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Bosonic nature of collective Cooper pairs

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ABSTRACT

Superconductivity is not considered as a Bose–Einstein condensation, because the creation and annihilation operators of Cooper pairs do not satisfy bosonic commutation relations. However, collective pairs can be constructed by a linear combination of Cooper pairs and we demonstrate in this Letter that these collective Cooper pairs have bosonic nature. In addition, the Bardeen–Cooper–Schrieffer (BCS) superconducting ground state can be built by means of these pairs and in consequence, could be treated as a Bose–Einstein condensate.

1. Introduction

The Cooper pairs have an integer total spin; however, they are not considered as bosons because their creation ($\hat{b}^\dagger_{\mathbf{k}} \equiv \hat{c}^\dagger_{\mathbf{k}\uparrow}\hat{c}^\dagger_{-\mathbf{k}\downarrow}$) and annihilation ($\hat{b}_{\mathbf{k}} \equiv \hat{c}_{-\mathbf{k}\downarrow}\hat{c}_{\mathbf{k}\uparrow}$) operators have the following commutation relations [1]

$$\begin{cases} [\hat{b}^\dagger_{\mathbf{k}}, \hat{b}^\dagger_{\mathbf{k}'}] \equiv \hat{b}^\dagger_{\mathbf{k}}\hat{b}^\dagger_{\mathbf{k}'} - \hat{b}^\dagger_{\mathbf{k}'}\hat{b}^\dagger_{\mathbf{k}} = [\hat{b}_{\mathbf{k}}, \hat{b}_{\mathbf{k}'}] = 0, \\ [\hat{b}_{\mathbf{k}}, \hat{b}^\dagger_{\mathbf{k}'}] = (1 - \hat{n}_{-\mathbf{k}\downarrow} - \hat{n}_{\mathbf{k}\uparrow})\delta_{\mathbf{k}\mathbf{k}'}, \end{cases} \tag{1}$$

where $\hat{n}_{\mathbf{k}\sigma} = \hat{c}^\dagger_{\mathbf{k}\sigma}\hat{c}_{\mathbf{k}\sigma}$ is the number operator of electrons, $\hat{c}^\dagger_{\mathbf{k}\sigma}$ and $\hat{c}_{\mathbf{k}\sigma}$ are the creation and annihilation operators of a single electron with linear momentum $\mathbf{k}$ and spin σ, respectively. In addition, it is easy to prove that $\hat{b}^\dagger_{\mathbf{k}}\hat{b}^\dagger_{\mathbf{k}} = \hat{b}_{\mathbf{k}}\hat{b}_{\mathbf{k}} = 0$, which even emphasizes their fermionic character.

There is a controversial about whether the superconductivity is a Bose–Einstein condensation [2] and experimental verifications through the Hanbury Brown–Twiss effect have been proposed [3]. Recently, Kaplan, Navarro and Sánchez [4] found that the commutation relations of Eq. (1) lead to a para-Fermi statistics of rank $p = 1$. On the other hand, Fujita and Morabito [5] pointed out that the number operator $\sum_{\mathbf{k}} \hat{b}^\dagger_{\mathbf{k}}\hat{b}_{\mathbf{k}}$ has eigenvalues of 0, 1, 2, ..., despite that the eigenvalues of $\hat{b}^\dagger_{\mathbf{k}}\hat{b}_{\mathbf{k}}$ can only be 0 or 1.

In this Letter, we introduce the concept of collective Cooper pairs as a linear combination of Cooper pairs and we demonstrate that they have bosonic nature.

2. Superconducting ground state

Let us start from the BCS ground state ($|G\rangle$), which can be written as [6],

$$|G\rangle = \prod_{\mathbf{k}} (u_{\mathbf{k}} + v_{\mathbf{k}}\hat{b}^\dagger_{\mathbf{k}})|0\rangle, \tag{2}$$

where $|v_{\mathbf{k}}|^2$ and $|u_{\mathbf{k}}|^2$ are respectively the occupied and unoccupied probabilities of the pair $(\mathbf{k}\uparrow, -\mathbf{k}\downarrow)$. Expanding Eq. (2) we find that for $u_{\mathbf{k}} \neq 0$,

$$\begin{aligned} |G\rangle &= \left[\prod_{\mathbf{k}} u_{\mathbf{k}}\right]\left[1 + \sum_{\mathbf{k}} \frac{v_{\mathbf{k}}}{u_{\mathbf{k}}}\hat{b}^\dagger_{\mathbf{k}} + \sum_{\mathbf{k}\mathbf{k}'} \frac{v_{\mathbf{k}}v_{\mathbf{k}'}}{u_{\mathbf{k}}u_{\mathbf{k}'}}\hat{b}^\dagger_{\mathbf{k}}\hat{b}^\dagger_{\mathbf{k}'} + \cdots\right]|0\rangle \\ &= \left[\prod_{\mathbf{k}} u_{\mathbf{k}}\right]\sum_{n=0}^{N}\left(\sum_{\mathbf{k}} \frac{v_{\mathbf{k}}}{u_{\mathbf{k}}}\hat{b}^\dagger_{\mathbf{k}}\right)^n |0\rangle, \end{aligned} \tag{3}$$

where $N \equiv \sum_{\mathbf{k}} 1$ is the maximum number of Cooper pairs.

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3. The state of *n* collective Cooper pairs

In Eq. (3), each term of the summation corresponds to a state of n pairs and then, we define the state of n collective Cooper pairs ($|n\rangle$) as

$$|n\rangle \equiv A_n\left(\sum_{\mathbf{k}}\frac{v_{\mathbf{k}}}{u_{\mathbf{k}}}\hat{b}^{\dagger}_{\mathbf{k}}\right)^n|0\rangle, \tag{4}$$

where

$$A_n \equiv \left(n!\sum_{\mathbf{k}_1}\cdots\sum_{\substack{\mathbf{k}_n \\ \mathbf{k}_i\neq\mathbf{k}_j}}\left|\frac{v_{\mathbf{k}_1}\cdots v_{\mathbf{k}_n}}{u_{\mathbf{k}_1}\cdots u_{\mathbf{k}_n}}\right|^2\right)^{-1/2} \tag{5}$$

is the normalization factor. Eq. (4) can be rewritten as

$$|n\rangle = A_n\sum_{\mathbf{k}_1}\cdots\sum_{\substack{\mathbf{k}_n \\ \mathbf{k}_i\neq\mathbf{k}_j}}\frac{v_{\mathbf{k}_1}\cdots v_{\mathbf{k}_n}}{u_{\mathbf{k}_1}\cdots u_{\mathbf{k}_n}}\hat{b}^{\dagger}_{\mathbf{k}_1}\cdots\hat{b}^{\dagger}_{\mathbf{k}_n}|0\rangle, \tag{6}$$

since $\hat{b}^{\dagger}_{\mathbf{k}}\hat{b}^{\dagger}_{\mathbf{k}} = 0$. From Eqs. (1) and (6) it can be proved that

$$\sum_{\mathbf{k}}\hat{b}^{\dagger}_{\mathbf{k}}\hat{b}_{\mathbf{k}}|n\rangle = n|n\rangle. \tag{7}$$

This fact shows that $|n\rangle$ is the eigenfunction expected in Ref. [5].

4. Creation and annihilation operators of collective Cooper pairs

From Eq. (4) we define the creation ($\hat{a}^{\dagger}$) and annihilation ($\hat{a}$) operators of collective Cooper pairs as

$$\hat{a}^{\dagger} = \sum_{\mathbf{k}}\frac{v_{\mathbf{k}}}{u_{\mathbf{k}}}\hat{b}^{\dagger}_{\mathbf{k}}\hat{\eta}, \tag{8}$$

$$\hat{a} = \sum_{\mathbf{k}}\frac{u_{\mathbf{k}}}{v_{\mathbf{k}}}\hat{b}_{\mathbf{k}}\hat{\beta}, \tag{9}$$

where $\hat{\eta}$ and $\hat{\beta}$ are operators that provide information about the number of pairs in the state $|n\rangle$, which are their eigenfunctions, i.e.,

$$\hat{\eta}|n\rangle = \eta_n|n\rangle \tag{10}$$

and

$$\hat{\beta}|n\rangle = \beta_n|n\rangle. \tag{11}$$

The eigenvalues in Eqs. (10) and (11) are

$$\eta_n = \sqrt{n+1}\frac{A_{n+1}}{A_n} \tag{12}$$

and

$$\beta_n = \frac{\sqrt{n}}{n(N-n+1)}\frac{A_{n-1}}{A_n}. \tag{13}$$

Therefore, we obtain (see Appendix A)

$$\hat{a}^{\dagger}|n\rangle = \sqrt{n+1}|n+1\rangle \tag{14}$$

and

$$\hat{a}|n\rangle = \sqrt{n}|n-1\rangle. \tag{15}$$

In consequence, the number operator of collective Cooper pairs ($\hat{n}$) is

$$\hat{n}|n\rangle \equiv \hat{a}^{\dagger}\hat{a}|n\rangle = n|n\rangle. \tag{16}$$

Furthermore, Eqs. (14) and (15) yield to

$$\left[\hat{a}, \hat{a}^{\dagger}\right]|n\rangle = |n\rangle \tag{17}$$

and

$$\left[\hat{a}^{\dagger}, \hat{a}^{\dagger}\right] = [\hat{a}, \hat{a}] = 0, \tag{18}$$

which are the bosonic commutation relations. In other words, the creation and annihilation operators of collective Cooper pairs have bosonic nature.

5. Summary

We have introduced a new excitation named collective Cooper pair, which is a linear combination of Cooper pairs. We proved that its creation and annihilation operators accomplish bosonic commutation relations. This bosonic nature of collective pairs is given rise from their diffuse character on the Cooper pairs, which permits the accumulation of many collective pairs at a single quantum state with a null center-of-mass momentum of pair.

It is worth mentioning that the BCS superconducting ground state $|G\rangle$ can be expressed in terms of $|n\rangle$, i.e.,

$$|G\rangle = \sum_{n=0}^{N}\frac{\prod_{\mathbf{k}} u_{\mathbf{k}}}{A_n}|n\rangle, \tag{19}$$

where

$$|n\rangle = \frac{(\hat{a}^{\dagger})^n}{\sqrt{n!}}|0\rangle. \tag{20}$$

Hence, the superconducting states can be considered as a set of collective Cooper pairs, which could lead to a Bose–Einstein condensation due to their bosonic nature. The present analysis can be extended to the case with the center-of-mass momentum of pair different from zero, which is currently under study.

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Appendix A

From Eqs. (4) and (8) we have

$$\hat{a}^{\dagger}|n\rangle = \eta_n A_n\sum_{\mathbf{k}}\frac{v_{\mathbf{k}}}{u_{\mathbf{k}}}\hat{b}^{\dagger}_{\mathbf{k}}\left(\sum_{\mathbf{k}}\frac{v_{\mathbf{k}}}{u_{\mathbf{k}}}\hat{b}^{\dagger}_{\mathbf{k}}\right)^n|0\rangle = \eta_n\frac{A_n}{A_{n+1}}|n+1\rangle, \tag{A.1}$$

which together with Eq. (12) leads to Eq. (14).

On the other hand, for the annihilation operator ($\hat{a}$), Eqs. (6) and (9) yield to

$$\hat{a}|n\rangle = \beta_n A_n\sum_{\mathbf{k}}\frac{u_{\mathbf{k}}}{v_{\mathbf{k}}}\hat{b}_{\mathbf{k}}\sum_{\mathbf{k}_1}\cdots\sum_{\substack{\mathbf{k}_n \\ \mathbf{k}_i\neq\mathbf{k}_j}}\frac{v_{\mathbf{k}_1}\cdots v_{\mathbf{k}_n}}{u_{\mathbf{k}_1}\cdots u_{\mathbf{k}_n}}\hat{b}^{\dagger}_{\mathbf{k}_1}\cdots\hat{b}^{\dagger}_{\mathbf{k}_n}|0\rangle, \tag{A.2}$$

where $\hat{b}_{\mathbf{k}}\hat{b}^{\dagger}_{\mathbf{k}_1}\cdots\hat{b}^{\dagger}_{\mathbf{k}_n} = 0$ if $\mathbf{k}\neq\mathbf{k}_i$, being $i = 1,\ldots,n$. For $\mathbf{k} = \mathbf{k}_i$, the summation of $\mathbf{k}$ is reduced to n terms and each term has the same value due to the fact that $\mathbf{k}_i$ is a dummy variable, i.e.,

$$\sum_{\mathbf{k}}\hat{b}_{\mathbf{k}}\sum_{\mathbf{k}_1}\cdots\sum_{\substack{\mathbf{k}_n \\ \mathbf{k}_i\neq\mathbf{k}_j}}\hat{b}^{\dagger}_{\mathbf{k}_1}\cdots\hat{b}^{\dagger}_{\mathbf{k}_n}|0\rangle$$

$$= n\sum_{\mathbf{k}_1}\cdots\sum_{\substack{\mathbf{k}_n \\ \mathbf{k}_i\neq\mathbf{k}_j}}\hat{b}^{\dagger}_{\mathbf{k}_1}\cdots\hat{b}^{\dagger}_{\mathbf{k}_{n-1}}\hat{b}_{\mathbf{k}_n}\hat{b}^{\dagger}_{\mathbf{k}_n}|0\rangle. \tag{A.3}$$

From Eq. (1), $\hat{b}_{\mathbf{k}}\hat{b}^{\dagger}_{\mathbf{k}} = 1 - \hat{n}_{-\mathbf{k}\downarrow} - \hat{n}_{\mathbf{k}\uparrow} + \hat{b}^{\dagger}_{\mathbf{k}}\hat{b}_{\mathbf{k}}$, and $\hat{n}_{\mathbf{k}\sigma}|0\rangle = \hat{b}^{\dagger}_{\mathbf{k}}\hat{b}_{\mathbf{k}}|0\rangle = 0$, the summation of $\mathbf{k}_n$ in Eq. (A.3) gives $N-(n-1)$ since $\mathbf{k}_n \neq \mathbf{k}_1,\ldots,\mathbf{k}_{n-1}$, thus

$$\sum_{\mathbf{k}} \hat{b}_{\mathbf{k}} \sum_{\mathbf{k}_1} \cdots \sum_{\substack{\mathbf{k}_n \\ \mathbf{k}_i \neq \mathbf{k}_j}} \hat{b}^{\dagger}_{\mathbf{k}_1} \cdots \hat{b}^{\dagger}_{\mathbf{k}_n} |0\rangle$$
$$= n(N-n+1) \sum_{\mathbf{k}_1} \cdots \sum_{\substack{\mathbf{k}_{n-1} \\ \mathbf{k}_i \neq \mathbf{k}_j}} \hat{b}^{\dagger}_{\mathbf{k}_1} \cdots \hat{b}^{\dagger}_{\mathbf{k}_{n-1}} |0\rangle. \qquad \text{(A.4)}$$

Therefore, Eq. (A.2) can be rewritten as

$$\hat{a}|n\rangle = \beta_n n(N-n+1) \frac{A_n}{A_{n-1}} |n-1\rangle. \qquad \text{(A.5)}$$

Finally, Eq. (15) is obtained by using Eqs. (13) and (A.5).

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