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# Helical anisotropy and magnetoimpedance of CoFeSiB wires under torsional stress

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#### Abstract

Recent measurements of the magnetoimpedance (at a fixed frequency of 1 MHz) of cobalt-rich wires subjected to torsion stress show an asymmetry as a function of torsion angle stemming from residual anisotropies induced during wire fabrication. We interpret these measurements with a simple model based on the competition between a circumferential magnetic anisotropy and another one induced by torsional or residual stress. This allows extraction of the physical parameters of the wire and explains the positive and negative torsion cases. The agreement between theoretical and experimental results provides a firm support for the model describing the behavior of the anisotropy field versus static magnetic field for all torsion angles.

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#### 1. Introduction

The giant magnetoimpedance effect (GMI) in amorphous ribbons, wires and thin films has become a topic of growing interest for a wide variety of prospective applications in storage information technology and sensors possessing high sensitivity and fast response [1,2].

Magnetoimpedance effect (MI) consists in the change of impedance introduced by a low-amplitude AC current  $I_{\rm AC}$  flowing through a magnetic conductor under application of a static magnetic field  $H_{\rm DC}$  (usually parallel to the direction of the AC current). The origin of this behavior is related to the relative magnetic permeability  $\mu_{\rm r}$  and the direction of the magnetic anisotropy field  $H_{\rm K}$ . When MI measurements are performed on wires, injecting an AC current in the wire and applying a DC magnetic field  $H_{\rm DC}$  along the wire axis, one probes the wire rotational permeability wire that controls the behavior of the MI.

Ordinarily, the MI curve is symmetric with respect to the DC magnetic field  $H_{\rm DC}$  with a single (at  $H_{\rm DC}=0$ ) or a double peak (when  $H_{\rm DC}=\pm H_{\rm max}$ ), the anisotropy field of the wire being considered as the value of  $H_{\rm max}$ . Applying a torsion stress to the wire alters its MI behavior versus field  $H_{\rm DC}$  in a way such that one observes, for instance, a single peak only for positive stress and a double peak for negative stress.

Recent work by Betancourt and Valenzuela (BV) [3] tackled MI measurements of CoFeBSi wires subjected to torsion stress [4,5], at a fixed frequency of 1 MHz, obtaining the aforementioned asymmetry [6,7] of the MI profile with respect to torsion stress. Interpreting the asymmetry in terms of residual anisotropies induced during wire fabrication [3] leads to an effect that, respectively, counterbalances or enhances the wire circumferential anisotropy with positive or negative torsional stress.

The wires were prepared by the in-rotating waterquenching technique and have the nominal composition  $(Co_{94}Fe_6)_{72.5}Si_{12.5}B_{15}$ . Their typical dimensions are 10 cm in length and 120 µm for diameter. They possess a characteristic domain structure dictated by the sign of the

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magnetostriction coefficient  $\lambda_s^R$  which is expected to have a small value ( $|\lambda_s^R| \sim 10^{-7}$  in these Cobalt-rich alloys). When  $\lambda_s^R$  is negative, an axially oriented magnetization core exists surrounded by circular domains [1,2], whereas in the opposite case, radially oriented magnetization states exist such as those observed in Fe-rich alloys [7]. After performing several measurements such as Barkhausen jump and magnetization reversal measurements [3], BV favor the negative sign for the  $\lambda_s^R$  coefficient and presence of helical anisotropy [4].

In this paper, a simple model for the anisotropy field, based on the competition between a fabrication-induced circumferential anisotropy and a torsion-induced one with presence of fabrication-induced residual torsional stress, is proposed. This model allows the extraction of the relevant physical parameters of the wire and explains the observed behavior in the cited work [3] as a function of the static magnetic field for all torsion angles.

### 2. Theory

In order to reveal the residual stress within the wires, positive and negative torsion stress [4] are applied and the effect is analyzed with MI response. Following the work of Favieres et al. [8] on cylindrical CoP amorphous multilayers electrodeposited on copper wires, we use a stress-induced easy axis making an arbitrary angle  $\delta_T$  [4] with the circumferential direction (see Fig. 1). We extend this model to include the presence of residual torsion stress induced by fabrication. The total magnetoelastic energy in presence of applied and residual torsional stress ( $\sigma_T$ ) is

$$E = K_{\rm c} \sin^2(\theta) + \frac{3}{2} \lambda_{\rm s}^{\rm R} (\sigma + \sigma_{\rm r}) \sin^2(\delta_{\rm T} - \theta), \tag{1}$$

where  $K_c$  is the fabrication-induced circumferential anisotropy of the wire and  $\lambda_s^R$  the rotational saturation magnetostriction constant [9] (see Fig. 1). We are in the linear case where the magnetoelastic energy is proportional to stress  $\sigma$ . Our energy functional does not include demagnetization nor Zeeman terms originating from the DC field  $H_{DC}$  applied along the wire axis. Energy due to the AC circular field  $H_{AC}$  created by the injected AC current  $I_{AC}$  is neglected. The anisotropy field as a function of stress is determined by identifying it, experimentally, with the value of the DC field  $H_{\rm max}$  where the MI curve displays a peak. Previously Makhnovskiy et al. [10] introduced explicitly these terms in the energy functional with a single helical anisotropy [4] term in contrast with our work dealing with several competing (fabrication and torsion-induced) anisotropies. The stress  $\sigma$  (on the surface of the wire) is related to the torsion angle  $\delta$  through:

$$\sigma = (Ga\delta)/l,\tag{2}$$

where G is the shear modulus, a is the radius of the wire and l the length (see Fig. 2). In general,  $\sigma$  has a radial dependence since the wire has an inner core with an axial magnetization surrounded by circular domains [1,2] when

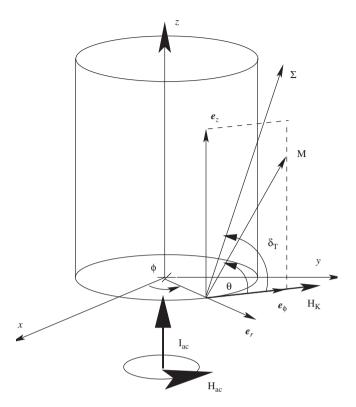


Fig. 1. Geometry of the anisotropy and stress-induced easy axis. The circumferential anisotropy axis induced by fabrication is along  $\mathbf{e}_{\phi}$  and the torsion stress-induced easy axis  $\Sigma$  makes an angle  $\delta_T$  with  $\mathbf{e}_{\phi}$ . The zero-stress circular anisotropy field  $H_K$  is along  $\mathbf{e}_{\phi}$ . In the helical anisotropy case, the magnetization  $\mathbf{M}$  lies in the  $\mathbf{e}_{\phi}$ ,  $\mathbf{e}_z$  plane making a non-zero angle  $\theta$  with the circumferential basis vector  $\mathbf{e}_{\phi}$ , otherwise, both the magnetization  $\mathbf{M}$  and anisotropy axis lie in the  $\mathbf{e}_r$ ,  $\mathbf{e}_{\phi}$  plane.

 $\lambda_{\rm s}^{\rm R}$  is small and negative. We neglect such dependence and consider, for simplicity, the value of stress on the surface [11] of the wire. Introducing the saturation magnetization of the samples  $M_{\rm s}$  (measured with a vibrating sample magnetometer [3] as  $\mu_0 M_{\rm s} = 0.8\,{\rm T}$ ), the energy may be rewritten as

$$E = (M_s/2)[H_K \sin^2(\theta) + H_\sigma \sin^2(\theta - \delta_T)]. \tag{3}$$

This shows that we have a competition between a circumferential anisotropy field  $H_{\rm K}=2K_{\rm c}/M_{\rm s}$  and a torsion stress-induced one (called henceforth twist field including the residual stress)  $H_{\sigma}=3\lambda_{\rm s}^{\rm R}(\sigma+\sigma_{\rm r})/M_{\rm s}$ . Using a Stoner–Wohlfarth approach [12,13], this amounts to rotate the astroid equation from a set of perpendicular fields  $H_x$ ,  $H_y$ :

$$H_x^{2/3} + H_y^{2/3} = H_K^{2/3} \tag{4}$$

to another by an angle  $\theta^*$  resulting in another astroid. This angle is the same that gives the new equilibrium orientation of the magnetization and is simply obtained from the condition minimizing the energy, i.e.  $[\partial E/\partial \theta]_{\theta=\theta^*}=0$ . We obtain

$$\tan(2\theta^*) = \frac{H_{\sigma}\sin(2\delta_{\rm T})}{H_{\rm K} + H_{\sigma}\cos(2\delta_{\rm T})}.$$
 (5)

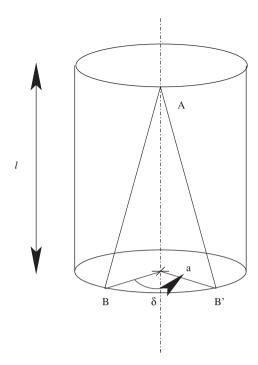


Fig. 2. Applying a torsion stress to a wire moves the point B to B' by angle  $\delta$ . The deformation is the displacement per unit length, i.e.  $\varepsilon = BB'/l = a\delta/l$  where l is the length of the wire. By Hooke's law, stress and deformation are related by the shear modulus G, hence  $\sigma = G\varepsilon = G(a\delta/l)$ .

Using this expression of  $\theta^*$ , the resulting anisotropy field  $H^*$  is written as

$$H^* = \sqrt{H_{\rm K}^2 + H_{\sigma}^2 + 2H_{\rm K}H_{\sigma}\cos(2\delta_{\rm T})}.$$
 (6)

The total energy after this transformation is written as

$$E = (M_{\rm s}/2)[H^* \sin^2(\theta - \theta^*) - H^* \sin^2(\theta^*) + H_{\sigma} \sin^2(\delta_{\rm T})]$$
 (7)

indicating the presence of helical [4] anisotropy (since we have an angle  $\delta_T$  in general different from  $0^\circ$  and  $90^\circ$ ) resulting from the competing circumferential and torsion (applied + residual) induced one. In fact we ought to find from the MI experiment the angles bounding the value of  $\delta_T$ .

The MI peak value versus field and torsion angle should correspond to the anisotropy field  $H^*$  given by the above formula (Eq. (6)) as explored in the next section.

## 3. Numerical procedure

The MI maximum value field  $H_{\rm max}$  is to be fitted to the theoretical expression of  $H^*$ . The latter depends on four parameters,  $H_{\rm K}$  the zero-stress circumferential anisotropy field,  $b=3\lambda_{\rm s}^{\rm R}/M_{\rm s}$  the normalized magnetostriction constant, the angle  $\delta_{\rm T}$  and the residual torsional stress  $\sigma_{\rm r}$ . These can be reduced to three by normalising all quantities by  $H_{\rm K}$ .

We use a procedure based on a least squares minimization procedure of the curve  $H^*$  versus  $\sigma$  to the set of experimental

measurements  $[\sigma_i, y_i]_{i=1,n}$  where  $y_i = H^*(\alpha; \sigma_i)$ .  $\alpha$  represents the set of parameters  $b, \delta_T$  and  $\sigma_r$ . The slope of the curve  $H^*$  versus  $\sigma$  that appears to be linear is used in the fitting. This choice is sufficient for the description of the method we use, nonetheless our algorithm allows us to select any coherent set of criteria we choose to fit the data.

Hence the couple of minimum equations for the data points and the slope are

$$\frac{1}{n} \sum_{i=1}^{n} [H^*(\alpha; \sigma_i) - y_i]^2$$
 (8)

$$\left| \frac{1}{n} \sum_{i=1}^{n} \frac{\left[ b^2 (\sigma_i + \sigma_r) + b H_K \cos(2\delta_T) \right]}{H^*(\boldsymbol{\alpha}; \sigma_i)} - s_e \right|, \tag{9}$$

where  $s_e$  is the overall experimental slope. The fitting method is based on the Broyden algorithm which is the generalization to higher dimension of the one-dimensional secant method [14] and allow us to determine in principle two unknowns out of three b,  $\sigma_r$  and  $\delta_T$ . Broyden method is chosen because it can handle underdetermined problems (since we have two equations and three unknowns).

We consider two cases:

Case A: Absence of residual stress ( $\sigma_r = 0$ ), then  $H_K$  is determined as  $H_K = H^*(\alpha; 0)$  and b,  $\delta_T$  are obtained with Broyden algorithm.

Case P: Presence of residual stress, then  $H_{\rm K}$  is a normalising parameter and b,  $\sigma_{\rm r}$  are evaluated with Broyden algorithm.

The parameter  $Ga/l = 60.82 \,\mathrm{MPa/rad}$  is obtained directly by averaging over the torsion angles and the torsion stress as done in Ref. [3]. Using a wire radius of  $a = 60 \,\mathrm{\mu m}$  and a length of  $l = 10 \,\mathrm{cm}$ , we obtain  $G = 100 \,\mathrm{GPa}$  which is a little different from the value used in Ref. [3] (60 GPa).

The fitting shown in Fig. 3 (corresponding to case P as explained in the next section) allows us to predict the anisotropy field  $H^*$ , in the negative stress torsion case, from the experimental results of MI measurements; that is interesting enough, since the single peak is dominant (at  $H_{\rm DC}=0$ ) and the double peak is barely noticeable in the MI curve versus  $H_{\rm DC}$ . As an example, we can infer that the anisotropy field  $H^*$  is zero for a negative value of stress  $\sigma \sim -150\,{\rm MPa}$ .

## 4. Results and discussion

The fitting procedure in case A, discriminates among the positive and negative  $\lambda_s^R$  values in favor of the negative value (implying presence of circular magnetization pattern in the wire). The fitting quality (equivalent to a  $\chi^2$  test) is 0.094 when  $b=1.39\,(\text{A/m})/\text{MPa}$  (roughly equal to the slope of  $H^*$  versus  $\sigma$ ) the angle  $\delta_T$  is almost 180° and  $\lambda_s^R=4.68\times 10^{-7}$ . On the other hand, the fitting quality is  $1.8\times 10^{-3}$ , with  $b=1.70\,(\text{A/m})/\text{MPa}$ , and  $\delta_T{\sim}0^\circ$ . In this case  $\lambda_s^R=-4.55\times 10^{-7}$  agreeing with the value found by BV [3].

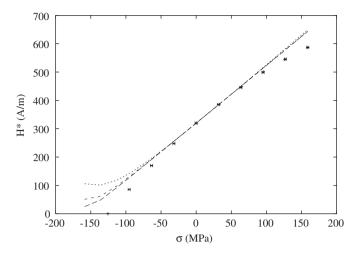


Fig. 3. Experimental results of the anisotropy field  $H^*$  versus stress as obtained by BV [3] from the field value of  $H_{\rm DC}$  at MI peak value and theoretical lines for variable circular anisotropy field  $H_{\rm K}$  at zero-stress. As  $H_{\rm K}$  is decreased from  $100\,{\rm A/m}$  (upper line) to  $50\,{\rm A/m}$  and finally  $25\,{\rm A/m}$  (lowest line) we get a weaker tapering off of the line for negative stress  $\sigma$  and better fitting of the experimental results. The residual torsion stress is  $\sim 150\,{\rm MPa}$ .

In spite of the successful fit, the result is not acceptable for at least two reasons: firstly, with torsional stress the angle  $\delta_{\rm T}$  should be at least 45° (or between 45° and 90° as in Favieres et al. [8] and Makhnovskiy et al. [10]), Secondly the zero-stress anisotropy field  $H_{\rm K}$  ( $H_{\rm K}=320\,({\rm A/m})$  [3] is too large for these wires (For Cobalt-rich wires  $H_{\rm K}$  is on the order of a few tens of A/m).

Then we move on to discuss the next case P. Starting with  $H_{\rm K}=100\,{\rm A/m}$ , we perform Broyden minimization obtaining:  $b=2.13\,({\rm A/m})/{\rm MPa}$  yielding  $|\lambda_{\rm s}^{\rm R}|=7.13\times 10^{-7}$  while  $\sigma_{\rm r}=142.4\,{\rm MPa}$ . For  $H_{\rm K}=50\,{\rm A/m}$  we obtain:  $b=2.05\,({\rm A/m})/{\rm MPa}$  yielding  $|\lambda_{\rm s}^{\rm R}|=6.87\times 10^{-7}$  while  $\sigma_{\rm r}=154.1\,{\rm MPa}$ . It is remarkable to notice that the fitting curve tapers off for negative stress (see Fig. 3) while getting better for smaller values of  $H_{\rm K}$ . For  $H_{\rm K}=25\,{\rm A/m}$  we obtain:  $b=2.03\,({\rm A/m})/{\rm MPa}$  yielding  $|\lambda_{\rm s}^{\rm R}|=6.83\times 10^{-7}$  while  $\sigma_{\rm r}=156.5\,{\rm MPa}$ .

The value  $|\lambda_s^R| = 6.83 \times 10^{-7}$  is slightly larger than found previously by BV. However, we agree with BV [3] interpretation of the MI asymmetry in terms of a residual torsion stress (~150 MPa) that is enhanced or counterbalanced (since  $H^* = 0$  for  $\sigma = -150$  MPa) with an applied positive or negative torsional stress. The origin of the asymmetry was interpreted by Freijo et al. [15] on the basis of the exchange coupling between the wire axis core, axially magnetized, and the rest of the volume carrying a circular magnetization. During cooling the tubular domain wall structure induced between both regions retain an anisotropy equivalent to the observed helical.

We conclude that our results are similar to those obtained with the A case and agree as well with those obtained by BV despite the use in both cases of the linear fit:  $H^* = H_{\rm K} + b\sigma$  that allows to extract from the slope of

the  $H^*$  versus  $\sigma$  the magnetostriction coefficient with the formula [16]:

$$\lambda_{\rm s}^{\rm R} = -(\mu_0 M_{\rm s}/3)({\rm d}H^*/{\rm d}\sigma).$$
 (10)

In order to examine the general case with  $\cos(2\delta_T) \neq \pm 1$ , we observe that when the magnitude of the twist field  $H_{\sigma}$  is small with respect to the zero-stress anisotropy field  $H_K$  one may expand, to fourth order (for instance), the anisotropy field  $H^*$  with respect to  $\sigma$  obtaining

$$H^*/H_{\rm K} = 1 + \frac{H_{\sigma}}{H_{\rm K}}\cos(2\delta_{\rm T}) + \frac{1}{2}\left[\frac{H_{\sigma}}{H_{\rm K}}\right]^2\sin^2(\delta_{\rm T})$$
$$-\frac{1}{2}\left[\frac{H_{\sigma}}{H_{\rm K}}\right]^3\cos(2\delta_{\rm T})\sin^2(\delta_{\rm T}) + O(\sigma^4). \tag{11}$$

One may also derive a similar expansion for  $\lambda_s^R$  as a function of  $\sigma$  [16]. Using the above relationship Eq. (10), one gets

$$\lambda_{\rm s}^{\rm R} = -\left(\mu_0 M_{\rm s} b/3\right) \left\{ \cos(2\delta_{\rm T}) + \frac{H_{\sigma}}{H_{\rm K}} \sin^2(\delta_{\rm T}) - \frac{3}{2} \left[ \frac{H_{\sigma}}{H_{\rm K}} \right]^2 \cos(2\delta_{\rm T}) \sin^2(\delta_{\rm T}) + O(\sigma^3) \right\}. \tag{12}$$

Previously, this kind of expansion has been performed by several workers and particularly by Zhukov et al. [17] who made also a fitting of the expansion coefficients as a function of frequency. This might be included easily in our approach by letting  $H_{\sigma}$  vary with frequency.

Generally speaking, the linear regime  $H^* = H_{\rm K} \pm b\sigma$  should not be observed when the angle  $\delta_{\rm T}$  is different from  $0^{\circ}$  and  $90^{\circ}$  ( $\pm$  an integer multiple of  $90^{\circ}$ ). Then the relationship  $H^* = \sqrt{H_{\rm K}^2 + H_{\sigma}^2 + 2H_{\rm K}H_{\sigma}\cos(2\delta_{\rm T})}$  should apply and when the twist field is small  $(H_{\sigma} \ll H_{\rm K})$ , the above expressions Eqs. (11), and (12) apply.

Experiments versus frequency involving wires with offwire axis torsion stress conditions ought to be done in order to test the general non-linear case (where  $H_{\sigma}$  not necessarily being very small with respect to  $H_{\rm K}$ ), and precision in measuring  $H^*$  versus  $\sigma$  should be increased (in the positive torsion angle case of BV [3]) in order to be able to locate accurately the peak of the MI versus field  $H_{\rm DC}$ and test the above formula. This work is in progress.

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