International Journal of Modern Physics B
Vol. 22, Nos. 25 & 26 (2008) 4386–4397

World Scientific
www.worldscientific.com

DC RESISTIVITY OF CHARGED COOPER PAIRS IN A SIMPLE BOSON-FERMION MODEL OF SUPERCONDUCTORS

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Received 31 July 2008

An analytic expression for the contribution $\sigma_B(\lambda, T)$ to the conductivity from charged bosonic Cooper pairs (CPs) is derived via two-time Green function techniques as a function of the BCS interelectron interaction model parameter λ and temperature T. Within the framework of a binary boson-fermion gas mixture model, it is shown that a self-consistent description of the resistivity data observed in high-temperature superconductors is possible *only* by assuming the presence of a finite gap between the energy spectra of free fermions and bosonic CPs.

Keywords: High-T_c superconductivity; boson-fermion models; charged boson conductivity; gapped Cooper-pair dispersion.

1. Introduction

Boson-fermion (BF) statistical models of superconductivity (SC) as a Bose-Einstein condensation (BEC) began to be studied in the mid-1950s, predating even the BCS-Bogoliubov theory of SC. However, the successes of the BCS theory in describing properties of traditional low-temperature SCs left BF-models neglected for many years. But the discovery of copper-oxide SCs, and discovering that it is impossible to describe the peculiarities of cuprates within the framework of BCS model, led to revisiting many traditonal SC scenarios. Because of the short coherence length of Cooper pairs (CPs), as well as explaining very naturally the pseudogap phenomenon observed in high-temperature superconductors (HTSCs) in terms of preformed CPs, BF models became attractive candidates to examine the physics of HTSCs. BCS

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theory merely contemplates the presence of Cooper "*correlations*" of single-fermion states. By contrast, BF models[1,2,3] posit the existence of actual bosonic CPs. For example, one may assume that the subsystem of electrons lying within the spherical shell $E_F - \hbar\omega_D \leq \epsilon \leq E_F + \hbar\omega_D$ about the Fermi energy E_F of the ideal Fermi gas in single-electron ϵ energy space, with $\hbar\omega_D$ the Debye energy parameter of the BCS model interaction, consists of two coexisting and dynamically interacting subsystems: Fermi particles (or pairable but *unpaired* fermions), and individual pair bosonic CP entities made up of two mutually confined fermions. The simplest Hamiltonian describing a binary mixture of interacting fermions with bosons as suggested in Refs. 1, 2, 3 has been applied in an effort to understand the properties of HTSCs. This Hamiltonian is

$$H = H_e^o + H_B^o + H_{int} \tag{1}$$

$$H_e^o \equiv \sum_{\mathbf{k},\sigma} \xi_{\mathbf{k}} a_{\mathbf{k}\sigma}^+ a_{\mathbf{k}\sigma} \qquad \text{and} \qquad H_B^o \equiv \sum_{\mathbf{K}} \mathcal{E}_{\mathbf{K}} b_{\mathbf{K}}^+ b_{\mathbf{K}} \tag{2}$$

where H_e^o and H_B^o are zeroeth-order Hamiltonians of free (pairable but unpaired) fermions and composite-boson CPs. Here $a_{\mathbf{k}\sigma}^+$ and $a_{\mathbf{k}\sigma}$ are the usual fermion creation and annihilation operators for individual electrons of momenta $\mathbf{k}$ and spin $\sigma = \uparrow$ or $\downarrow$ while $b_{\mathbf{K}}^+$ and $b_{\mathbf{K}}$ are postulated[4,5] to be bosonic operators associated with CPs of definite total, or center-of-mass momentum (CMM), wavevector $\mathbf{K}$. Fermion $\xi_{\mathbf{k}} \equiv \epsilon_{\mathbf{k}} - \mu$ and boson energies $\mathcal{E}_{\mathbf{K}}$ are measured from μ and 2μ, respectively, where μ is the fermionic chemical potential defined in Ref. 7. Processes of formation/disintegration of bosons are then represented by an interaction Hamiltonian reminiscent of an analogous Fröhlich electron-phonon expression

$$H_{int} \equiv \frac{f}{L^{d/2}} \sum_{\mathbf{q},\mathbf{K}} \left(b_{\mathbf{K}}^+ a_{\mathbf{q}+\mathbf{K}/2\uparrow} a_{-\mathbf{q}+\mathbf{K}/2\downarrow} + b_{\mathbf{K}} a_{-\mathbf{q}+\mathbf{K}/2\downarrow}^+ a_{\mathbf{q}+\mathbf{K}/2\uparrow}^+ \right) \tag{3}$$

where f is a phenomenological BF vertex interaction form factor coupling parameter, related with the attractive interelectron (*four*-fermion) interaction strength V of the s-wave BCS model interaction through[4,5] $f = \sqrt{2\hbar\omega_D V}$. Here L is the system size in d dimensions.

The new ingredient in the BF model (1) comes from Ref. 8 where it was shown that introducing an *attractive* interaction between electrons in the gas of electrons leads to the formation a new type of lower-energy mixture state with bosonic excitations *above the Fermi sea* of unpaired elecrons. Competition between electrons to occupy energy levels below E_F so as to minimize the volume of the Fermi sea leads to pushing away from that sea attractively-interacting charge carrier levels and raising them above E_F. These "raised" carriers appear outside the Fermi sea confined into positive-energy resonant CPs. Processes of pair formation and their subsequent disintegration into two unpaired electrons given by (3) were crucial[8] in getting a BF mixture state with positive-energy bosonic-excitations. Owing to these continual formation/disintegration processes, the total energy E of a mixture becomes *lower* compared with the energy of a single-component Fermi system of

interacting fermions *without* explicit bosons. Separation of the initial attractively-interacting-fermion system into bosons and fermions with spectra shifted with respect to each other by the coupling-dependent, positive-energy gap were anticipated in Ref. 9. Linearly-dispersive (in CMM wavevector K) and gapped by $2\Delta(\lambda)$, with $\Delta(\lambda)$ the BCS single-fermion gap, describe our composite-boson CPs as found in Ref. 9 via a Bethe-Salpeter integral equation in the ladder approximation for coupled two-particle and two-hole wave functions. In Ref. 8 it was proved there that such a state with resonant bosons, i.e., bosons rising *above* the Fermi sea, may be energetically favorable if and only if H_{int} is included in (1). It was shown[7] that the separation by $\Delta(\lambda)$ between spectra of bosons and fermions provides, in contrast to BCS theory, a continuous decrease upon cooling of the chemical potential below the chemical potential E_F associated with interactionless fermions at zero temperature T, i.e., the Fermi energy. In other words, the BF mixture state develops gradually as T is lowered. It emerges from the system of unpaired electrons by forming *incoherent* CPs (with a λ- and T-dependent number density) *one by one*, and therefore differing from the BCS state where a phase transition to a *coherent* state occurs abruptly at T_c. Physically, the BF mixture includes *nonzero*-CMM pairs whereas BCS theory does not. Owing to the gapped boson spectrum it was convenient in Ref. 7 to define two characteristic temperatures. Firstly, a depairing temperature T^* below which the electronic chemical potential $\mu(\lambda, T)$ first becomes less than E_F of the interactionless electrons; below this T^* the first CPs begin to appear in the system. The condition $E_F - \mu(\lambda, T^*) = 0$ yields the T^* below which a transition occurs from normal state with no composite bosons to one with such bosons, i.e., the subsystem becomes an incoherent binary BF mixture. Secondly, the BEC temperature T_c at which a singularity in the total number density of bosons $n_B(\lambda, T_c) \equiv (1/L^d)\sum_{\mathbf{K}}(e^{\Omega_{\mathbf{K}}/k_B T_c} - 1)^{-1}$ first occurs, where $\Omega_{\mathbf{K}}$ is the boson energy $\mathcal{E}_{\mathbf{K}}$ renormalized by (3). At temperatures below T^* ($T^* > T_c$), $2e$-charged bosons contribute to a charge current in the presence of an applied electric field. In this sense, the bosons of the present model resemble CP fluctuations above T_c in the Aslamazov-Larkin theory.[10] The perturbation H_{int} (3) in (1) is necessary in yielding a lower-energy ground-state corresponding to the BF mixture; a finite boson lifetime will produce a resistivity in the boson gas. There may be boson scatterings in the subsystem other than those associated with H_{int} in (1), such as scattering from ionic-lattice irregularities. However, these events are important in accounting for the *total* resistivity and are of no direct concern for the mechanism due to the separation of the many-electron system into bosons and fermions; they can be considered as part of an *improper* part of the resistivity. In this paper we investigate the *self*-resistivity $\rho_B(\lambda, T)$ caused solely by the formation/disintegration of bosons without which a BF mixture state (with positive-energy resonant bosons) cannot be realized.

2. Basic Equations

According to (C.3) the Green function $\left\langle\left\langle b^+_{\mathbf{p}} b_{\mathbf{p+k}} \mid \rho_B(-\mathbf{k})\right\rangle\right\rangle_\omega$ determines the response of the subsystem of composite CPs to an external electric field, i.e., describes the transport due to $2e$-charged CPs. The limit when $\omega = 0$ and $k \to 0$ defines the bosonic part of the conductivity associated with a static and longwave electric field. The equation to determine this function $\left\langle\left\langle b^+_{\mathbf{p}} b_{\mathbf{p+k}} \mid \rho_B(-\mathbf{k})\right\rangle\right\rangle_\omega$ may be obtained by choosing $A \equiv b^+_{\mathbf{p}} b_{\mathbf{p+k}}$ and $B \equiv \rho_B(-\mathbf{k})$ in

$$\omega \left\langle\left\langle A \,|B\right\rangle\right\rangle_\omega = \left\langle [A, B]\right\rangle_H + \left\langle\left\langle [A, H] \,|B\right\rangle\right\rangle_\omega \tag{4}$$

which is the first of the well-known infinite chain of equations for the Green functions.[12] The relevant expression becomes

$$\omega \left\langle\left\langle b^+_{\mathbf{p}} b_{\mathbf{p+k}} \mid \rho_B(-\mathbf{k})\right\rangle\right\rangle_\omega = \left\langle [b^+_{\mathbf{p}} b_{\mathbf{p+k}}, \rho_B(-\mathbf{k})]\right\rangle + \left\langle\left\langle [b^+_{\mathbf{p}} b_{\mathbf{p+k}}, H] \mid \rho_B(-\mathbf{k})\right\rangle\right\rangle_\omega \tag{5}$$

where ω is henceforth in energy units. Putting (A.3) in the first term on rhs of (5) and inserting (1) in the second one, and using relations **a)** $[b^+_{\mathbf{p}} b_{\mathbf{p+k}}, \rho_B(-\mathbf{k})] = b^+_{\mathbf{p}} b_{\mathbf{p}} - b^+_{\mathbf{p+k}} b_{\mathbf{p+k}}$; **b)** $[b^+_{\mathbf{p}} b_{\mathbf{p+k}}, H^o_{el}] = 0$; and **c)** $[b^+_{\mathbf{p}} b_{\mathbf{p+k}}, H^o_B] = (\mathcal{E}_{\mathbf{p+k}} - \mathcal{E}_{\mathbf{p}}) b^+_{\mathbf{p}} b_{\mathbf{p+k}}$, transforms (5) into

$$\begin{aligned}(\omega - \mathcal{E}_{\mathbf{pk}}) \left\langle\left\langle b^+_{\mathbf{p}} b_{\mathbf{p+k}} \mid \rho_B(-\mathbf{k})\right\rangle\right\rangle_\omega &= n_B(\mathbf{p}) - n_B(\mathbf{p}+\mathbf{k}) \\ &\quad + \left\langle\left\langle [b^+_{\mathbf{p}} b_{\mathbf{p+k}}, H_{int}] \mid \rho_B(-\mathbf{k})\right\rangle\right\rangle_\omega . \end{aligned} \tag{6}$$

Here $\mathcal{E}_{\mathbf{pk}} \equiv \mathcal{E}_{\mathbf{p+k}} - \mathcal{E}_{\mathbf{p}}$ are energy differences associated with transitions between bosonic states with center-of-mass momentum (CMM) wavevectors $\mathbf{p}$ and $\mathbf{p}+\mathbf{k}$ which are caused by the applied electric field of wavevector $\mathbf{k}$; $n_B(\mathbf{p}) \equiv \left\langle b^+_{\mathbf{p}} b_{\mathbf{p}}\right\rangle$ and $n_B(\mathbf{p}+\mathbf{k}) \equiv \left\langle b^+_{\mathbf{p+k}} b_{\mathbf{p+k}}\right\rangle$ are Bose occupation numbers $[\exp(\mathcal{E}/k_BT) - 1]^{-1}$ of corresponding energy states $\mathcal{E}_{\mathbf{p}}$ and $\mathcal{E}_{\mathbf{p+k}}$.[7] The formal solution for (6) can be written through a mass operator $M_{\mathbf{p}}(\mathbf{k},\omega)$ defined as

$$\left\langle\left\langle [b^+_{\mathbf{p}} b_{\mathbf{p+k}}, H_{int}] \mid \rho_B(-\mathbf{k})\right\rangle\right\rangle_\omega \equiv M_{\mathbf{p}}(\mathbf{k},\omega) \left\langle\left\langle b^+_{\mathbf{p}} b_{\mathbf{p+k}} \mid \rho_B(-\mathbf{k})\right\rangle\right\rangle_\omega . \tag{7}$$

It relates the higher-order Green functions on the rhs of (6) in terms of $\left\langle\left\langle b^+_{\mathbf{p}} b_{\mathbf{p+k}} \mid \rho_B(-\mathbf{k})\right\rangle\right\rangle_\omega$. From (6) and (7) we have

$$G_{\mathbf{p}}(\mathbf{k},\omega) \equiv \left\langle\left\langle b^+_{\mathbf{p}} b_{\mathbf{p+k}} \mid \rho_B(-\mathbf{k})\right\rangle\right\rangle_\omega = \frac{n_B(\mathbf{p}) - n_B(\mathbf{p}+\mathbf{k})}{\omega - \mathcal{E}_{\mathbf{pk}} - M_{\mathbf{p}}(\mathbf{k},\omega)}. \tag{8}$$

The problem now is to develop some physically reasonable approximation to determine $M_{\mathbf{p}}(\mathbf{k},\omega)$ in (8). To do this, we must write the next equation in the chain of equations (4) for $\left\langle\left\langle [b^+_{\mathbf{p}} b_{\mathbf{p+k}}, H_{int}] \mid \rho_B(-\mathbf{k})\right\rangle\right\rangle$ which is on rhs of (6). Calculating $[b^+_{\mathbf{p}} b_{\mathbf{p+k}}, H_{int}]$ and inserting it into (6) gives

$$\begin{aligned}&(\omega - \mathcal{E}_{\mathbf{pk}}) G_{\mathbf{p}}(\mathbf{k},\omega) = n_B(\mathbf{p}) - n_B(\mathbf{p}+\mathbf{k}) \\ &+ fL^{-d/2} \sum_{\mathbf{q}} \left\{ \left\langle\left\langle b^+_{\mathbf{p}} a_{\mathbf{q+(p+k)/2}\uparrow} a_{-\mathbf{q}+(\mathbf{p}+\mathbf{k})/\mathbf{2}\downarrow} \mid \rho_B(-\mathbf{k})\right\rangle\right\rangle_\omega \\ &- \left\langle\left\langle b_{\mathbf{p+k}} a^+_{-\mathbf{q}+\mathbf{p}/\mathbf{2}\downarrow} a^+_{\mathbf{q}+\mathbf{p}/\mathbf{2}\uparrow} \mid \rho_B(-\mathbf{k})\right\rangle\right\rangle_\omega \right\} . \end{aligned} \tag{9}$$

Equations for the higher-order Green functions appearing on the rhs of (9) may be written by choosing in (4) $B \equiv \rho_B(-\mathbf{k})$, *first* $A \equiv b^+_{\mathbf{p}} a_{\mathbf{q}+(\mathbf{p}+\mathbf{k})/2\uparrow} a_{-\mathbf{q}+(\mathbf{p}+\mathbf{k})/2\downarrow}$ and *then* $A \equiv b_{\mathbf{p}+\mathbf{k}} a^+_{-\mathbf{q}+\mathbf{p}/2\downarrow} a^+_{\mathbf{q}+\mathbf{p}/2\uparrow}$. This leads to

$$\begin{aligned}
&\omega \left\langle\left\langle b^+_{\mathbf{p}} a_{\mathbf{q}+(\mathbf{p}+\mathbf{k})/2\uparrow} a_{-\mathbf{q}+(\mathbf{p}+\mathbf{k})/2\downarrow} \mid \rho_B(-\mathbf{k})\right\rangle\right\rangle_\omega = \\
&\left\langle [b^+_{\mathbf{p}}, \rho_B(-\mathbf{k})] a_{\mathbf{q}+(\mathbf{p}+\mathbf{k})/2\uparrow} a_{-\mathbf{q}+(\mathbf{p}+\mathbf{k})/2\downarrow} \right\rangle_H \\
&+ \left\langle\left\langle [b^+_{\mathbf{p}} a_{\mathbf{q}+(\mathbf{p}+\mathbf{k})/2\uparrow} a_{-\mathbf{q}+(\mathbf{p}+\mathbf{k})/2\downarrow}, H] \mid \rho_B(-\mathbf{k})\right\rangle\right\rangle_\omega
\end{aligned} \tag{10}$$

$$\begin{aligned}
&\omega \left\langle\left\langle b_{\mathbf{p}+\mathbf{k}} a^+_{-\mathbf{q}+\mathbf{p}/2\downarrow} a^+_{\mathbf{q}+\mathbf{p}/2\uparrow} \mid \rho_B(-\mathbf{k})\right\rangle\right\rangle_E = \\
&\left\langle [b_{\mathbf{p}+\mathbf{k}}, \rho_B(-\mathbf{k})] a^+_{-\mathbf{q}+\mathbf{p}/2\downarrow} a^+_{\mathbf{q}+\mathbf{p}/2\uparrow} \right\rangle_H \\
&+ \left\langle\left\langle [b_{\mathbf{p}+\mathbf{k}} a^+_{-\mathbf{q}+\mathbf{p}/2\downarrow} a^+_{\mathbf{q}+\mathbf{p}/2\uparrow}, H] \mid \rho_B(-\mathbf{k})\right\rangle\right\rangle_\omega .
\end{aligned} \tag{11}$$

Here, the averages $\langle ... \rangle_H$ on the rhs (10) and (11) are simplified by applying $[b^+_{\mathbf{p}}, \rho_B(-\mathbf{k})] \equiv -b^+_{\mathbf{p}+\mathbf{k}}$ and $[b_{\mathbf{p}+\mathbf{k}}, \rho_B(-\mathbf{k})] \equiv b_{\mathbf{p}}$. The calculation of $[\cdots, H]$ in (10) and (11) are straightforward. In the rhs of (10) and (11) consider the Green functions originating from the terms $[\cdots, H^o_e]$ and $[\cdots, H^o_B]$ separately, and ignore the Green functions stemming from the commutators $[\cdots, H_{int}]$ which produce expressions such as a^+a^+aa and b^+ba^+a and lead to the higher-order Green functions which turned out *not* to be reducible to lower-order functions.[13] The mathematical justification of ignoring Green functions coming from $[\cdots, H_{int}]$ is not trivial. Formally, omitted terms on the rhs of (10) and (11) are $O(f^3)$ and so contain an extra power f with respect to the lhs ones. Long, tedious manupulations finally give

$$\left\langle\left\langle b^+_{\mathbf{p}} a_{\mathbf{q}+(\mathbf{p}+\mathbf{k})/2\uparrow} a_{-\mathbf{q}+(\mathbf{p}+\mathbf{k})/2\downarrow} \mid \rho_B(-\mathbf{k})\right\rangle\right\rangle = \frac{-\left\langle b^+_{\mathbf{p}+\mathbf{k}} a_{\mathbf{q}+(\mathbf{p}+\mathbf{k})/2\uparrow} a_{-\mathbf{q}+(\mathbf{p}+\mathbf{k})/2\downarrow} \right\rangle}{\omega - [\xi_{\mathbf{q}+(\mathbf{p}+\mathbf{k})/2} + \xi_{-\mathbf{q}+(\mathbf{p}+\mathbf{k})/2} - \mathcal{E}_{\mathbf{p}}]} \tag{12}$$

$$\left\langle\left\langle b_{\mathbf{p}+\mathbf{k}} a^+_{-\mathbf{q}+\mathbf{p}/2\downarrow} a^+_{\mathbf{q}+\mathbf{p}/2\uparrow} \mid \rho_B(-\mathbf{k})\right\rangle\right\rangle = \frac{\left\langle b_{\mathbf{p}} a^+_{-\mathbf{q}+\mathbf{p}/2\downarrow} a^+_{\mathbf{q}+\mathbf{p}/2\uparrow} \right\rangle}{\omega + [\xi_{\mathbf{q}+\mathbf{p}/2} + \xi_{-\mathbf{q}+\mathbf{p}/2} - \mathcal{E}_{\mathbf{p}+\mathbf{k}}]}. \tag{13}$$

Putting (12) and (13) into (9) and using (7) leads to

$$\begin{aligned}
M_{\mathbf{p}}(\mathbf{k},\omega) G_{\mathbf{p}}(\mathbf{k},\omega) = -f L^{-d/2} \sum_{\mathbf{q}} \Bigg[&\frac{\left\langle b^+_{\mathbf{p}+\mathbf{k}} a_{\mathbf{q}+(\mathbf{p}+\mathbf{k})/2\uparrow} a_{-\mathbf{q}+(\mathbf{p}+\mathbf{k})/2\downarrow} \right\rangle}{\omega - [\xi_{\mathbf{q}+(\mathbf{p}+\mathbf{k})/2} + \xi_{-\mathbf{q}+(\mathbf{p}+\mathbf{k})/2} - \mathcal{E}_{\mathbf{p}}]} \\
&+ \frac{\left\langle b_{\mathbf{p}} a^+_{-\mathbf{q}+\mathbf{p}/2\downarrow} a^+_{\mathbf{q}+\mathbf{p}/2\uparrow} \right\rangle}{\omega + [\xi_{\mathbf{q}+\mathbf{p}/2} + \xi_{-\mathbf{q}+\mathbf{p}/2} - \mathcal{E}_{\mathbf{p}+\mathbf{k}}]} \Bigg]
\end{aligned} \tag{14}$$

which must be solved together with (8). Some manipulation gives

$$M_{\mathbf{p}}(\mathbf{k},\omega) = \frac{(\omega - \mathcal{E}_{\mathbf{pk}}) L_{\mathbf{p}}(\mathbf{k},\omega)}{n_B(\mathbf{p}) - n_B(\mathbf{p}+\mathbf{k}) + (\omega - \mathcal{E}_{\mathbf{pk}}) L_{\mathbf{p}}(\mathbf{k},\omega)} \tag{15}$$

where the rhs of (14) is designated as $L_{\mathbf{p}}(\mathbf{k},\omega)$. Note that the denominators in $L_{\mathbf{p}}(\mathbf{k},\omega)$ are associated with the transitions between the two single-electron energies $\xi_{\mathbf{q+Q/2}}$ and $\xi_{\mathbf{-q+Q/2}}$ as well as boson states of energy $\mathcal{E}_{\mathbf{Q}}$ where $\mathbf{Q}$ takes on values $\mathbf{p}+\mathbf{k}$ and $\mathbf{p}$. In the longwavelength and static limits (14) becomes

$$M_{\mathbf{p}}(\omega) = f[\partial n_B(\mathcal{E})/\partial\mathcal{E}]^{-1}L^{-d/2}\sum_{\mathbf{q}}\left[\frac{\left\langle b^+_{\mathbf{p}}a_{\mathbf{q+p/2}\uparrow}a_{\mathbf{-q+p/2}\downarrow}\right\rangle}{\omega-[\xi_{\mathbf{q+p/2}}+\xi_{\mathbf{-q+p/2}}-\mathcal{E}_{\mathbf{p}}]}\right.$$
$$\left.+\frac{\left\langle b_{\mathbf{p}}a^+_{\mathbf{-q+p/2}\downarrow}a^+_{\mathbf{q+p/2}\uparrow}\right\rangle}{\omega+[\xi_{\mathbf{q+p/2}}+\xi_{\mathbf{-q+p/2}}-\mathcal{E}_{\mathbf{p}}]}\right] \tag{16}$$

where $M_{\mathbf{p}}(\omega)\equiv M_{\mathbf{p}}(\mathbf{0},\omega)$ since

$$n_B(\mathbf{p})-n_B(\mathbf{p}+\mathbf{k})\simeq-\frac{\partial n_B(\mathcal{E})}{\partial\mathcal{E}}(\frac{\partial\mathcal{E}_{\mathbf{p}}}{\partial\mathbf{p}}\cdot\mathbf{k}) \qquad \text{and} \qquad \mathcal{E}_{\mathbf{pk}}\equiv\mathcal{E}_{\mathbf{p+k}}-\mathcal{E}_{\mathbf{p}}\simeq\frac{\partial\mathcal{E}_{\mathbf{p}}}{\partial\mathbf{p}}\cdot\mathbf{k}.$$

To determine $M_{\mathbf{p}}(\omega)$ one must first find the averages $\langle b^+_{\mathbf{p}}a_{\mathbf{q+p/2}\uparrow}a_{\mathbf{-q+p/2}\downarrow}\rangle$ and $\langle b_{\mathbf{p}}a^+_{\mathbf{-q+p/2}\downarrow}a^+_{\mathbf{q+p/2}\uparrow}\rangle$ in the rhs of (16). These may be expressed in terms of the Green function $\langle\langle a_{\mathbf{q+Q/2}}a_{\mathbf{-q+}Q/\mathbf{2}\downarrow}|b^+_{\mathbf{Q}}\rangle\rangle$ given by Eq. (B9) of Ref. 7. However, to obtain a quick result we approximate $\langle b^+_{\mathbf{p}}a_{\mathbf{q+p/2}\uparrow}a_{\mathbf{-q+p/2}\downarrow}\rangle\approx\langle b^+_{\mathbf{p}}\rangle\langle a_{\mathbf{q+p/2}\uparrow}a_{\mathbf{-q+p/2}\downarrow}\rangle$ and $\langle b_{\mathbf{p}}a^+_{\mathbf{-q+p/2}\downarrow}a^+_{\mathbf{q+p/2}\uparrow}\rangle\approx\langle b_{\mathbf{p}}\rangle\langle a^+_{\mathbf{-q+p/2}\downarrow}a^+_{\mathbf{q+p/2}\uparrow}\rangle$ which would otherwise be exact for the non-interacting statistical BF mixture. At temperatures T within the interval $T_c<T<T^*$ where the number density of condensed bosons n_{B0} is negligible, using Eqs. (A14) and (A15) as established in Ref. 7, we obtain

$$\left\langle a^+_{\mathbf{q+p/2}\uparrow}a^+_{\mathbf{-q+p/2}\downarrow}\right\rangle=\frac{f}{2L^{d/2}}\frac{\left\langle b^+_{\mathbf{p}}\right\rangle}{\xi_{\mathbf{q+p/2}}}[\tanh(\xi_{\mathbf{q+p/2}}/2k_BT)$$
$$-\frac{\xi_{\mathbf{-q+p/2}}\tanh(\xi_{\mathbf{q+p/2}}/2k_BT)-\xi_{\mathbf{q+p/2}}\tanh(\xi_{\mathbf{-q+p/2}}/2k_BT)}{\xi_{\mathbf{q+p/2}}+\xi_{\mathbf{-q+p/2}}}]. \tag{17}$$

It is easy to see that contributions to (16) associated with terms such as the second in square brackets in (17) may be neglected. Indeed, these terms with small $\mathbf{q}$ cancel out in (17). As to terms with large $\mathbf{q}$ in (16) from the second term in square brackets in (17), they vanish since $p\ll q$ with p and q being respectively boson CMM and electron wavenumbers. Assuming $p\ll q$ in the main part of the integration interval in (16) we may substitute $\left\langle b^+_{\mathbf{p}}\right\rangle/\xi_{\mathbf{q}}$ for the prefactor $\left\langle b^+_{\mathbf{p}}\right\rangle/\xi_{\mathbf{q+p/2}}$ before the square brackets in (17). Then the second term in square brackets in (17) changes its sign when $\mathbf{q}\to-\mathbf{q}$ and therefore does not contribute to (16); the only contribution to it comes from the first term in (17) marked below as $\langle...\rangle^{(1)}_{\mathcal{H}}$, i.e., from

$$\langle a^+_{\mathbf{-q+p/2}\downarrow}a^+_{\mathbf{q+p/2}\uparrow}\rangle^{(1)}_{\mathcal{H}}=-\frac{f}{2L^{d/2}}\frac{\left\langle b^+_{\mathbf{p}}\right\rangle}{\xi_{\mathbf{q+p/2}}}\tanh(\xi_{\mathbf{q+p/2}}/2k_BT) \tag{18}$$

where the sign change is due to Fermi commutation relations and

$$\left\langle a_{\mathbf{q+p/2}\uparrow}a_{\mathbf{-q+p/2}\downarrow}\right\rangle^{(1)}_{\mathcal{H}}=-\frac{f}{2L^{d/2}}\frac{\left\langle b_{\mathbf{p}}\right\rangle}{\xi_{\mathbf{q+p/2}}}\tanh(\xi_{\mathbf{q+p/2}}/2k_BT). \tag{19}$$

which can be obtained from (18) by complex conjugation. Substituting (18) and (19) into (16), where we put $n_B(\mathbf{p}) \approx \langle b_{\mathbf{p}}^{+} \rangle \langle b_{\mathbf{p}} \rangle$, and using the fact that $p \ll q$ (p and q being respectively boson CMM and electron wavenumbers) yields

$$M_{\mathbf{p}}(\omega) = \frac{f^2 n_B(\mathbf{p})}{2L^d[\partial n_B(\mathcal{E}_{\mathbf{p}})/\partial \mathcal{E}_{\mathbf{p}}]} \sum_{\mathbf{q}} \xi_{\mathbf{q}}^{-1} \tanh(\xi_{\mathbf{q}}/2k_BT) \left(\frac{1}{\omega - \omega_{\mathbf{qp}}^{BF}} - \frac{1}{\omega + \omega_{\mathbf{qp}}^{BF}} \right) \tag{20}$$

where we define $\omega_{\mathbf{qp}}^{BF} \equiv 2\xi_{\mathbf{q}} - \mathcal{E}_{\mathbf{p}}$. Replacing the summation in (20) over fermion momenta $\mathbf{q}$ by an energy ξ integration and then using the identity $(x \pm i0)^{-1} = P(1/x) \mp \pi i \delta(x)$ one easily finds the *imaginary* part $\Gamma_{\mathbf{p}}(\omega)$ of (20) which is responsible for the resistivity caused by boson depairing, namely

$$\Gamma_{\mathbf{p}}(\omega) = \frac{\pi N(E_F) f^2 n_B(\mathbf{p})}{2[-\partial n_B(\mathcal{E}_{\mathbf{p}})/\partial \mathcal{E}_{\mathbf{p}}]} \left(\frac{\tanh[(\omega + \mathcal{E}_{\mathbf{p}})/4k_BT]}{\omega + \mathcal{E}_{\mathbf{p}}} + \frac{\tanh[(\omega - \mathcal{E}_{\mathbf{p}})/4k_BT]}{\omega - \mathcal{E}_{\mathbf{p}}} \right). \tag{21}$$

Here $N(E_F)$ is the electronic density-of-states (for each spin *and* per unit volume) at the Fermi surface. In the limit of a static external field (21) becomes

$$\Gamma_{\mathbf{p}}(0) = 2\pi\hbar\omega_D \lambda \frac{n_B(\mathbf{p})}{[-\partial n_B(\mathcal{E}_{\mathbf{p}})/\partial \mathcal{E}_{\mathbf{p}}]} \frac{\tanh(\mathcal{E}_{\mathbf{p}}/4k_BT)}{\mathcal{E}_{\mathbf{p}}} \tag{22}$$

where we have put $f = \sqrt{2\hbar\omega_D V}$[4,5] and defined $N(E_F)V \equiv \lambda$. The function $\Gamma_{\mathbf{p}}(0)$ depends weakly on $\mathbf{p}$. This is because the boson-fermion vertex coupling f in (1) which is responsible for the smearing out of bosonic linewidths, is assumed *not* to be CMM $\mathbf{p}$-dependent and because the of structure of $\mathcal{E}_{\mathbf{p}}$ itself. In general, bosons energies may be written as[7,8]

$$\mathcal{E}_{\mathbf{p}} = 2[E_F - \mu(\lambda, T)] + 2\Delta + E(\mathbf{p}). \tag{23}$$

Here $E_F - \mu(\lambda, T)$ is the shift due to bosonization of the chemical potential $\mu(\lambda, T)$ with respect to the E_F of noninteracting $T = 0$ fermions, namely

$$E_F - \mu(\lambda, T) = -\Delta(\lambda) + \frac{\lambda\hbar\omega_D}{2} \int_{-\hbar\omega_D}^{\hbar\omega_D} \frac{dx}{\sqrt{x^2 + f^2 n_{B0}(T)}} \tanh \frac{\sqrt{x^2 + f^2 n_{B0}(T)}}{2k_BT} \tag{24}$$

as established in Ref. 7, Eq. (28), $E(\mathbf{p})$ is the CP dispersion relation, and the generally coupling-dependent 2Δ is a quantity describing the bosonic gap. In the absence of *no* $\mathbf{p}$-dependent leading terms in (23), the sensitivity of $\Gamma_{\mathbf{p}}(0)$ to changes of $\mathbf{p}$ appears significantly reduced. We take $\mathbf{p} = 0$ in (22), i.e., write $\Gamma_{\mathbf{p}}(0) \approx \overline{\Gamma}$ to get

$$\tau_B^{tr} \equiv \hbar/\overline{\Gamma} = \frac{1}{2\pi\lambda\omega_D(k_BT)} \frac{\mathcal{E}_{\mathbf{0}}}{\tanh(\mathcal{E}_{\mathbf{0}}/4k_BT)} \frac{\exp(\mathcal{E}_{\mathbf{0}}/k_BT)}{\exp(\mathcal{E}_{\mathbf{0}}/k_BT) - 1}. \tag{25}$$

This is the mean value for the boson transport relaxation time which in turn determines the boson-induced Drude-like[14] electrical conductivity

$$\sigma_B = \frac{(2e)^2 n_B(\lambda, T)}{M_B} \tau_B^{tr}. \tag{26}$$

Note that $\mathcal{E}_{\mathbf{0}}$ in (25) is independent of the assumed gap Δ in the boson spectrum (23) and, according to (24), is given by

$$\mathcal{E}_{\mathbf{0}}(\lambda, \omega_D, T) = \lambda \hbar \omega_D \int_{-\hbar\omega_D}^{\hbar\omega_D} \frac{dx}{x} \tanh(x/2k_B T). \tag{27}$$

3. Discussion

If $\Delta \leq 0$ in (24) one immediately has $E_F - \mu(\lambda, T) > 0$ which implies that at any arbitrarily large T the Hamiltonian (1) describes a BF mixture. However, only a $\Delta > 0$ in (23) gives a *finite* T^* which as approached from above implies a transition from a normal metal to a BF mixture. Only positive Δ leads to finite bosonic resistivity $\rho_B \equiv 1/\sigma_B$ and which diverges at T^*—therefore, providing the continuous crossing between the resistivity in normal and pseudogapped phases as observed in HTSCs.[15] Analysis, details of which will be given elsewhere, of τ_B^{tr} as a function of the coupling parameter λ for different ratios of Debye-to-Fermi temperatures Θ_D/T_F shows a very strong λ-dependence. A specific feature found from (25) and (26) is a rapid decrease in the mean effective time between succesive collisions as λ increases. In particular, for $\Theta_D/T_F = 0.1$, not atypical of quasi 2D HTSCs, τ_B^{tr} is $\simeq 10^{-12}s$ for small λ and becomes $\lesssim 10^{-14}s$ for $\lambda \simeq 1$. Scattering effects due to bosons are distinct from those due to fermions: τ_B^{tr} rises linearly with normal-state charge density n^o thus reducing the boson scattering rate $1/\tau_B^{tr}$ in the presence of densely-populated fermion states, as compared with the dilute Fermi sea of electrons. In Fig. 1, the temperature behavior of the resistivity ρ_B caused by boson scattering is displayed for zero and nonzero values of Δ for fixed $\lambda = 0.35$, $\Theta_D/T_F = 0.1$ and $n^o = 10^{21} cm^{-3}$. Considered a free parameter $\Delta/k_B T_F$ is chosen to be 0 (full curve), 0.01 (dashed curve) and 0.011 (dotted curve). For $\Delta \neq 0$ (dotted and dashed curves) the number density of CPs $n_B(\lambda, T)$ vanishes at some characteristic T^* marked in Fig. 1 by vertical arrows. For temperatures immediately above T_c (which appears less sensitive to exact Δ than T^*) and for any Δ, as in the Aslamazov-Larkin theory,[10] ρ_B is nearly linear in T . However, depending on the assumed value of Δ, the later deviation of T from T_c leads to qualitatively different ρ_B. For $\Delta = 0$ (full curve in Fig. 1) ρ_B reaches a maximum and then decreases slowly over a broad range of T without any sign of a BF mixture-to-normal metal transition. For $\Delta < 0$ the situation is qualitatively the same as for $\Delta = 0$, but ρ_B is smaller. However, for $\Delta > 0$ (dashed- and dotted-curves in Fig. 1) there is a finite T^* on approaching of which the boson resistivity diverges so that for temperatures above T^* the total resistivity is determined only by the contribution of unpaired electrons, thus reflecting the main peculiarity found in all experiments (see, for

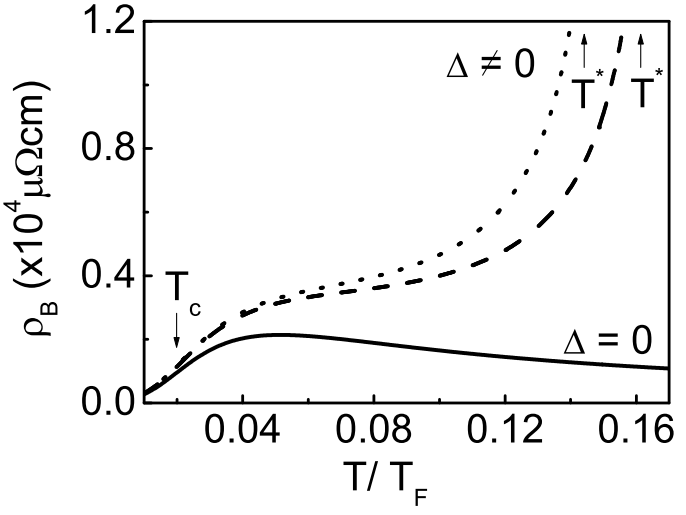

Fig. 1. Temperature behavior (in units of the Fermi temperature T_F) of the resisitivity ρ_B caused by bosonic CPs scatterings for the BCS model interaction with $\lambda = 0.35$, $\Theta_D/T_F = 0.1$ and $n^o = 10^{23} cm^{-3}$. Full, dashed and dotted curves correspond to $\Delta/k_B T_F = 0, 0.01$ and 0.011, respectively.

example, Ref. 15) Also from Fig. 1 a decrease in Δ (though still positive) leads to a shift of T^* to the higher Ts, while for $\Delta \geq 0$, T^* is finite and ρ_B increases with T becoming infinitely large at T^*. The increase of ρ_B with T may be understood in terms of temperature depairing effects.

Furthermore, the number density of bosons $n_B(\lambda, T)$ radically decreases with T, leading to enormous values for ρ_B around the value T^*. However, if there were $\Delta \leq 0$ in (26) then according to (24) the characteristic T^* becomes infinitely large, i.e., not depending how high T is, the system appears in a BF mixture state. In the case of $\Delta \leq 0$ the number density of bosons $n_B(\lambda, T)$ becomes less sensitive to temperature changes than for $\Delta > 0$. For example, if $\Delta \leq 0$ in (23) then at $T = 0$ depairing does not occur at all because of the absence of free fermionic states to be occupied as a result of boson breakups. Owing to the exclusion principle CP breakups occur more rarely for $\Delta \leq 0$. With increasing T the mobility of CPs increases which explains the monotonic decrease in ρ_B (full curve) in Fig. 1.

4. Conclusion

We have shown that within the framework of a binary BF mixture gas model it is possible to get a self-consistent description of the resistivity data observed in HTSCs only by assuming the presence of a finite positive gap between the energy spectra of free fermions and of bosons. Otherwise, calculations contradict with the experimental findings in HTSCs of a finite T^* below which the resistivity differs from that of a normal metal.

Acknowledgments

TAM thanks the Institute of Physics of Azerbaijan for granting him academic leave at the Baskent University of Turkey. MdeLl thanks UNAM-DGAPA-PAPIIT (Mexico) IN106401 for partial support; he is also grateful for travel support through a grant to Southern Illinois University at Carbondale from the U.S. Army Research Office.

Appendix A. Current Density Operator for 2*e*-Charged Bosons

The second-quantized boson density operator is defined as

$$\rho_B(\mathbf{r}) \equiv \phi^+(\mathbf{r})\phi(\mathbf{r}) \tag{A.1}$$

with $\phi^+(\mathbf{r})$ and $\phi(\mathbf{r})$ being phenomenological local boson field operators. These can be decomposed into Fourier components $b_{\mathbf{k}}$ in a volume L^d with periodic boundary conditions

$$\phi(\mathbf{r}) = L^{-d/2}\sum_{\mathbf{k}} b_{\mathbf{k}} \exp i\mathbf{k}\cdot\mathbf{r}. \tag{A.2}$$

In terms of the boson creation $b^+_{\mathbf{k}}$ and annihilation $b_{\mathbf{k}}$ operators, (A.1) becomes

$$\rho_B(\mathbf{r}) = L^{-d}\sum_{\mathbf{q}} \rho_B(\mathbf{q}) \exp i\mathbf{q}\cdot\mathbf{r} \quad \text{with} \quad \rho_B(\mathbf{q}) \equiv \sum_{\mathbf{k}} b^+_{\mathbf{k}} b_{\mathbf{k}+\mathbf{q}} \tag{A.3}$$

being a Fourier component of $\rho_B(\mathbf{r})$. The classical wave functions $\Psi(r)$ become operators $\phi(\mathbf{r})$ in the second quantized representation. In particular, to get a boson current density, $\mathbf{j}_B(\mathbf{r})$, in the second quantized representation, the functions $\Psi(r)$ in the quantum mechanical current density

$$\mathbf{j}(\mathbf{r}) = \alpha \mathbf{Re}(\Psi^*(\mathbf{r})\frac{-i\nabla}{M}\Psi(\mathbf{r})) = \frac{i\alpha}{2M}[\{\nabla\Psi^*(\mathbf{r})\}\Psi(\mathbf{r}) - \Psi^*(\mathbf{r})\nabla\Psi(\mathbf{r})] \tag{A.4}$$

must be replaced by the operators $\phi(\mathbf{r})$. Here α and M are the charge and mass of carriers, respectively. For 2*e*-charged bosons we have $\alpha \equiv 2e$ with $M = 2m$ the boson mass. Using (A.2) in (A.4), $\mathbf{j}_B(\mathbf{r})$ takes a form

$$\mathbf{j}_B(\mathbf{r}) = L^{-d}\sum_{\mathbf{q}} \mathbf{j}_B(\mathbf{q}) \exp i\mathbf{q}\cdot\mathbf{r} \quad \text{with} \quad \mathbf{j}_B(\mathbf{q}) = \frac{\alpha}{M}\sum_{\mathbf{k}} (\mathbf{k} + \frac{\mathbf{q}}{2}) b^+_{\mathbf{k}} b_{\mathbf{k}+\mathbf{q}} \tag{A.5}$$

where symbols in bold stand for vector quantities.

Appendix B. Subsystem Response to External Field and Green Functions

A longitudinal external electrical field $\mathbf{E}(\mathbf{r}, t)$ may then be written in terms of a coordinate- and time-dependent scalar potential $\varphi(\mathbf{r}, t)$ as $\mathbf{E}(\mathbf{r}, t) = -\nabla\varphi(\mathbf{r}, t)$. The Fourier transforms of $\varphi(\mathbf{r}, t)$ and $E(\mathbf{r}, t)$ are, say,

$$X(\mathbf{r}, t) = \frac{e^{\varepsilon t}}{2\pi}\int_{-\infty}^{+\infty} d\omega \exp(-i\omega t) L^{-d}\sum_{\mathbf{k}} X(\mathbf{k}, \omega) \exp i\mathbf{k}\cdot\mathbf{r} \tag{B.1}$$

where $X(\mathbf{r},t)$ can be chosen to be $\varphi(\mathbf{r},t)$ or $E(\mathbf{r},t)$. There is then the simple relation $\mathbf{E}(\mathbf{k},\omega) = -i\mathbf{k}\varphi(\mathbf{k},\omega)$. We assume that $\mathbf{E}(\mathbf{r},t)$ is adiabatically "switched on" at time $t = -\infty$. To get the correct asymptotic behavior for $\mathbf{j}_B(\mathbf{r})$, vanishing at $t = -\infty$, one introduces a prefactor $e^{\varepsilon t}$ with infinitesimal $\varepsilon > 0$ in (B.1).[11] Up to linear-order terms in $\varphi(\mathbf{r},t)$, the Hamiltonian $H^1(t)$ which is associated with the interaction between bosons and the external electric field can be expressed as

$$H^1(t) = \alpha \int d\mathbf{r}\rho_B(\mathbf{r})\varphi(\mathbf{r},t). \tag{B.2}$$

Inserting the expressions for $\rho_B(\mathbf{r})$ and $\varphi(\mathbf{r},t)$ into $H^1(t)$ and changing the order of integrations leaves

$$H^1(t) = \alpha e^{\varepsilon t} \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \left\{ L^{-d} \sum_{\mathbf{k}} \varphi(\mathbf{k},\omega)\rho_B(-\mathbf{k}) \right\} \exp(-i\omega t), \quad \varepsilon \to 0. \tag{B.3}$$

We rewrite $H^1(t)$ in terms of its Fourier transform as

$$\begin{aligned} H^1(t) &= e^{\varepsilon t} \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} H^1(\omega) \exp(-i\omega t), \\ H^1(\omega) &= \alpha L^{-d} \sum_{\mathbf{q}} \varphi(\mathbf{q},\omega)\rho_B(-\mathbf{q}). \end{aligned} \tag{B.4}$$

Consider now the average value $\overline{A(t)} \equiv Tr\{\rho(t)A\}$ of a dynamical operator A defined through the statistical operator $\rho(t)$ which satisfies the Liouville equation $i\partial\rho(t)/\partial t = [H + H^1(t), \rho(t)]$ with the Hamiltonian $H + H^1(t)$, and an equilibrium thermal average $\langle A \rangle_H$ carried out with the time-independent operator $\rho^o \equiv e^{-H/k_BT}/Tr\{e^{-H/k_BT}\}$ which in turn is a solution of $[H, \rho^o] \equiv H\rho^o - \rho^o H = 0$. It can be shown (see, e.g., Ref. 11) that the change $\overline{\Delta A(t)} \equiv \overline{A(t)} - \langle A \rangle_H$ in $\langle A \rangle_H$ due to switching on an interaction $H^1(t)$ is directly related with the Fourier component of the retarded Green function $\langle\langle A|H^1(\omega)\rangle\rangle^{ret}$ by

$$\overline{\Delta A(t)} = e^{\varepsilon t} \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \langle\langle A|H^1(\omega)\rangle\rangle_{\omega+i0}^{ret} \exp(-i\omega t). \tag{B.5}$$

That is, the Fourier-component $\overline{\Delta A(\omega)}$ of $\overline{\Delta A(t)}$ is given by

$$\overline{\Delta A(\omega)} \equiv \langle\langle A|H^1(\omega)\rangle\rangle_{\omega}^{ret}. \tag{B.6}$$

Appendix C. Charged-Boson Currents and Conductivity

Let us define the conductivity $\sigma_B(\mathbf{q},\omega)$ associated with the current of CPs in analogy with the conductivity in ordinary electron gas, i.e., as

$$\overline{\mathbf{j}_B(\mathbf{q},\omega)} = \sigma_B(\mathbf{q},\omega)\mathbf{E}(\mathbf{q},\omega). \tag{C.1}$$

Setting $A \equiv \mathbf{j}_B(\mathbf{q})$ in (B.6) we get the Fourier component of the average current density $\overline{\mathbf{j}_B(\mathbf{q},\omega)} \equiv \left\langle\left\langle \mathbf{j}_B(\mathbf{q})|H^1(\omega)\right\rangle\right\rangle_\omega^{ret}$ caused by the external electric field. Because of (B.4) this becomes

$$\overline{\mathbf{j}_B(\mathbf{q},\omega)} \equiv \alpha L^{-d} \left\langle\left\langle \mathbf{j}_B(\mathbf{q})|\rho_B(-\mathbf{q})\right\rangle\right\rangle_\omega^{ret} \varphi(\mathbf{q},\omega). \tag{C.2}$$

Here $\mathbf{j}_B(\mathbf{q})$ is defined by (A.5). Comparing (C.1) and (C.2) yields an expression

$$\sigma_B(\mathbf{q},\omega) = \frac{i\alpha^2}{Mq^2} L^{-d} \sum_{\mathbf{p}} \mathbf{q}\cdot(\mathbf{p}+\frac{\mathbf{q}}{2}) \left\langle\left\langle b_{\mathbf{p}}^+ b_{\mathbf{p}+\mathbf{q}}|\rho_B(-\mathbf{q})\right\rangle\right\rangle_\omega^{ret} \tag{C.3}$$

to calculate the conductivity provided by CPs, and where the dot in (C.3) is the usual dot product of two vectors. The applied external electric field $\mathbf{E}(\mathbf{r},t)$ is assumed to change so slowly that its variation over the spatial extent of a CP is negligible.

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