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# Anomalous Hall effect induced by a pure longitudinal spin injection

S.Y. Liu[a,*], Norman J.M. Horing[b], X.L. Lei[a], B.M. Monroy[c]

[a]*Department of Physics, Shanghai Jiaotong University, Shanghai 200030, China*
[b]*Department of Physics and Engineering Physics, Stevens Institute of Technology, Hoboken, NJ 07030, USA*
[c]*Inst. Investig. Materiales, Universidad Nacional Autonoma de México, Mexico D.F., Mexico*

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**Abstract**

Recently, a nonvanishing anomalous Hall resistivity was observed accompanying a pure spin injection into aluminum through a ferromagnetic contact [Nature 442, (2006) 176]. In this paper, we present a quantitative explanation of this experimental observation considering intrinsic spin–orbit coupling, as well as an extrinsic spin–orbit interaction induced by electron-impurity scattering. In connection with this, we evaluate the band structure and wave function of aluminum by the orthogonalized-plane-wave method and determine the contributions to anomalous Hall resistivity from side-jump and skew scatterings after numerically solving a spatially dependent Boltzmann equation. The obtained Hall resistivity is in good agreement with the experimental data.


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Spin–orbit interaction (SOI) in a spin-split system can give rise to a nonvanishing anomalous Hall current (AHC), in addition to the ordinary one [1]. To observe this anomalous Hall effect (AHE) in homogeneous systems, two nonvanishing quantities are required: magnetization and longitudinal current. Recently, Valenzuela and Tinkham presented a Hall measurement arising from spin injection into metal aluminum [2], finding a finite Hall resistivity in the case of *vanishing* net charge current. In this paper, we provide a realistic, microscopic quantitative theory to investigate the AHE due to spin injection considering the momentum dependencies of the scattering rates and of the side-jump (SJ) and skew scattering (SS) contributions to the AHC. The resulting Hall resistivities are in good agreement with the experimental data. We determine the energy band structure of electrons in aluminum using the standard OPW method in the absence of SO interaction, and present the electronic Bloch states as a mixture of spin up and spin down species:

$$\psi^{(\mu)}_{n\mathbf{p}}(\mathbf{r}) = \sum_{\mathbf{G}} [a^{(\mu)}_{n\mathbf{p}}(\mathbf{G})|\mu\rangle + b^{(\mu)}_{n\mathbf{p}}(\mathbf{G})|\bar{\mu}\rangle] \exp[\mathrm{i}(\mathbf{p}-\mathbf{G})\cdot\mathbf{r}], \tag{1}$$

with $\mu = 1, 2$ representing spinors ($\bar{\mu} = 3 - \mu$), $\mathbf{p}$ as lattice momentum confined to the first BZ, $n$ as band index, and $\mathbf{G}$ as reciprocal lattice vector. The coefficient $a^{(\mu)}_{n\mathbf{p}}(\mathbf{G})$ is determined by the pseudopotential form of the Schrödinger equation, while $b^{(\mu)}_{n\mathbf{p}}(\mathbf{G})$ is obtained from a first-order perturbation associated with the SO part of the pseudopotential, $\lambda\hat{\mathbf{L}}\cdot\hat{\mathbf{S}}P_1$ ($\lambda$ is the SO coupling constant and $P_1$ is the operator for projection on the state $l = 1$). To take account of extrinsic SOI, the electron-impurity scattering potential, $V(\mathbf{p},\mathbf{k})$, involves not only a spin-independent part, $V_0(\mathbf{p},\mathbf{k})$, but also a SO part, $V_{\mathrm{SO}}(\mathbf{p},\mathbf{k})$: $V^{\mu\nu}_{\mathrm{SO}}(\mathbf{p},\mathbf{k}) \equiv A(p,k)\mathbf{p}\times\mathbf{k}\cdot \langle\nu|\vec{\sigma}|\mu\rangle$ ($\vec{\sigma}$ is the Pauli matrices).

In the presence of spin diffusion, the transport properties of solids may be described by a Wigner distribution function, $\hat{\rho}(n,\mathbf{p},\mathbf{R})$, which is a $2\times 2$ diagonal matrix depending on the band index $n$, on microscopic electron momentum $\mathbf{p}$, and on macroscopic space coordinate

*Corresponding author
*E-mail addresses:* liusy@sjtu.edu.cn (S.Y. Liu), marel@ciencias.unam.mx (B.M. Monroy).

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$\mathbf{R} \equiv (x, y, z)$. This function obeys a kinetic (Boltzmann) equation given by

$$\frac{\mathbf{p}}{m} \cdot \nabla_{\mathbf{R}} \hat{\rho}_{\mu\mu}(n, \mathbf{p}, \mathbf{R}) = \sum_{\mathbf{p},\mathbf{k},n'} \{ W^{nn'}_{\mu\mu}(\mathbf{p}, \mathbf{k})[\hat{\rho}_{\mu\mu}(n', \mathbf{k}, \mathbf{R}) - \hat{\rho}_{\mu\mu}(n, \mathbf{p}, \mathbf{R})] + W^{nn'}_{\mu\bar{\mu}}(\mathbf{p}, \mathbf{k})[\hat{\rho}_{\bar{\mu}\bar{\mu}}(n', \mathbf{k}, \mathbf{R}) - \hat{\rho}_{\mu\mu}(n, \mathbf{p}, \mathbf{R})]\}, \tag{2}$$

with, $W^{nn'}_{\mu\mu}(\mathbf{p}, \mathbf{k})$ and $W^{nn'}_{\mu\bar{\mu}}(\mathbf{p}, \mathbf{k})$, respectively, as the non-spin-flip and spin-flip scattering rates, depending on $a^{(\mu)}_{n\mathbf{p}}$ and $b^{(\mu)}_{n\mathbf{p}}$.

The extrinsic SOI makes nonvanishing contributions to AHC through a SJ process proposed by Berger and a SS given by Smit. Applying the general rules of nonequilibrium Green's functions, we find that the SJ current, $\mathbf{J}^{\mathrm{SJ}}(\mathbf{R})$, and the SS AHC, $\mathbf{J}^{\mathrm{SS}}(\mathbf{R})$, are determined by [ $\mathbf{v}_{n\mathbf{p}} \equiv \mathrm{d}\varepsilon_{n\mathbf{p}}/\mathrm{d}\mathbf{p}$, $\tau(n', \mathbf{k})$ is the transport relaxation time, and $\varepsilon_{\alpha\beta z}$ is the totally antisymmetric tensor]

$$J^{\mathrm{SJ}}_{\alpha}(\mathbf{R}) = -\mathrm{i}\pi e N_i \sum_{\substack{\mathbf{G}_1,\mathbf{G}_2,\mathbf{G}_3,\mathbf{G}_4 \\ \mathbf{p},\mathbf{k},\mu,n,n'}} (-1)^{\mu} \bar{A}^{(\mu)}(n\mathbf{p}, n'\mathbf{k}, \mathbf{G}_1, \mathbf{G}_2) \times \bar{V}^{(\mu)}_0(n'\mathbf{k}, n\mathbf{p}, \mathbf{G}_3, \mathbf{G}_4)\varepsilon_{\alpha\beta z}[(\mathbf{k} - \mathbf{G}_2) - (\mathbf{p} - \mathbf{G}_1)]_{\beta}\delta(\varepsilon_{n'\mathbf{k}} - \varepsilon_{n\mathbf{p}}) \times [\hat{\rho}_{\mu\mu}(n, \mathbf{p}, \mathbf{R}) - \hat{\rho}_{\mu\mu}(n', \mathbf{k}, \mathbf{R})],$$

$$\mathbf{J}^{\mathrm{SS}}(\mathbf{R}) = \mathrm{i}4\pi^2 e N_i \sum_{\substack{\mathbf{p},\mathbf{k},\mathbf{q} \\ \mu,n,n',n''}} \sum_{\substack{\mathbf{G}_1,\mathbf{G}_2,\mathbf{G}_3 \\ \mathbf{G}_4,\mathbf{G}_5,\mathbf{G}_6}} \mathbf{v}_{n\mathbf{p}}(-1)^{\mu}\delta(\varepsilon_{n'\mathbf{k}} - \varepsilon_{nbfp}) \times \delta(\varepsilon_{n''\mathbf{q}} - \varepsilon_{n\mathbf{p}})\tau(n', \mathbf{k})\{\bar{A}^{(\mu)}(n\mathbf{p}, n'\mathbf{k}, \mathbf{G}_1, \mathbf{G}_2)V^{(\mu)}_0(n'\mathbf{k}, n''\mathbf{q}, \mathbf{G}_3, \mathbf{G}_4) \times \varepsilon_{\alpha\beta z}[\mathbf{p} - \mathbf{G}_1]_{\alpha}[\mathbf{k} - \mathbf{G}_2]_{\beta} \times V^{(\mu)}_0(n''\mathbf{q}, n\mathbf{p}, \mathbf{G}_5, \mathbf{G}_6) + \begin{bmatrix} n\mathbf{p} & \to & n'\mathbf{k} \\ n'\mathbf{k} & \to & n''\mathbf{q} \\ n''\mathbf{q} & \to & n\mathbf{p} \end{bmatrix} + \begin{bmatrix} n\mathbf{p} & \to & n''\mathbf{q} \\ n'\mathbf{k} & \to & n\mathbf{p} \\ n''\mathbf{q} & \to & n'\mathbf{k} \end{bmatrix} \Bigg\} \hat{\rho}_{\mu\mu}(n', \mathbf{k}, \mathbf{R}),$$

with $N_i$ as the impurity density, $\bar{A}^{(\mu)}(n\mathbf{p}, n'\mathbf{k}, \mathbf{G}_1, \mathbf{G}_2) \equiv [a^{(\mu)}_{n'\mathbf{k}}(\mathbf{G}_2)]^* a^{(\mu)}_{n\mathbf{p}}(\mathbf{G}_1) A(|\mathbf{p} - \mathbf{G}_1|, |\mathbf{k} - \mathbf{G}_2|)$, and $\bar{V}^{(\mu)}_0(nbfp, n'\mathbf{k}, \mathbf{G}_1, \mathbf{G}_2) \equiv [a^{(\mu)}_{n'\mathbf{k}}(\mathbf{G}_2)]^* a^{(\mu)}_{n\mathbf{p}}(\mathbf{G}_1) V_0(\mathbf{p} - \mathbf{G}_1, \mathbf{k} - \mathbf{G}_2)$.

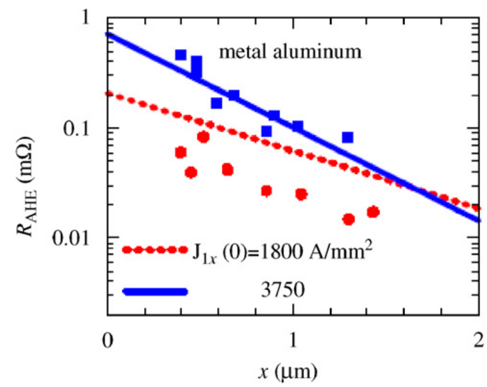


Fig. 1. Anomalous Hall resistivities, $R_{\mathrm{AHE}}$, as functions of coordinate $x$ for several different initial values of $J_{\uparrow x}(x)$: $J_{\uparrow x}(0) = 3750$, $1800\,\mathrm{A}/\mu\mathrm{m}^2$. The solid and dotted lines are our results, and the squares (circles) are the experimental data (Ref. [2]).

Using these formulas, we have calculated the Hall resistivities for two initial values of spin-up current $J_{\uparrow x}(x)$ and have compared the obtained results with the experimental data. The results are plotted in Fig. 1. It is evident that our theoretical results are in good agreement with the experimental data.

In conclusion, we have presented a fully microscopic theory to analyze the AHE observed in the spin-injection experiment in aluminum. Performing realistic calculations for aluminum, we have determined anomalous Hall resistivities that are in good agreement with the experimental data.

## References

[1] For a recent review, see J. Sinova, et al., Int. J. Mod. Phys. B 18 (2004) 1083.

[2] S.O. Valenzuela, M. Tinkham, Nature 442 (2006) 176.