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FURTHER EVIDENCE FOR LINEARLY-DISPERSIVE COOPER PAIRS

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A recent Bose–Einstein condensation (BEC) model of several cuprate superconductors is based on bosonic Cooper pairs (CPs) moving in 3D with a *quadratic* energy-momentum (dispersion) relation. The 3D BEC condensate-fraction versus temperature formula poorly fits penetration-depth data for two cuprates in the range $1/2 < T/T_c \leq 1$ where T_c is the BEC transition temperature. We show how these fits are dramatically improved, assuming cuprates to be quasi-2D, and how equally good fits are obtained for conventional 3D and quasi-1D nanotube superconducting data, provided the correct *linear* CP dispersion is assumed in BEC at their assumed corresponding dimensionalities. This is offered as additional concrete empirical evidence for linearly-dispersive pairs in another recent BEC scenario of superconductors within which a BCS condensate turns out to be a very special case.

Keywords: Cooper pairs; dispersion relation; Bose–Einstein condensation; lower dimensions.

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1. Introduction

A Bose–Einstein condensation (BEC) model was applied by Rosencwaig in Ref. 1 to address seven cuprate superconductors (SCs) with transition temperatures T_c at optimal doping ranging from 22 K to 133 K. These are: $La_{2-x}Sr_xCuO_4$ (LSCO), $Nd_{2-x}Ce_xCuO_4$ (NCCO), $YBa_2Cu_3O_{7-y}$ Y123, $Bi_2Sr_2CaCu_2O_{8-y}$ Bi2212, $Bi_2Sr_2Ca_2Cu_3O_{10-y}$ (Bi2223), $HgBa_2CaCu_2O_{7-y}$ (Hg1212) and $HgBa_2Ca_2Cu_3O_{9-y}$ (Hg1223). His starting point is the well-known fact that BEC in an ideal Bose gas occurs below temperatures T such that the thermal wavelength $\lambda \equiv h/\sqrt{2\pi m_B k_B T}$ becomes larger than the average inter-bosonic separation, with m_B the boson mass, and h, k_B the Planck and Boltzmann constants, respectively. More exactly, BEC sets in whenever

$$n_B\lambda^3 > 2.612\,, \tag{1}$$

where n_B is the boson number density, and λ is taken as the bosonic quasiparticle diameter. This leads to a critical temperature T_c given by the familiar formula

$$T_c = \frac{2\pi\hbar^2 n_B^{2/3}}{(2.612)^{2/3} m_B k_B} \simeq \frac{3.31\hbar^2 n_B^{2/3}}{m_B k_B} \propto n_B^{2/3}\,, \tag{2}$$

of conventional BEC theory. He identifies an interaction distance with λ, which thus becomes T-independent, while the number-density n_B of weakly-interacting "preformed" electron or hole pairs acquires a T-dependence. Associated with (2) is the BE condensate fraction

$$N_0(T)/N_0(0) = \begin{cases} 1 - (T/T_c)^{3/2} & \text{for } T \leq T_c\,, \\ 0 & \text{otherwise}\,, \end{cases} \tag{3}$$

where $N_0(T)$ is the number of bosonic pairs at temperature T in the lowest-energy state with (total, or) center-of-mass momentum wavenumber $K = 0$, and $N_0(0)$ is that same number at $T = 0$.

2. Cooper Pair Dispersion

All of this assumes three dimensions (3D) and that boson excitation energies are given by

$$\varepsilon_K = \hbar^2 K^2 / 2m_B\,. \tag{4}$$

This would hold if the composite bosons moved *in vacuo* such as, say, a deuteron of mass $m_B = m_p + m_n$ in empty space, with m_p and m_n being the proton and neutron masses. However, in the presence of the Fermi sea of the other single charge carriers, the bosonic "dispersion relation" becomes *linear*[2,4] in leading order rather than quadratic as in (4). Reference 2 first mentioned, and Refs. 3 and 7 later discussed this linearity in detail. It is associated with the original Cooper pair (CP) problem[8] of two electrons (or holes) above (or below) the Fermi surface of the remaining system electrons. It was also found in a more general view of CPs in Refs. 9 and 10 within a many-body Green's function formalism treating both electron- and hole-pairs on an equal footing. For either ordinary or generalized CPs, the leading term in the K-expansion is linear. This linearly-dispersive "moving CP" object is often confused in the literature with the more common Anderson–Bogoliubov–Higgs (ABH)[11] (Ref. 12, p. 44)[13] collective excitation which is also linear in leading order, but which is just the sound mode of the many-fermion system. By contrast, in a many-boson system, these two modes, the "particle" and "sound" modes, are apparently identical.[14,15]

A particularly clear example comparing linear and quadratic dispersion is perhaps the analytical result of Ref. 6 in 2D for an attractive delta potential assumed between electrons. This interfermion interaction mimics the net effect of Coulomb repulsion plus attractive, say, electron-phonon interactions. The 2D delta potential well, which otherwise supports an infinite number of bound levels, is imagined "regularized"[16] to support a single bound level of energy B_2 as occurs,[17] e.g., with the two-parameter Cooper/BCS[8,19] model interelectronic interaction. Miyake used this interaction to obtain[18] both the zero-temperature BCS gap Δ and the chemical potential μ analytically in terms of B_2. Since the regularized delta well turns out to be infinitesimally weak, its 0^+ strength can be eliminated[6] in favor of B_2 which then plays the role of the coupling constant, with $0 \leq B_2 < \infty$ spanning weak to strong coupling. Instead of (4), a more general analytical expression found in Ref. 6 that includes the Fermi sea is

$$\varepsilon_K = \frac{2}{\pi}\hbar v_F K + \left[1 - \left\{2 - \left(\frac{4}{\pi}\right)^2\right\}\frac{E_F}{B_2}\right]\frac{\hbar^2 K^2}{2(2m_e)} + O(K^3)\,, \tag{5}$$

where v_F is the Fermi velocity defined through the Fermi surface energy $E_F \equiv m_e v_F^2/2$ and m_e is the effective electron mass. The leading term is linear, and *only* in the vacuum limit ($v_F \to 0$, implying $E_F \to 0$) does it precisely become the quadratic (4) with $m_B = 2m_e$ expected physically *for any fixed coupling* B_2. Figure 1 of Ref. 6 exhibits the smooth crossover in 2D from a purely linear to a purely quadratic form, as one increases coupling and/or as one "switches off" the Fermi sea medium (non-zero E_F) in which the pair propagates down to the pure vacuum (zero E_F) medium. A very similar behavior was also observed in 3D,[7] but only numerically.

3. Bose–Einstein Condensation

Expressions more general than (2) and (3), but reducing to them, are known[20] in any dimensionality $d > 0$ (integer or not) and for any dispersion relation

$$\varepsilon_K = C_s K^s, \qquad \text{with} \quad s > 0. \tag{6}$$

They are

$$T_c = \frac{C_s}{k_B}\left[\frac{s\Gamma(d/2)(2\pi)^d n_B}{2\pi^{d/2}\Gamma(d/s)g_{d/s}(1)}\right]^{s/d} \quad \propto \quad n_B^{s/d}\,, \tag{7}$$

and

$$N_0(T)/N_0(0) = \begin{cases} 1 - (T/T_c)^{d/s} & \text{for } T \leq T_c \\ 0 & \text{otherwise.} \end{cases} \tag{8}$$

Here $g_\sigma(1)$ are the Bose integrals which for $\sigma > 1$ coincides with the Riemann Zeta-function $\zeta(\sigma)$ and diverges for $\sigma \leq 1$. $\Gamma(\sigma)$ is the gamma function, and $n_B \equiv N/L^d$ is the d-dimensional boson number density. The divergence of $g_\sigma(1)$ for $\sigma \leq 1$ ensures from (7) that $T_c \equiv 0$ for all $d \leq s$, but that otherwise T_c is non-vanishing. In $d = 3$ and quadratic dispersion $s = 2$ and, if $C_2 = \hbar^2/2m_B$, (7) and (8) respectively become (2) and (3), as $g_{3/2}(1) \equiv \zeta(3/2) \simeq 2.612$. However, for $s = 1$ BEC *can* occur for all $d > 1$. This coincides, fortuitously, with *all* dimensions where actual superconductors have been found to exist, down to the quasi-one-dimensional organics[21,23] consisting of parallel chains of molecules. As regards to dimensionality, therefore, the BEC picture contrasts sharply with the BCS scheme where T_c is non-vanishing for all $d > 0$ even though no exactly 1D superconductors have been found to date. In fact, beautiful experiments[24,25] with nanowires of different thicknesses sputter-coated with an amorphous superconductor ($T_c \simeq 5.5K$) have shown how superconductivity is extinguished for the smallest nanowire diameters interpreted as approaching precisely 1D.

Although the creation/annihilation operators of BCS pairs do *not* obey the usual Bose commutation rules [see Eqs. (2.11) to (2.13) of Ref. 19; see also p. 38 of Ref. 2], CPs in fact *satisfy BE statistics.* Indeed, BCS pairs and CPs are *distinct.* A BCS pair is defined with fixed total (or center-of-mass) momentum wavevector $\mathbf{K} \equiv \mathbf{k_1} + \mathbf{k_2}$ *and* fixed relative-momentum wavevector $\mathbf{k} \equiv (\mathbf{k_1} - \mathbf{k_2})/2$, whereas a CP is defined with fixed $\mathbf{K}$ only, since a sum over $\mathbf{k}$ is implied in any conceivable formulation of CPs. This is because in the thermodynamic limit an indefinitely large number of BCS pairs, each with fixed momenta $\hbar\mathbf{k_1}$ and $\hbar\mathbf{k_2}$, correspond to different relative momenta $\hbar\mathbf{k}$ but whose $\mathbf{k_1}$ and $\mathbf{k_2}$ add vectorially to the *same* total $\mathbf{K}$. These remarks apply even when only $\mathbf{K} = \mathbf{0}$ pairs were considered in Ref. 19.

4. Results

Empirical evidence for the *linearly-dispersive* nature of CPs in BSCCO has been argued by Wilson[26] as being suggested by the scanning tunneling microscope conductance scattering data obtained by Davis and coworkers[27,28] in this cuprate. In Fig. 1 we present additional evidence, based on experimental data from penetration-

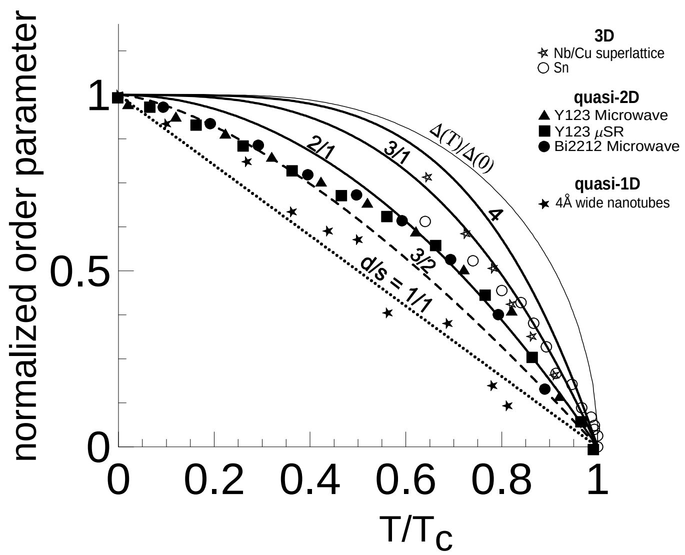

Fig. 1. BE condensate-fraction curves for bosons in $d = 3$, 2, or 1.

depth measurements in two 3D SCs[29,30] and two quasi-2D cuprates,[31,33] as well as from gap measurements in a quasi-1D nanotube SC.[34] When plotted as a presumably universal "normalized order parameter" the data depart substantially (at least in 3D and 2D) from the BCS normalized gap order parameter, but are seen to agree quite well, at least for $T > 0.5T_c$, with the *pure-phase* (only $2e$- *or* $2h$-CP) BEC condensate-fraction formula (8) for d = 3, 2 and 1, *provided one assumes* $s = 1$. For lower T's, one can argue[35] that a *mixed* BEC phase containing both $2e$- and $2h$-CPs becomes more stable (i.e., has lower Helmholtz free energy) so that the simple pure-phase formula (8) is no longer strictly valid. Indeed, (8) applies to the CPs because in the binary boson-fermion gas mixture — say, a Cooper/BCS model interaction forming the bosonic CPs with a maximum allowed[39,40] coupling of $\lambda = 1/2$ — only a minuscule fraction ($< 0.1\%$)[41] of the individual fermion charge carriers are paired up into CPs, ensuring that a substantial Fermi sea is still present. Such tiny fractions are consistent with some very recent far-infrared charge-dynamics measurements[42,43] in LSCO.

In Fig. 1 we show BE condensate-fraction curves (in thick) $1 - (T/T_c)^{d/s}$ for bosons in d = 3, 2, or 1, assuming dispersion relation (6) for s = 2 and 1, for a *pure* phase of either $2e$- or $2h$-CPs, compared to empirical data for 3D SCs (Nb/Cu and Sn); for two quasi-2D SCs (Y123 and Bi2212 with $T_c \simeq 93$ K and 91 K, respectively); and a quasi-1D SC (4 Å-wide nanotubes with $T_c \simeq 15$ K). The dashed curve labeled $d/s = 3/2$ appears in Fig. 6 of Ref. 1 and seems to provide the only adjustable-parameter-free comparison with experimental data in that paper. The ordinate axis refers to a universal "normalized order parameter." Data for the 3D and 2D SCs refer to penetration depth measurements. Nanotube data are gap $\Delta(T)$ measurements giving $\Delta(T)/\Delta(0)$ but are plotted as $[\Delta(T)/\Delta(0)]^2$ so as to coincide with the $2h$-CP condensate fraction $m_0(T)/m_0(0)$ according to the relation $\Delta(T) = f\sqrt{m_0(T)}$, with f a boson-fermion coupling constant (which drops out from the normalized order parameter), that follows for $2h$-CP condensates from the generalized BEC theory of Ref. 36. The dotted straight line marked $d/s = 1/1$ strictly corresponds from (7) to $T_c \equiv 0$. However, it serves as a lower bound to all curves with $d/s = (1+\epsilon)/1 > 1$ for small but non-zero ϵ, implying *quasi*-1D geometries for which $T_c > 0$. Also shown for reference are the two-fluid model[37] curve $1-(T/T_c)^4$ and the BCS normalized gap $\Delta(T)/\Delta(0)$ order-parameter curve.[38]

5. Conclusions

To conclude, we have presented normalized order-parameter data based on penetration-depth and gap measurements that strongly suggest a *linear* energy versus center-of-mass-momentum (dispersion) relation for Cooper pairs in various materials that can be viewed as 3D, quasi-2D and quasi-1D superconductors (SCs). The linearity is a manifestation of the Fermi sea background in which the pairs propagate, as opposed to the quadratic relation of composite bosons moving *in* *vacuo.* It ensures that a BEC picture of SCs is applicable over all dimensionalities in which SCs occur.

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