$ when the tip of their relative-momentum wavevector $\mathbf{k} \equiv \frac{1}{2}(\mathbf{k}_1 - \mathbf{k}_2)$ points anywhere inside the shaded overlap volume in k -space of the two spherical shells in Fig. 1 whose center-to-center separation is $\mathbf{K} \equiv \mathbf{k}_1 + \mathbf{k}_2$, the total (or center-of-mass) momentum wavevector for the pair. Otherwise, there is no interaction and hence no pairing. For neutral-fermion systems there is no cutoff $\hbar\omega_D$ and V is *not* constant in \mathbf{k} and \mathbf{k}' , except when the interaction range is much shorter than the average fermion spacing as in a trapped Fermi gas where it has become customary to employ contact (or δ) interactions. Such are a special case of the more general finite-ranged, separable interaction, as is the interaction (1), introduced in Ref. 9, and for which the conclusions below will continue to apply.

Vectors \mathbf{k} ending at all points of a simple-cubic lattice in k -space (in the 3D case) with lattice spacing $2\pi/L$ with L the system size, ensure that the interaction (1) is nonzero provided such points are in the shaded overlap volume of Fig. 1. The endpoints of two vectors \mathbf{k} and \mathbf{k}' are illustrated in Fig. 2. In the thermodynamic limit there are infinitely many acceptable \mathbf{k} values allowed for each fixed value of \mathbf{K} for the $K \geq 0$ CP eigenvalue equation^{21,50} with the interaction (1), namely

$$V \sum_{\mathbf{k}}' (\hbar^2 k^2/m + \hbar^2 K^2/4m - 2E_F - \mathcal{E}_K)^{-1} = 1. \quad (2)$$

The prime on the summation sign signifies the restrictions in the interaction (1) using k_F as lower limit. For CPs consisting of equal and opposite momentum electrons $K = 0$, solving Eq. (2) analytically gives the familiar result of infinite-lifetime, negatively-bound (with respect to $2E_F$) CPs

$$\mathcal{E}_0 = -2\hbar\omega_D / [\exp\{2/N(E_F)V\} - 1] \xrightarrow{\lambda \rightarrow 0} -2\hbar\omega_D e^{-2/\lambda} \quad (3)$$

where $\lambda \equiv N(E_F)V \geq 0$ with $N(E_F)$ the density of electronic states at the Fermi energy. The first expression for \mathcal{E}_0 is exact for all values λ in 2D [where $N(\epsilon)$ is

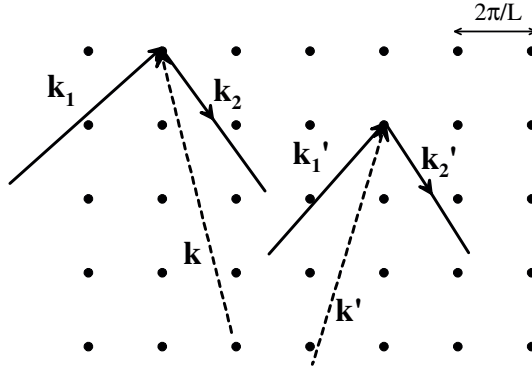


Fig. 2. Two distinct relative-momentum wavevector values $\mathbf{k} \equiv \frac{1}{2}(\mathbf{k}_1 - \mathbf{k}_2)$ and $\mathbf{k}' \equiv \frac{1}{2}(\mathbf{k}'_1 - \mathbf{k}'_2)$ (dashed lines) whose tips are within the overlap volume of Fig. 1. They contribute to the summation in (2) or to the integral in (5) to determine a CP energy \mathcal{E}_K . This illustrates why the $\delta_{\mathbf{k},\mathbf{k}'}$ term for BCS pairs in (15) is *always* zero for CPs, with the Pauli Principle being strictly satisfied.

constant] and is otherwise a good approximation if $\hbar\omega_D \ll E_F$ as occurs in metals. If hole CPs are included in the BS scheme along with electron CPs as in Eq. (2) and Eq. (3), instead of Eq. (3) one obtains the evidently unphysical purely-imaginary result

$$\mathcal{E}_0 = \pm i2\hbar\omega_D/\sqrt{e^{2/\lambda} - 1} \tag{4}$$

(see detailed treatment in Ref. 27). Replacing the IFG sea by the BCS ground-state sea, instead of Eq. (2) the eigenvalue equation for \mathcal{E}_K in 2D (Ref. 44 (and a very similar one in 3D (Ref. 43) and in 1D (Ref. 45))) is rather

$$\frac{1}{2\pi} \lambda \hbar v_F \int_{k_F - k_D}^{k_F + k_D} dk \int_0^{2\pi} d\varphi u_{\mathbf{K}/2+\mathbf{k}} v_{\mathbf{K}/2-\mathbf{k}} \times \{u_{\mathbf{K}/2-\mathbf{k}} v_{\mathbf{K}/2+\mathbf{k}} - u_{\mathbf{K}/2+\mathbf{k}} v_{\mathbf{K}/2-\mathbf{k}}\} \times \frac{E_{\mathbf{K}/2+\mathbf{k}} + E_{\mathbf{K}/2-\mathbf{k}}}{-\mathcal{E}_K^2 + (E_{\mathbf{K}/2+\mathbf{k}} + E_{\mathbf{K}/2-\mathbf{k}})^2} = 1 \tag{5}$$

where $v_F \equiv \sqrt{2E_F/m}$, φ is the angle between \mathbf{K} and \mathbf{k} , k_D is defined by $k_D/k_F \equiv \hbar\omega_D/2E_F \ll 1$, $E_{\mathbf{k}} \equiv \sqrt{\xi_{\mathbf{k}}^2 + \Delta^2}$ with $\xi_{\mathbf{k}} \equiv \hbar^2 k^2/2m - E_F$ and Δ is the electronic gap, while $v_k^2 \equiv \frac{1}{2}(1 - \xi_k/E_{\mathbf{k}})$ and $u_k^2 \equiv 1 - v_k^2$ are the usual Bogoliubov functions.⁵¹ Note that the lower limit in the interaction (1) is being taken as $\sqrt{k_F^2 - k_D^2}$ as in BCS theory²² which does not exclude hole pairings. Besides endowing CPs with the physically expected *finite* lifetimes, the more general BS treatment yields the nontrivial eigenvalue solution for $K = 0$ given by $\mathcal{E}_0 = 2\Delta$, with $\Delta \equiv \hbar\omega_D/\sinh(1/\lambda) \xrightarrow{\lambda \rightarrow 0} 2\hbar\omega_D e^{-1/\lambda}$ the single-electron BCS energy gap, as opposed to the ordinary CP problem giving Eq. (3). [Note the factor of two difference in the exponential compared with Eq. (3) due to excluding²¹ or not²² hole pairs.] The nontrivial (new) BS solution governed by Eq. (5) has been called⁴³⁻⁴⁵ a “moving CP” composite-boson excitation mode to distinguish it from another (a

trivial or known) sound wave solution sometimes called the Anderson–Bogoliubov–Higgs (Ref. 40 p. 44), Ref. 52 and⁵³ excitation mode which in contrast with the moving CP mode vanishes as $K \rightarrow 0$ for any finite coupling, i.e. is gapless, and is governed by another eigenvalue equation⁴³⁴⁴ different from Eq. (5). In the pure boson gas, on the other hand, both “particle” and “sound” solutions are indistinguishable.^{54,55} In the fermion case of interest here it should be feasible to search for, identify and distinguish both particle and sound modes experimentally, e.g. see Ref. 56.

3. BCS Pairs with Nonzero Total Momentum

Whether the pairwise interfermion interaction is between charge carriers or between neutral atoms, a CP state of energy \mathcal{E}_K , defined via eigenvalue equation (2) or (5), will clearly be characterized *only by a definite \mathbf{K} but not definite \mathbf{k}* . Although elementary, this is our main point. It contrasts with that of a “BCS pair” defined as a dimer with fixed \mathbf{K} and \mathbf{k} (or equivalently fixed \mathbf{k}_1 and \mathbf{k}_2), even though only the case $\mathbf{K} = 0$ is considered in Ref. 22, for which their annihilation $b_{\mathbf{k}}$ and creation $b_{\mathbf{k}}^\dagger$ operators are not quite bosonic since they obey the relations, Ref. 22, Eqs. (2.11)–(2.13),

$$[b_{\mathbf{k}}, b_{\mathbf{k}'}^\dagger] = (1 - n_{-\mathbf{k}\downarrow} - n_{\mathbf{k}\uparrow})\delta_{\mathbf{k}\mathbf{k}'} \quad (6)$$

$$[b_{\mathbf{k}}^\dagger, b_{\mathbf{k}'}^\dagger] = [b_{\mathbf{k}}, b_{\mathbf{k}'}] = 0 \quad (7)$$

where $n_{\pm\mathbf{k}s} \equiv a_{\pm\mathbf{k}s}^\dagger a_{\pm\mathbf{k}s}$ are fermion number operators, with creation $a_{\mathbf{k}_1s}^\dagger$ and annihilation $a_{\mathbf{k}_1s}$ operators referring to the fermions, and

$$\{b_{\mathbf{k}}, b_{\mathbf{k}'}\} = 2b_{\mathbf{k}}b_{\mathbf{k}'}(1 - \delta_{\mathbf{k}\mathbf{k}'}) \quad (8)$$

which is not quite fermionic, unless $\mathbf{k} = \mathbf{k}'$ when Eq. (6) is not bosonic. The precise Bose commutation relations are, of course,

$$[b_{\mathbf{k}}, b_{\mathbf{k}'}^\dagger] = \delta_{\mathbf{k}\mathbf{k}'} \quad (9)$$

$$[b_{\mathbf{k}}^\dagger, b_{\mathbf{k}'}^\dagger] = [b_{\mathbf{k}}, b_{\mathbf{k}'}] = 0 \quad (10)$$

with Eq. (9) differing sharply from Eq. (6). The fermion creation $a_{\mathbf{k}_1s}^\dagger$ and annihilation $a_{\mathbf{k}_1s}$ operators satisfy the usual Fermi anti-commutation relations

$$\{a_{\mathbf{k}_1s}^\dagger, a_{\mathbf{k}'_1s'}^\dagger\} = \{a_{\mathbf{k}_1s}, a_{\mathbf{k}'_1s'}\} = 0 \quad (11)$$

$$\{a_{\mathbf{k}_1s}, a_{\mathbf{k}'_1s'}^\dagger\} = \delta_{\mathbf{k}_1\mathbf{k}'_1} \delta_{ss'} \quad (12)$$

The distinction between BCS pairs and CPs holds for the original or “ordinary” CPs Ref. 21 as in Eqs. (2) and (3). It also applies to the generalized BS CPs defined more consistently without excluding two-hole pairs when the lower limit in the interaction (1) is taken as $\sqrt{k_F^2 - k_D^2}$ as in BCS theory.²²

The BCS-pair annihilation and creation operators for any $\mathbf{K} > 0$ can be defined quite generally as

$$b_{\mathbf{k}K} \equiv a_{\mathbf{k}_2\downarrow} a_{\mathbf{k}_1\uparrow} \quad \text{and} \quad b_{\mathbf{k}K}^\dagger \equiv a_{\mathbf{k}_1\uparrow}^\dagger a_{\mathbf{k}_2\downarrow}^\dagger \tag{13}$$

where $a_{\mathbf{k}_1s}^\dagger$ and $a_{\mathbf{k}_1s}$ obey Eqs. (11) and (12), and as before $\mathbf{k} \equiv \frac{1}{2}(\mathbf{k}_1 - \mathbf{k}_2)$ and $\mathbf{K} \equiv \mathbf{k}_1 + \mathbf{k}_2$ are the relative and total (or center-of-mass) momentum wavevectors, respectively, associated with two fermions with wavevectors

$$\mathbf{k}_1 = \mathbf{K}/2 + \mathbf{k} \quad \text{and} \quad \mathbf{k}_2 = \mathbf{K}/2 - \mathbf{k}. \tag{14}$$

Using the same techniques to derive Eqs. (6)–(8) valid for $\mathbf{K} = 0$, the operators $b_{\mathbf{k}\mathbf{K}}$ and $b_{\mathbf{k}\mathbf{K}}^\dagger$ are found to satisfy: (a) the “pseudo-boson” commutation relations

$$[b_{\mathbf{k}K}, b_{\mathbf{k}'K}^\dagger] = (1 - n_{\mathbf{K}/2-\mathbf{k}\downarrow} - n_{\mathbf{K}/2+\mathbf{k}\uparrow})\delta_{\mathbf{k}\mathbf{k}'} \tag{15}$$

$$[b_{\mathbf{k}\mathbf{K}}^\dagger, b_{\mathbf{k}'\mathbf{K}}^\dagger] = [b_{\mathbf{k}\mathbf{K}}, b_{\mathbf{k}'\mathbf{K}}] = 0 \tag{16}$$

where $n_{\mathbf{K}/2\pm\mathbf{k}s} \equiv a_{\mathbf{K}/2\pm\mathbf{k}s}^\dagger a_{\mathbf{K}/2\pm\mathbf{k}s}$ are fermion number operators; as well as (b) a “pseudo-fermion” anti-commutation relation

$$\{b_{\mathbf{k}\mathbf{K}}, b_{\mathbf{k}'\mathbf{K}}\} = 2b_{\mathbf{k}\mathbf{K}} b_{\mathbf{k}'\mathbf{K}} (1 - \delta_{\mathbf{k}\mathbf{k}'}). \tag{17}$$

Our only restriction was that $\mathbf{K} \equiv \mathbf{k}_1 + \mathbf{k}_2 = \mathbf{k}'_1 + \mathbf{k}'_2$. If $\mathbf{K} = 0$ so that $\mathbf{k}_1 = -\mathbf{k}_2 = \mathbf{k}$ (the only case considered by BCS), and calling $b_{\mathbf{k}\mathbf{K}=0} \equiv b_{\mathbf{k}}$, etc., Eqs. (15)–(17) become Eqs. (6)–(8), as they should. So, neither BCS pairs with $\mathbf{K} \geq 0$ are bosons as the relation (15) contains additional terms not⁵⁸ present in the the usual boson commutation relations analogous to Eq. (9).

To our knowledge, no one has yet succeeded in constructing CP creation and annihilation operators that obey Bose commutation relations, starting from fermion creation $a_{\mathbf{k}_1s}^\dagger$ and annihilation $a_{\mathbf{k}_1s}$ operators, as is *postulated* in Refs. 2 and 3 in a generalized BEC theory that contains BCS theory as a special case. This postulation is grounded in magnetic-flux quantization experiments^{59–61} that establish the presence of charged pairs — albeit without being able⁶² to specify the *sign* of those charges. However, although the eigenvalues of $b_{\mathbf{k}\mathbf{K}}^\dagger b_{\mathbf{k}\mathbf{K}}$ are 0 or 1 in keeping with the Pauli Exclusion Principle, those of $\sum_{\mathbf{k}} b_{\mathbf{k}\mathbf{K}}^\dagger b_{\mathbf{k}\mathbf{K}}$ are evidently 0, 1, 2, ... because of the indefinitely many values taken on by the summation index \mathbf{k} . This implies BE statistics and corroborates the qualitative conclusions reached above. A discussion in greater detail of this is found in Refs. 63 and 64.

4. Conclusion

In conclusion, in the thermodynamic limit *any* number of CPs with a definite total (or center-of-mass) wavevector \mathbf{K} can occupy a *single* state of CP energy \mathcal{E}_K and thus obey BE statistics. Hence, CPs can undergo a BEC. All this holds for *any* coupling, i.e. regardless of the size of the CPs and of their mutual overlap. As in liquid ⁴He, the BEC statistical mechanism may serve as a starting point —

even before the effects of interboson interactions (excluded also in BCS theory) are accounted⁶⁵ for — in constructing a microscopic theory of superconductivity, or of neutral-fermion superfluidity, that will eventually lead to T_c values calculated from first principles without adjustable parameters and that hopefully agree with observed ones.

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