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Generalized BEC in superconductivity

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Abstract

A generalized Bose–Einstein condensation (GBEC) statistical theory of superconductors accounts not for BB interactions but rather for boson–fermion (BF) interactions. It extends the 1989 Friedberg–Lee BEC theory by including as bosons two-hole (2h) singlet Cooper pairs (CPs) in addition to the usual two-electron (2e) ones. It contains BCS theory when both kinds of pairs are equal in the BE condensate and in excited states—at least as far as *identically* reproducing the BCS gap equation for all temperatures T as well as the $T=0$ BCS condensation energy for all couplings. As a ternary BF model with BF interactions, it yields T_cs one-to-three orders-of-magnitude higher than BCS theory with the same Cooper/BCS model electron–phonon interaction. These T_cs appear to be surprisingly insensitive to the BF interaction.

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The GBEC theory is described in detail in Ref. [1]; it is defined by a Hamiltonian of the form $H=H_0+H_{\rm int}$ where

$$H_0=\sum_{\mathbf{k}_1,s_1}\epsilon_{\mathbf{k}_1}a^+_{\mathbf{k}_1s_1}a_{\mathbf{k}_1s_1}+\sum_{\mathbf{K}}E_+(K)b^+_{\mathbf{K}}b_{\mathbf{K}}-\sum_{\mathbf{K}}E_-(K)c^+_{\mathbf{K}}c_{\mathbf{K}}$$

and $\mathbf{K}\equiv\mathbf{k_1}+\mathbf{k_2}$ is the total momentum wavevector of the pair, while $\epsilon_{\mathbf{k}_1}\equiv\hbar^2k_1^2/2m$ are the single-electron, and $E_\pm(K)\equiv E_\pm(0)\pm\varepsilon_K$ the 2e- and 2h-CP *phenomenological*, energies. Here $a^+_{\mathbf{k}_1s_1}(a_{\mathbf{k}_1s_1})$ are creation (annihilation) operators for fermions and similarly $b^+_{\mathbf{K}}(b_{\mathbf{K}})$ and $c^+_{\mathbf{K}}(c_{\mathbf{K}})$ for two-electron (2e-) and 2h-CPs, respectively. The postulated b and c operators depend only on $\mathbf{K}$ and *not* also on the relative $\mathbf{k}\equiv\frac{1}{2}(\mathbf{k_1}-\mathbf{k_2})$ wavevector as in the pairs of Ref. [2] Eqs. (2.9)–(2.13), included only for the particular case of $\mathbf{K}=0$, which are *not* bosons. The CPs in H_0 can be shown (Ref. [1, App. A]) to obey Bose–Einstein statistics. The unperturbed Hamiltonian H_0 thus describes a *non-Fermi-liquid*. The interaction Hamiltonian $H_{\rm int}$ in the expression $H=H_0+H_{\rm int}$ is, neglecting $\mathbf{K}>0$ CPs,

$$H_{\rm int}=L^{-3/2}\sum_{\mathbf{k}}\{f_+(k)[a^+_{\mathbf{k}\uparrow}a^+_{-\mathbf{k}\downarrow}b_{\mathbf{0}}+a_{-\mathbf{k}\downarrow}a_{\mathbf{k}\uparrow}b^+_{\mathbf{0}}]+f_-(k)[a^+_{\mathbf{k}\uparrow}a^+_{-\mathbf{k}\downarrow}c^+_{\mathbf{0}}a_{-\mathbf{k}\downarrow}a_{\mathbf{k}\uparrow}c_{\mathbf{0}}]\},$$

where L is the 3D system size. The BF interaction vertex form factors $f_\pm(k)$ are taken as in Ref. [1]; this gives a single BF coupling parameter f. A convenient energy scale $E_f\equiv\frac{1}{4}[E_+(0)+E_-(0)]$ is then introduced which in general *differs* from the Fermi energy $E_{\rm F}=\frac{1}{2}mv_{\rm F}^2\equiv k_{\rm B}T_{\rm F}$ where $T_{\rm F}$ is the Fermi temperature. The total number-density of charge-carrier mobile electrons $n\equiv N/L^3$ is related via $E_{\rm F}=(\hbar^2/2m)\ (3\pi^2n)^{2/3}$, while E_f is of the same form but with n replaced by, say, n_f, which in turn serves as convenient electron-density scale. The grand thermodynamic potential $\Omega=-PL^3=F-\mu N=E-TS-\mu N$ is then constructed exactly where F is the Helmholtz free energy, P the pressure, μ the electron chemical potential, E the internal energy and S the entropy. The conditions $\partial F/\partial N_0=0$, $\partial F/\partial M_0=0$ and $\partial\Omega/\partial\mu=-N$ where $N_0(T)$

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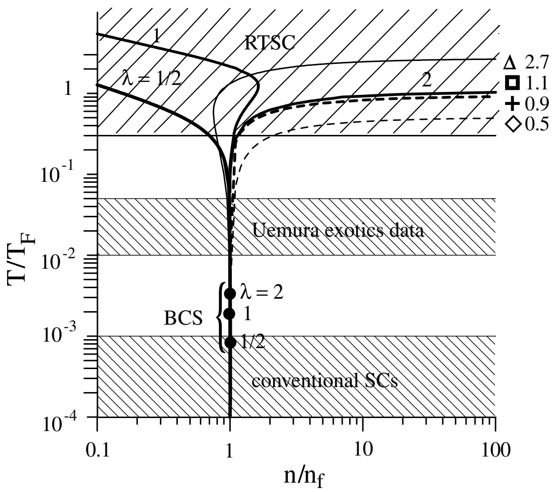

Fig. 1. Phase diagram in 3D for temperature (in units of T_F) *vs* electron density n (in units of n_f) for $f = 0$ and $\hbar\omega_D/E_F = 0.005$. Thin curves refer to quadratic boson dispersion ε_K, full to 2h-CP BEC and dashed to 2e-CP BEC. Thick curves refer to linear boson dispersion ε_K [6,7] for $\lambda \equiv N(0)V$ values indicated, again full to 2h-CP and dashed to 2e-CP BECs. Symbols $\triangle$ etc. refer to $n/n_f = \infty$ limits.

and $M_0(T)$ are the number of BE-condensed 2e- and 2h-CPs, respectively, then yield *three* BE-condensed equilibrium phases [3] besides the normal phase. Only the two pure BEC phases defined by $N_0(T_c) \equiv 0$ and $M_0(T_c) \equiv 0$ are found to display T_c values higher than the corresponding BCS value, while a mixed phase gives lower T_cs including the BCS value.

The very existence of these BEC phases is not inconsistent with a proof [4] that "Bose condensation of fermion pairs ... is impossible." A clear-cut distinction between these pairs which depend on both **K** *and* **k** and which therefore do *not* obey Bose commutation relations, and CPs such as those in H_0 which at least obey BE statistics, clarifies this point. The GBEC theory leads to (a) the BCS gap equation for equal numbers of both kinds of pairs, both in the $\mathbf{K} = 0$ state and in all $\mathbf{K} \neq 0$ states taken collectively, and in weak coupling, *regardless of CP overlaps*; and also to (b) the precise familiar BEC T_c formula in the strong-coupling limit. Also obtained as a special case when 2h-CPs are ignored are the equations of the Friedberg–Lee model [5]. Fig. 1 displays BEC phase boundaries for zero BF interaction ($f = 0$) for the 2h-CP (full curves) and 2e-CP (dashed curves) pure phases. Thick curves are for linearly-dispersive $\varepsilon_K = \lambda\hbar v_F K/4$ [6] bosons moving in the Fermi sea, while thin curves are for quadratic $\varepsilon_K = \hbar^2K^2/4m$ bosons as in Refs. [3,5]; the latter is appropriate *in vacuo* (see Ref. [7, Eq. (8)]) and obviously independent of coupling λ. Black dots are the BCS values $T_c/T_F = 1.134(\hbar\omega_D/E_F)\exp(-1/\lambda)$. RTSC stands for room-temperature superconductivity in a material with $T_F = 10^3$ K. Curves twisting to the left are unphysical "catastrophes" in that T_c *diverges* as $n/n_f \to 0$.

The results of Fig. 1 differ little if f is nonzero and identified with the BCS interaction parameters V and $\hbar\omega_D$ via $f = \sqrt{2V\hbar\omega_D}$, the shift in T_c/T_F being less than 2×10^{-4} for $n/n_f \geqslant 2$. The GBEC theory reproduces [8] the precise BCS gap equation for all T, as well as the $T = 0$ BCS condensation energy per unit volume (Ref. [2, Eq. (2.42)]), *for any coupling*.

In summary, the GBEC theory consists of an unperturbed Hamiltonian H_0 describing a non-Fermi-liquid that is a ternary gas of 2e- and 2h-Cooper pairs along with unpaired electrons, plus a perturbed Hamiltonian H_{int} accounting for boson-fermion interactions that cause formation and disintegration of both kinds of pairs. The unperturbed H_0 seems to contain most of the physics controlling the magnitude of T_c. Finally, RTSC is predicted to be possible in principle with phonons.

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