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Generalized Superconducting Gap in an Anisotropic Boson–Fermion Mixture with a Uniform Coulomb Field

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A boson–fermion (BF) quantum-statistical binary gas mixture model of high-T_c superconductors (HTSCs) recently introduced consists of *resonant* bosonic Cooper electron pairs (CPs) in chemical and thermal equilibrium with single unpaired electrons. Here, this model is refined and extended to include: i) the *anisotropy* of the original BF vertex interaction causing boson formation/disintegration and ii) momentum-independent Coulomb repulsions between electron charge carriers. It is shown that pair breakings due to Coulomb repulsion depend on the separation between boson and fermion spectra. Specifically, as such a separation shrinks, the pair-breaking ability of the Coulomb interaction weakens and disappears altogether at the Bose–Einstein condensation (BEC) T_c, i.e., at the temperature at which a complete softening of bosons occurs due to boson self-energy renormalization. Simultaneous inclusion of both effects produces "islands" in momentum space of *incoherent* CPs above the Fermi sea as temperature is lowered. These islands grow upon further cooling and merge together just before T_c is reached. The BF model thusly extended now predicts a pseudogap phase in 2D HTSCs with lines of points on the Fermi surface along which the pseudogap vanishes, hence explaining the origin of the temperature-dependent "Fermi arcs" observed in experiments.

KEYWORDS: boson–fermion models, preformed Cooper pairs, Bose–Einstein condensation, pseudogap

1. Introduction

The opening of a so-called *pseudogap* in the electronic spectrum of high-temperature superconductors (HTSCs)[1,2] well above their critical temperature T_c has spurred intense debate on its origins, see e.g. ref. 3. A possible origin[4] of the phenomenon is the formation of so-called *preformed* but incoherent Cooper pairs (CPs) above T_c. Thus, various boson–fermion (BF) models[5–11] became natural candidates to describe the novel features of HTSCs. These models are based on the notion that in the presence of an effective interfermion attractive interaction the gas of single fermionic charge carriers in an ionic lattice can evolve into both pairable but unpaired (or itinerant) fermions *plus* individual bosonic CPs.

Perhaps the simplest hamiltonian describing a binary mixture of fermions interacting with bosons in a d-dimensional cubical box of volume L^d is

$$\begin{aligned}\mathcal{H} &\equiv \mathcal{H}_e + \mathcal{H}_B + \mathcal{H}_{int}\\ &= \sum_{\mathbf{k},\sigma} \xi_{\mathbf{k}} a^+_{\mathbf{k}\sigma} a_{\mathbf{k}\sigma} + \sum_{\mathbf{K}} \mathcal{E}_{\mathbf{K}} b^+_{\mathbf{K}} b_{\mathbf{K}}\\ &\quad + fL^{-d}\sum_{\mathbf{q},\mathbf{K}} (b^+_{\mathbf{K}} a_{\mathbf{q}+\mathbf{K}/2\uparrow} a_{-\mathbf{q}+\mathbf{K}/2\downarrow} + \text{h.c.}). \qquad (1)\end{aligned}$$

The first two terms are, respectively, the hamiltonians of free (pairable but unpaired) fermions H_e and of composite-boson CPs $\mathcal{H}_B$, where $a^+_{\mathbf{k}\sigma}$ and $a_{\mathbf{k}\sigma}$ are the usual fermion creation and annihilation operators for individual electrons of momenta $\mathbf{k}$ and spin $\sigma = \uparrow$ or $\downarrow$ while $b^+_{\mathbf{K}}$ and $b_{\mathbf{K}}$ are postulated[12,13] (for a brief review, see ref. 14) to be bosonic operators associated with CPs of definite total, or center-of-mass momentum (CMM), wavevector $\mathbf{K} \equiv \mathbf{k}_1 + \mathbf{k}_2$ being the sum of the wavevectors of two electrons.

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The last term in eq. (1) $\mathcal{H}_{int}$ describes processes of boson formation/disintegration where f is a phenomenological BF coupling constant, nonzero only in the electron-energy range $E_F - \hbar\omega_D \le \epsilon \le E_F + \hbar\omega_D$ about the Fermi energy E_F of the ideal Fermi gas and $\hbar\omega_D$ is the Debye energy. The full hamiltonian $\mathcal{H}$ was first used in refs. 7–10. In (1) fermion $\xi_{\mathbf{k}} = \epsilon_{\mathbf{k}} - \mu$ and boson $\mathcal{E}_{\mathbf{K}}$ energies are measured from μ and 2μ, respectively, where the electronic chemical potential μ is fixed from the constancy of the total electron number whose operator $N \equiv \sum_{\mathbf{k},\sigma} a^+_{\mathbf{k}\sigma} a_{\mathbf{k}\sigma} + 2\sum_{\mathbf{K}} b^+_{\mathbf{K}} b_{\mathbf{K}}$ includes both the number of unpaired fermions and twice the number of bosons. It commutes with (1) and is therefore an invariant of motion for the BF mixture state.

BF models have been devised to investigate various properties of HTSCs such as thermodynamic, transport, pseudogap, single-particle and collective spectra (see, e.g., ref. 15 and references therein). In particular, by using (1) it was shown in refs. 16 and 17 that an upward shift of the boson relative to the fermion spectra leads upon cooling to a continuous decrease of the fermion chemical potential $\mu(\lambda, T)$ with respect to the value E_F associated with free fermions, all as a result of the BF state lying lower in energy than the interactionless fermion state.[18] Thus, as T is decreased the BF binary-gas mixture state develops from the attractively-interacting fermion gas as it gradually *bosonizes*. Accordingly, the single-fermion spectrum becomes gapped as the difference $E_F - \mu(\lambda, T)$ grows from zero already beginning at and below a temperature $T = T^* > T_c$ so that a minimum energy given by a generalized gap $E_g(\lambda, T)$ is required to excite single fermions from the subsystem of unpaired fermions in the BF mixture. An analytic expression for $E_g(\lambda, T)$ embodying this bosonization was derived[16,17] via two-time Green functions.[19] This generalized gap vanishes above the specific "depairing" or "pseudogap" temperature $T^* > T_c$, where T_c is the critical Bose–Einstein condensation (BEC) temperature singularity associated with the BF binary gas, but is nonzero for all $T < T^*$ due initially to the formation of preformed pairs.

In this paper we generalize the results of refs. 16 and 17 in the two important ways just mentioned. In §2 these generalizations are introduced in the BF hamiltonian (1); in §3 the two-time Green function technique to obtain the relevant occupation numbers is introduced; in §4 the role of *renormalized* boson energies in the consequent BEC is discussed; in §5 we analyze the role of a uniform Coulomb interaction in forming the BF mixture properties; and in §6 concluding remarks are stated.

2. Generalization

First, we modify the last term in eq. (1) to contain so-called anisotropy factors $\phi_{\mathbf{q}}$[20] writing it as

$$H_f \equiv fL^{-d}\sum_{\mathbf{q},\mathbf{K}}(\phi_{\mathbf{q}} b^+_{\mathbf{K}} a_{\mathbf{q}+\mathbf{K}/2\uparrow} a_{-\mathbf{q}+\mathbf{K}/2\downarrow} + \text{h.c.}) \tag{2}$$

where $\phi_{\mathbf{q}} = \phi_{-\mathbf{q}}$. The factors $\phi_{\mathbf{q}}$ in (2) originate from the anisotropy of the BF interaction leading to CP formation or from the anisotropy of the Fermi surface, etc. Second, we add to eq. (1) the term

$$H_U \equiv U_0 L^{-d} \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} a^+_{\mathbf{k}+\mathbf{q}/2\uparrow} a^+_{-\mathbf{k}+\mathbf{q}/2\downarrow} a_{-\mathbf{k}'+\mathbf{q}/2\downarrow} a_{\mathbf{k}'+\mathbf{q}/2\uparrow} \tag{3}$$

stemming from the Coulomb interaction between fermions modelled as a spatially uniform repulsive field of strength $U_0 \geq 0$. Thus, instead of $\mathcal{H}_{\text{int}}$ in eq. (1) one now has

$$\mathcal{H}'_{\text{int}} \equiv H_U + H_f. \tag{4}$$

Instead of (1) as treated in refs. 7–10, 16, and 17 we study the new total hamiltonian

$$\mathcal{H} = \mathcal{H}_{\text{e}} + \mathcal{H}_{\text{B}} + \mathcal{H}'_{\text{int}} \tag{5}$$

where $\mathcal{H}_{\text{e}}$ and $\mathcal{H}_{\text{B}}$ are the first two terms in eq. (1) while $\mathcal{H}'_{\text{int}}$ is given by (4). We shall examine the distribution of the free fermions below some T^* defined as a critical temperature below which the many-fermion gas separates into a system of coexisting fermions and bosons which mutually convert into one another. We note that the effect of on-site Coulomb repulsion in a BF model was previously addressed by Domanski[20] within a mean-field-approximation (MFA) where, following the Bogoliubov recipe, zero-momentum boson operators are replaced by c-numbers and boson self-energies are neglected (see below). The contribution to (1) from a term such as (3), but only when $U_0 \leq 0$, was reported in refs. 21 and 22.

Properties of the electronic subsystem are functionals of the bosonic variables which in turn depend on coupling, say $\lambda \geqslant 0$ which is dimensionless, and on absolute temperature T. We assume that the distribution of fermions at any λ and T depends on the boson distribution at the same λ and T. Therefore, to consider properties related with the fermionic subsystem of a BF mixture one must use in eq. (5) not the energy $\mathcal{E}_{\mathbf{K}}$ of "bare bosons" but λ- and T-dependent energies $\Omega_{\mathbf{K}}$ of bosonic CPs "dressed" due to their interaction with fermions.[11] In an isotropic model as described by eq. (1) without an explicit Coulombic term, an *implicit* equation to determine $\Omega_{\mathbf{K}}$ was derived in ref. 11, namely

$$\Omega_{\mathbf{K}} = \mathcal{E}_{\mathbf{K}} + f^2 L^{-d} \sum_{\mathbf{q}} \frac{1 - n_{\mathbf{q}+\mathbf{K}/2\uparrow} - n_{-\mathbf{q}+\mathbf{K}/2\downarrow}}{\Omega_{\mathbf{K}} - (\xi_{\mathbf{q}+\mathbf{K}/2} + \xi_{-\mathbf{q}+\mathbf{K}/2})}. \tag{6}$$

Here

$$n_{\mathbf{k},\sigma} = \frac{1}{2}\left[1 - \frac{\xi_{\mathbf{k}}}{E_{\mathbf{k}}}\tanh\left(\frac{E_{\mathbf{k}}}{2k_{\text{B}}T}\right)\right] \tag{7}$$

with

$$E_{\mathbf{k}} \equiv \sqrt{\xi^2_{\mathbf{q}} + 2\lambda(\hbar\omega_{\text{D}})(E_{\text{F}} - \mu)}. \tag{8}$$

are T-dependent occupation numbers $n_{\mathbf{k},\sigma}$ of unpaired electrons in a state with momentum wavevector $\mathbf{k}$ and spin σ, while $\xi_{\mathbf{k}} = \epsilon_{\mathbf{k}} - \mu$ are the single-fermion energy levels. Processes contributing to the energy $\mathcal{E}_{\mathbf{K}}$ of "bare" bosonic CPs in eq. (6) are given by the term describing formation/disintegration of bosons in eq. (1). In our qualitative discussion we shall use eq. (6) for the boson energies. The crucial role of a boson-energy renormalization such as eq. (6), caused by the boson self-energy, but ignored in ref. 20, was deduced by Alexandrov. In particular in ref. 23 he points out that superconductivity in BF models is not BCS-like but driven by the BEC of bosons which arises due to the complete softening of their spectrum at T_{c}, and is possible when going beyond the MFA. Doing this via two-time Green-function (GF) techniques to investigate the charge-carrier distribution in the generalized hamiltonian BF model (5) is now sketched.

3. Two-Time Green Functions

Whatever distribution of free carriers occurs in a superconductor can be addressed by starting from $n_{\mathbf{k},\sigma} \equiv \langle a^+_{\mathbf{k},\sigma} a_{\mathbf{k},\sigma}\rangle$. These c-numbers $n_{\mathbf{k},\sigma}$ are then obtained, e.g., from an infinite chain of equations for two-time retarded GF $\langle\langle A(t) \mid B(t')\rangle\rangle$ as defined in ref. 19 eq. (2.1b) for dynamical operators $a_{\mathbf{k}\sigma}(t)$ and $a^+_{\mathbf{k}'\sigma}(t')$ at times t and t' in the Heisenberg representation. If A and B are any two operators, the Fourier transform $\langle\langle A \mid B\rangle\rangle_\omega$ of $\langle\langle A(t) \mid B(t')\rangle\rangle$ satisfies the infinite chain of equations [see, e.g., eq. (A·2) in ref. 11].

$$\hbar\omega\langle\langle A \mid B\rangle\rangle_\omega = \langle [A, B]_\eta\rangle_{\mathcal{H}} + \langle\langle [A, \mathcal{H}]_- \mid B\rangle\rangle_\omega \tag{9}$$

where square brackets $[A, B]_\eta \equiv AB + \eta BA$ denote the commutator $(\eta = -1)$ or anticommutator $(\eta = +1)$ of operators A and B.

Choosing $A \equiv a_{\mathbf{k}\uparrow}$, $B \equiv a^+_{\mathbf{k}'\uparrow}$, $\eta = +1$ in eq. (9) gives for eq. (9)

$$\begin{aligned}(\hbar\omega - \xi_{\mathbf{k}})\langle\langle a_{\mathbf{k}\uparrow} \mid a^+_{\mathbf{k}'\uparrow}\rangle\rangle_\omega &= \delta_{\mathbf{k}\mathbf{k}'} - fL^{-d/2}\sum_{\mathbf{K}}\phi_{\mathbf{k}-\mathbf{K}/2}\langle\langle b_{\mathbf{K}} a^+_{-\mathbf{k}+\mathbf{K}\downarrow} \mid a^+_{\mathbf{k}'\uparrow}\rangle\rangle_\omega \\ &\quad + U_0 L^{-d/2}\sum_{\mathbf{p},\mathbf{q}}\langle\langle a^+_{-\mathbf{k}+\mathbf{q}\downarrow} a_{-\mathbf{p}+\mathbf{q}/2\downarrow} a_{\mathbf{p}+\mathbf{q}/2\uparrow} \mid a^+_{\mathbf{k}'\uparrow}\rangle\rangle_\omega\end{aligned} \tag{10}$$

since $[a_{\mathbf{k}\uparrow}, a^+_{\mathbf{k}'\uparrow}]_+ = \delta_{\mathbf{k}\mathbf{k}'}$. To obtain eq. (10) one uses the explicit expression for the commutator $[a_{\mathbf{k}\uparrow}, \mathcal{H}]$ in eq. (9). Thus eq. (10) relates the first-order GF on the lhs with *higher-order* GFs on the rhs. In analogy with the pure Bose gas where the emergence below a critical T_{c} of nonzero $\langle b_0\rangle$ and $\langle b^+_0\rangle$ signals the appearance of superfluidity,[24] here we expect nonzero $\langle b_{\mathbf{K}}\rangle$ and $\langle b^+_{\mathbf{K}}\rangle$ to presage the BF mixture state that emerges in an attractively-interacting fermion gas. Thus, we put

$$\langle\langle b_{\mathbf{K}} a^{+}_{-\mathbf{k}+\mathbf{K}\downarrow} \mid a^{+}_{\mathbf{k}'\uparrow}\rangle\rangle = \langle b_{\mathbf{K}}\rangle \langle\langle a^{+}_{-\mathbf{k}+\mathbf{K}\downarrow} \mid a^{+}_{\mathbf{k}'\uparrow}\rangle\rangle + \langle\langle (b_{\mathbf{K}} - \langle b_{\mathbf{K}}\rangle) a^{+}_{-\mathbf{k}+\mathbf{K}\downarrow} \mid a^{+}_{\mathbf{k}'\uparrow}\rangle\rangle \quad (11)$$

and retain only the term proportional to $\langle b_{\mathbf{K}}\rangle$. Many BF models deal with $\langle b_0\rangle \neq 0$, see e.g., ref. 20. We refer to $\langle b_{\mathbf{K}}\rangle \neq 0$ as a BF order parameter. Contributions beyond eq. (11) containing the difference $b^{+}_{\mathbf{K}} - \langle b^{+}_{\mathbf{K}}\rangle$ are neglected.[11,17] Note that the mean values $\langle b_{\mathbf{K}}\rangle$ and $\langle b^{+}_{\mathbf{K}}\rangle$ are identically zero for a ideal Bose gas described by the Hamiltonian $H^{o}_{B} \equiv \sum_{\mathbf{K}} E^{0}_{\mathbf{K}} b^{+}_{\mathbf{K}} b_{\mathbf{K}}$. In the BF mixture, however, $[N_B, \mathcal{H}] \neq 0$ implies a nonconstant $N_B(\lambda, T)$ that varies with coupling λ and temperature T. Thus, the term eq. (2) in eq. (5) breaks the degeneracy associated with the number conservation law and leads, in particular, to nonzero $\langle b_{\mathbf{K}}\rangle$ and $\langle b^{+}_{\mathbf{K}}\rangle$ in eq. (11).[17]

An additional GF on the rhs of (10) may be cast as a linear combination of the first-order GFs and of a so-called *irreducible* piece which by definition cannot be reduced to lower order GFs.[25] As in all first-order theories, we ignore that irreducible part write

$$\begin{aligned}\langle\langle a^{+}_{-\mathbf{k}+\mathbf{q}\downarrow} a_{-\mathbf{p}+\mathbf{q}/2\downarrow} a_{\mathbf{p}+\mathbf{q}/2\uparrow} \mid a^{+}_{\mathbf{k}'\uparrow}\rangle\rangle &= \langle a^{+}_{-\mathbf{k}+\mathbf{q}\downarrow} a_{-\mathbf{p}+\mathbf{q}/2\downarrow}\rangle \langle\langle a_{\mathbf{p}+\mathbf{q}/2\uparrow} \mid a^{+}_{\mathbf{k}'\uparrow}\rangle\rangle \\ &\quad - \langle a^{+}_{-\mathbf{k}+\mathbf{q}\downarrow} a_{\mathbf{p}+\mathbf{q}/2\uparrow}\rangle \langle\langle a_{-\mathbf{p}+\mathbf{q}/2\downarrow} \mid a^{+}_{\mathbf{k}'\uparrow}\rangle\rangle \\ &\quad + \langle a_{-\mathbf{p}+\mathbf{q}/2\downarrow} a_{\mathbf{p}+\mathbf{q}/2\uparrow}\rangle \langle\langle a^{+}_{-\mathbf{k}+\mathbf{q}\downarrow} \mid a^{+}_{\mathbf{k}'\uparrow}\rangle\rangle. \end{aligned} \quad (12)$$

An expression like eq. (12) is assumed in the statistical Wick–Bloch–de Dominicis theorem[26] which relates the average value $\langle A_1 A_2 \cdots A_{2p}\rangle$ of a product of creation and annihilation operators with a sum of *all* possible pairings of $\langle A_1 A_2 \cdots A_{2p}\rangle$.[27] Different terms on the rhs of (12) can be estimated as follows:

a) Appearance of nonzero averages $\langle a^{+}_{-\mathbf{k}+\mathbf{q}\downarrow} a_{-\mathbf{p}+\mathbf{q}/2\downarrow}\rangle$ with $-\mathbf{k}+\mathbf{q} \neq -\mathbf{p}+\mathbf{q}/2$ violates momentum conservation valid in a system with translational symmetry. To restore conservation of CMM we thus put $\langle a^{+}_{-\mathbf{k}+\mathbf{q}\downarrow} a_{-\mathbf{p}+\mathbf{q}/2\downarrow}\rangle = \delta_{-\mathbf{k}+\mathbf{q},-\mathbf{p}+\mathbf{q}/2} \langle a^{+}_{-\mathbf{k}+\mathbf{q}\downarrow} a_{-\mathbf{k}+\mathbf{q}\downarrow}\rangle$ in eq. (12).

b) The minus sign with the second contribution on the rhs of eq. (12) is due to the odd number transpositions necessary to re-arrange Fermi operators on the lhs to appear on the rhs. The prefactor $a^{+}_{-\mathbf{k}+\mathbf{q}\downarrow} a_{\mathbf{p}+\mathbf{q}/2\uparrow}$ in this term is due to processes accompanied by spin-flips. We assume $\langle a^{+}_{-\mathbf{k}+\mathbf{q}\downarrow} a_{\mathbf{p}+\mathbf{q}/2\uparrow}\rangle = 0$ and ignore all terms leading to such spin flips.

c) The last term on the rhs of (12) is most important. Being composed of *two* annihilating Fermi operators, the prefactor $\langle a_{-\mathbf{p}+\mathbf{q}/2\downarrow} a_{\mathbf{p}+\mathbf{q}/2\uparrow}\rangle$ of this contribution directly affects the formation of a boson with CMM equal to $\mathbf{q}$.

Substituting eqs. (11) and (12) into eq. (10) then yields

$$\begin{aligned}(\hbar\omega - \xi_{\mathbf{k}} - U_0 n/2)\langle\langle a_{\mathbf{k}\uparrow} \mid a^{+}_{\mathbf{k}'\uparrow}\rangle\rangle_{\omega} &= \delta_{\mathbf{k}\mathbf{k}'} - L^{-d/2} \sum_{\mathbf{q}} \Phi(\mathbf{k},\mathbf{q}) \langle\langle a^{+}_{-\mathbf{k}+\mathbf{q}\downarrow} \mid a^{+}_{\mathbf{k}'\uparrow}\rangle\rangle_{\omega}. \end{aligned} \quad (13)$$

Here $n \equiv n_{\uparrow} + n_{\downarrow}$ is the number density of fermions with both spins "$\uparrow$" and "$\downarrow$" and

$$\Phi(\mathbf{k},\mathbf{q}) \equiv f\phi_{\mathbf{k}-\mathbf{q}/2} \langle b_{\mathbf{q}}\rangle_{\mathcal{H}} - U_0 L^{-d/2} \sum_{\mathbf{p}} \langle a_{-\mathbf{p}+\mathbf{q}/2\downarrow} a_{\mathbf{p}+\mathbf{q}/2\uparrow}\rangle_{\mathcal{H}}. \quad (14)$$

One can now establish an equation for the terms $\langle\langle a^{+}_{-\mathbf{k}+\mathbf{q}\downarrow} \mid a^{+}_{\mathbf{k}'\uparrow}\rangle\rangle_{\omega}$ on the rhs of (13). Choosing in (9) $A \equiv a^{+}_{\mathbf{k}\downarrow}$ and $B \equiv a^{+}_{\mathbf{k}'\uparrow}$ and proceeding in a same manner as in obtaining eq. (13) gives

$$\begin{aligned}(\hbar\omega + \xi_{\mathbf{k}} + U_0 n/2)\langle\langle a^{+}_{\mathbf{k}\downarrow} \mid a^{+}_{\mathbf{k}'\uparrow}\rangle\rangle_{\omega} &= -L^{-d/2} \sum_{\mathbf{q}} \Phi^{*}(\mathbf{k},\mathbf{q}) \langle\langle a_{-\mathbf{k}+\mathbf{q}\uparrow} \mid a^{+}_{\mathbf{k}'\uparrow}\rangle\rangle. \end{aligned} \quad (15)$$

where $\Phi^{*}(\mathbf{k},\mathbf{q})$ is the complex conjugate of $\Phi(\mathbf{k},\mathbf{q})$. Note that the uniform Coulomb repulsion leads to a *permanent* shift *upward* of all single-fermion energy levels $\xi_{\mathbf{k}} = \epsilon_{\mathbf{k}} - \mu$. By setting $\mu \to \mu + U_0 n/2$, i.e., by referring $U_0 n/2$ to the ground-state energy associated with eq. (5), this permanent shift term of $U_0 n/2$ is dropped in eqs. (13) and (15). As to the coefficient functions Φ and Φ^{*}, they may be simplified by use of an exact relation[11] $\langle [b_{\mathbf{Q}}, \mathcal{H}]\rangle_{\mathcal{H}} \equiv 0$ which gives

$$L^{-d/2} \sum_{\mathbf{q}} \phi_{\mathbf{q}} \langle a_{\mathbf{q}+\mathbf{Q}/2\uparrow} a_{-\mathbf{q}+\mathbf{Q}/2\downarrow}\rangle_{\mathcal{H}} = -f^{-1} \Omega_{\mathbf{Q}} \langle b_{\mathbf{Q}}\rangle_{\mathcal{H}}, \quad (16)$$

where the average is performed over the new hamiltonian (5). With the mean-value theorem one may express the lhs of eq. (16) as $L^{-d/2} \tilde{\phi} \sum_{\mathbf{q}} \langle a_{\mathbf{q}+\mathbf{Q}/2\uparrow} a_{-\mathbf{q}+\mathbf{Q}/2\downarrow}\rangle$ where $\tilde{\phi}$ is some $\phi_{\mathbf{q}}$ from the region of integration over $\mathbf{q}$. Factors $\phi_{\mathbf{q}}$ [exactly $= 1$ in (2) if the system is isotropic] vary to modulate the $\mathbf{q}$-dependence of the interaction strength. However, the values of $\phi_{\mathbf{q}}$ remain within an interval near $\bar{\phi} = 1$ which is a normalized value of all anisotropy factors over the Fermi surface. Substituting $\bar{\phi}$ for $\tilde{\phi}$ in eq. (16) renders eq. (14) as

$$\Phi(\mathbf{k},\mathbf{q}) = \left[f\phi_{\mathbf{k}-\mathbf{q}/2} - \left(\frac{U_0 \Omega_{\mathbf{q}}}{f}\right)\right] \langle b_{\mathbf{q}}\rangle_{\mathcal{H}}.$$

Now (13) and (15) are integral equations expressing GFs $\langle\langle a_{\mathbf{k}\uparrow} \mid a^{+}_{\mathbf{k}'\uparrow}\rangle\rangle$ and $\langle\langle a^{+}_{\mathbf{k}\downarrow} \mid a^{+}_{\mathbf{k}'\uparrow}\rangle\rangle$ in terms of $\langle b^{+}_{\mathbf{q}}\rangle$ and $\langle b_{\mathbf{q}}\rangle$. However, these equations may be converted to a system of algebraic equations by considering the substantial difference in scale of *fermion* and the *BF* interaction energies.[17] Indeed, $\langle b_{\mathbf{q}}\rangle$ and $\langle b^{+}_{\mathbf{q}}\rangle$ differ from zero only for those $\mathbf{q}$ which are much smaller in magnitude than the characteristic fermionic wavenumbers $k \sim k_F$. This justifies putting $\mathbf{k} \pm \mathbf{q} \simeq \mathbf{k}$ in all expressions, where $\mathbf{k}$ and $\mathbf{q}$ are fermion and boson wavenumbers, respectively. Assuming $\Omega_{\mathbf{Q}} \simeq \Omega_0$,[20,28] (13) and (15) yield

$$(\hbar\omega - \xi_{\mathbf{k}})\langle\langle a_{\mathbf{k}\uparrow} \mid a^{+}_{\mathbf{k}'\uparrow}\rangle\rangle + S_{\mathbf{k}} \langle\langle a^{+}_{\mathbf{k}\downarrow} \mid a^{+}_{\mathbf{k}'\uparrow}\rangle\rangle = \delta_{\mathbf{k}\mathbf{k}'}$$

$$S^{*}_{\mathbf{k}} \langle\langle a_{\mathbf{k}\uparrow} \mid a^{+}_{\mathbf{k}'\uparrow}\rangle\rangle + (\hbar\omega + \xi_{\mathbf{k}})\langle\langle a^{+}_{\mathbf{k}\downarrow} \mid a^{+}_{\mathbf{k}'\uparrow}\rangle\rangle = 0$$

where we note that $\xi_{\mathbf{k}} = \xi_{-\mathbf{k}}$ and define $S_{\mathbf{k}} \equiv [f\phi_{\mathbf{k}} - (U_0\Omega_0/f)] \sum_{\mathbf{q}} \langle b_{\mathbf{q}}/L^{d/2}\rangle$. Finally, one gets

$$\langle\langle a_{\mathbf{k}\uparrow} \mid a^{+}_{\mathbf{k}'\uparrow}\rangle\rangle_{\omega} = \frac{\hbar\omega + \xi_{\mathbf{k}}}{(\hbar\omega)^2 - E^2_{\mathbf{k}}} \delta_{\mathbf{k}\mathbf{k}'}, \quad (17)$$

$$\langle\langle a^{+}_{-\mathbf{k}\downarrow} \mid a^{+}_{\mathbf{k}'\uparrow}\rangle\rangle_{\omega} = -\left[f\phi_{\mathbf{k}} - \left(\frac{U_0\Omega_0}{f}\right)\right] \times \frac{\delta_{\mathbf{k}\mathbf{k}'}}{(\hbar\omega)^2 - E^2_{\mathbf{k}}} \sum_{\mathbf{q}} \left\langle \frac{b^{+}_{\mathbf{q}}}{L^{d/2}} \right\rangle_{\mathcal{H}}. \quad (18)$$

Poles of the GFs eqs. (17) and (18) occur when

$$E_{\mathbf{k}} = \sqrt{(\epsilon_{\mathbf{k}} - \mu)^2 + E^2_{g\mathbf{k}}} \quad (19)$$

which defines the single-particle spectrum in the BF mixture phase. The spectrum of single fermions in the normal phase now appears *gapped* with the generalized gap

$$E_{g\mathbf{k}}(\lambda, T) \equiv f\left(\phi_{\mathbf{k}} - \frac{\Omega_0}{2\hbar\omega_D}\frac{U_0}{V}\right)\sqrt{n_B(\lambda, T)}, \quad (20)$$

where $n_B(\lambda, T)$ is the total number density of electron pairs. Recall that the BF coupling parameter f in eq. (20) was identified[12,13] with the attractive interelectron BCS interaction strength V through the relation $f = \sqrt{2\hbar\omega_D V}$ so as to recover the BCS gap equation as a special case of a BF model. Furthermore, on the rhs of eq. (20) we have put

$$\sum_{\mathbf{q},\mathbf{q}'}\left\langle\frac{b^+_{\mathbf{q}}}{L^{d/2}}\right\rangle_{\mathcal{H}}\left\langle\frac{b_{\mathbf{q}'}}{L^{d/2}}\right\rangle_{\mathcal{H}} = L^{-d}\sum_{\mathbf{q},\mathbf{q}'}\left\langle b^+_{\mathbf{q}}b_{\mathbf{q}'}\right\rangle_{\mathcal{H}} \simeq L^{-d}\sum_{\mathbf{q}}\left\langle b^+_{\mathbf{q}}b_{\mathbf{q}}\right\rangle_{H^o} \quad (21)$$

where the first equality can be justified[17,24] in the thermodynamic limit when $L^d \longrightarrow \infty$ and holds *only* for particles obeying Bose statistics and if in the second equality of eq. (21) the average over eq. (5) is approximated with the average over $H^o \equiv \mathcal{H}_e + \mathcal{H}_B$ of the interactionless BF mixture. The expression (20) for $E_{g\mathbf{k}}$ resembles eq. (21) of ref. 17. However, simultaneous inclusion of the anisotropy (2) and Coulombic repulsion (3) now leads to the replacement of f in eq. (21) of ref. 17 by the factor $f(\phi_{\mathbf{k}} - [(U_0/2V)(\Omega_0/\hbar\omega_D)])$ in eq. (20). The gap in the single-fermion spectrum $E_{g\mathbf{k}}(\lambda, T)$ thus emerges at and below temperatures T^* much higher than T_c associated with the BEC. Recalling that $\Delta(\lambda, T)$ is the BCS energy gap and $n_0(\lambda, T)$ the BEC condensate density associated with the zero-CMM state, the expression (20) thus generalizes the relation $\Delta(\lambda, T) \propto \sqrt{n_0(\lambda, T)}$ first found in refs. 7–9. The expression (20) contains an important new physical result, namely, that the pair-breaking ability of the Coulomb repulsion depends on the quantity $\Omega_0/\hbar\omega_D$ describing the degree of separation between boson and fermion spectra. Most affected by the Coulomb repulsion are those pairs that are well separated in energy from the single-fermion continuum. Larger $\Omega_0/\hbar\omega_D$ in eq. (20) enhances the effect of eq. (3). In contrast, smaller $\Omega_0/\hbar\omega_D$ weakens the pair-breaking ability of the Coulomb repulsion which even disappears altogether when $\Omega_0 = 0$, i.e., for pairs whose energy per fermion drops below the single-electron Fermi surface. This occurs as long as the Coulomb interaction is assumed a spatially-uniform field that mimics the repulsions between paired electrons within the field of the surrounding unpaired electrons. The isotropy associated with the uniform Coulomb field largely screens the direct interaction between electrons thus lowering the efficiency of direct repulsions. The pair-breaking ability dependence of the Coulomb repulsion on the separation between boson and fermion spectra appears crucial in understanding BF mixture properties.

4. Renormalized Boson Energies and BEC

BEC emerges upon simultaneously satisfying two conditions. First, a singularity must occur in the total number density of bosons

$$n_B(\lambda, T) \equiv L^{-d}\sum_{\mathbf{K}}\left[\exp\left(\frac{\Omega_{\mathbf{K}}}{k_B T}\right) - 1\right]^{-1} \quad (22)$$

where $\Omega_{\mathbf{K}}$ are the boson energies $\mathcal{E}_{\mathbf{K}}$ renormalized due to the BF interaction (6). This necessary condition occurs as the boson mode is softened, i.e., as $\Omega_{\mathbf{K}} \to 0$ which gives rise to the appearance of a nonzero number density $n_{B\mathbf{K}}$ of paired states of CMM K. Second, the value of T_c below which BEC occurs is related implicitly with the number density $n_B(\lambda, T_c)$ of bosons at T_c. In particular, assuming *a linear* boson dispersion of spherical symmetry over K the simple (but implicit) T_c formulas for 2D and 3D BF mixtures were reported [ref. 11 eq. (35)] to be

$$k_B T_c = \begin{cases} c_2 2\sqrt{3}\pi^{-1/2}\hbar v_F n_B(\lambda, T_c)^{1/2} & \text{for 2D} \\ c_3 \pi^{2/3}\zeta(3)^{-1/3}\hbar v_F n_B(\lambda, T_c)^{1/3} & \text{for 3D} \end{cases} \quad (23)$$

where c_d are dimensionality- and interaction-dependent dimensionless coefficients $c_2 = \lambda/2\pi$ in 2D[29] and $c_3 = \lambda/4$ in 3D.[30] Thus, in order for eq. (23) to hold, in addition to $\Omega_{\mathbf{K}} \to 0$ the difference $E_F - \mu(\lambda, T_c)$ which determines the total number of bosons $n_B(\lambda, T_c)$ of two-particle states must to be sufficiently large. For $K = 0$, taking into account the expressions (7) [ref. 11 eqs. (10) and (11)] and (6) becomes

$$\Omega_0 = \mathcal{E}_0 + f^2 L^{-d}\sum_{\mathbf{q}}\frac{1}{\Omega_0 - 2\xi_{\mathbf{q}}}\frac{\xi_{\mathbf{q}}}{E_{\mathbf{q}}}\tanh\left(\frac{E_{\mathbf{q}}}{2k_B T}\right). \quad (24)$$

Writing

$$\frac{\xi_{\mathbf{q}}}{\Omega_0 - 2\xi_{\mathbf{q}}} \equiv \frac{1}{2}\left(\frac{\Omega_0}{\Omega_0 - 2\xi_{\mathbf{q}}} - 1\right)$$

in eq. (24) immediately yields

$$\Omega_0(\lambda, T) = \frac{\mathcal{E}_0 - \lambda(\hbar\omega_D)\displaystyle\int_{-\hbar\omega_D}^{\hbar\omega_D}\frac{dx}{\sqrt{x^2 + 2\lambda(\hbar\omega_D)(E_F - \mu)}}\tanh\frac{1}{2}\beta\sqrt{x^2 + 2\lambda(\hbar\omega_D)(E_F - \mu)}}{1 - \lambda(\hbar\omega_D)\displaystyle\int_{-\hbar\omega_D}^{\hbar\omega_D}\frac{1}{\Omega_0(\lambda, T) - 2x}\frac{dx}{\sqrt{x^2 + 2\lambda(\hbar\omega_D)(E_F - \mu)}}\tanh\frac{1}{2}\beta\sqrt{x^2 + 2\lambda(\hbar\omega_D)(E_F - \mu)}} \quad (25)$$

where $\beta \equiv 1/k_B T$ and the explicit λ- and T-dependence of Ω_0 is emphasized. Through the relation $n_B(\lambda, T) = N(E_F)[E_F - \mu(\lambda, T)]$ [see eq. (20) of ref. 11], Ω_0 will also depend on $n_B(\lambda, T)$ where $N(E_F)$ is the electronic density-of-states per spin and per unit volume evaluated at E_F. In fact, (25) relates the boson energy $\Omega_0(\lambda, T)$ of CMM $\mathbf{K} = 0$ with the total boson number density $n_B(\lambda, T)$ in the BF mixture. In particular, when both $\Omega_0(\lambda, T) = 0$ and $E_F - \mu(\lambda, T) = 0$ eq. (25) provides the condition defining T^* at and below which the attractively-interacting fermion gas becomes a binary mixture of interacting fermions and bosons mutually converting into one another. Equating $\Omega_0(\lambda, T)$ to zero within the temperature range $T_c \le T \le T^*$ eq. (25) takes account of eq. (24) in ref. 17 which was used to determine the magnitude of isotropic generalized gap E_g (see Fig. 1 in ref. 17).

Figure 1 shows $\Omega_0(\lambda, T)/E_F$ for several fixed values of $(E_F - \mu)/E_F$ as a function of T/T_F. The parameters λ and

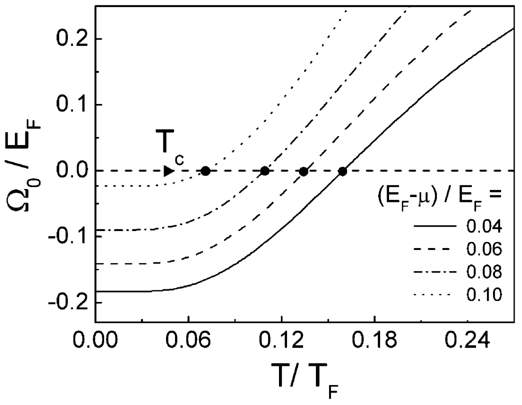

Fig. 1. Dimensionless remormalized bosonic CP energy $\Omega_0(\lambda, T)/E_F$ as a function of T/T_F for several *fixed* fractional number densities $(E_F-\mu)/E_F$ of bosons. Figure schematically shows how *non-temperature-dependent* "bare" bosonic CP energies $\mathcal{E}_0$ are renormalized by "switching-on" the BF interaction. However, the position of $\mathcal{E}_0$ measured from the total energy $2E_F$ of *two* interactionless fermions that make up a composite-boson is not shown. Defined in ref. 11 as $\mathcal{E}_0 = 2[E_F + \hbar\omega_D/\sinh(1/\lambda)]$, it is a coupling-dependent parameter, larger than $\Omega_0(\lambda, T)$ over the whole range of temperatures $T < T^*$. Here, λ and $\hbar\omega_D/E_F$ are chosen respectively to be 0.8 and 0.35. Black triangle marks associated BEC critical temperature.

$\hbar\omega_D/E_F$ are chosen to be respectively 0.8 and 0.35; this particular choice of λ and $\hbar\omega_D/E_F$ yields the value $T_c/T_F = 0.046$ previously found by applying for 2D superconductors eq. (37) of ref. 11. According to eq. (36) from this latter reference, the deviation of $\mu(\lambda, T)$ from E_F at T_c (in E_F units) necessary for BEC to occur is $(E_F-\mu)/E_F = 0.107$. Specifically, BEC does not occur until the fractional boson density (the ratio of the total boson number to the number of available states) $n_B(\lambda, T)/N(E_F)E_F$ reaches the critical value 0.107. Clearly, Ω_0 is positive at higher T. However, for any but fixed $n_B(\lambda, T)$ the boson energy $\Omega_0(\lambda, T)$ decreases monotonically upon cooling and passes through zero at a value of T/T_F determined by the value of $n_B(\lambda, T)$ alone. Physically, the composite-boson concentration $n_B(\lambda, T)$ in a BF mixture will increase at lower values of T, as expected. From Fig. 1 for higher $n_B(\lambda, T)$ the temperature T at which $\Omega_0(\lambda, T)$ changes sign (shown as dots in figure) shifts to lower T. However, BEC cannot occur until $n_B(\lambda, T)$ is at least as large as the critical $n_B(\lambda, T_c)$ satisfying the implicit condition (23). For the parameters chosen in Fig. 1 BEC does *not* occur while $n_B(\lambda, T)/N(E_F)E_F < 0.107$.

In a binary mixture of mutually-converting fermions and bosons $n_B(\lambda, T)$ is *not* fixed as in Fig. 1 but rather increases upon cooling and reaches the critical value $n_B(\lambda, T_c)$ immediately before the BEC. Owing to this continuous increase Ω_0 does not attain the value $\Omega_0(\lambda, T) = 0$ until T equals T_c. Thus, from Fig. 1 one concludes that in a BF mixture with varying boson density $n_B(\lambda, T)$ the temperature-dependent quantity Ω_0 changes sign precisely at the T_c associated with BEC (black triangle at $T_c/T_F = 0.046$ in Fig. 1). It should be noted that such a temperature-dependent behavior of boson energies found in this work rely on eq. (6) and is thus associated with boson formation/disintegration processes *alone*.

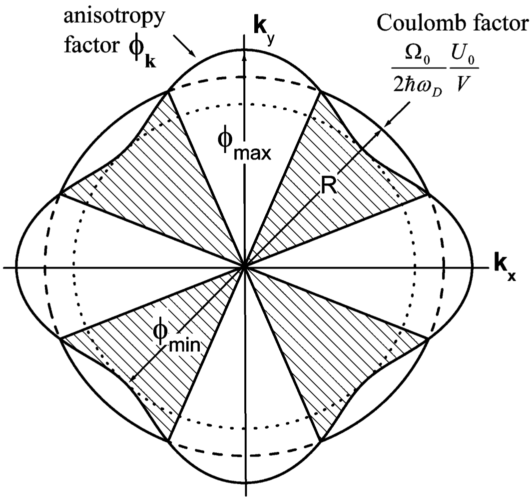

Fig. 2. Comparison of anisotropy factor $\phi_{\mathbf{k}}$ and the dimensionless uniform screened Coulomb potential $(\Omega_0/2\hbar\omega_D)(U_0/V) \equiv R$ shown as a function of wavevector $\mathbf{k}$. Projected onto the Fermi surface, i.e., areas where $\phi_{\mathbf{k}} \leq R$ and $\phi_{\mathbf{k}} > R$, define respectively so-called Fermi and BF arcs, i.e., disconnected segments of $\mathbf{k}$ wherein the energy-momentum dispersion resembles that in a normal metal or, analogous to a Bogoliubov dispersion relation but with a generalized gap eq. (20). Lowering T is followed by a reduction in R or a broadening of the BF arcs which merge into each other by satisfying the condition $\phi_{min} = R$ schematically depicted as the dashed curve. Below the temperature at which the Coulomb factor R passes through the minimum of the anisotropy factor $\phi_{\mathbf{k}}$ the areas with normal distribution of fermions, i.e., Fermi arcs, disappear entirely and the attractively-interacting Fermi gas becomes an anisotropic mixture of fermions and bosons mutually converting into each other.

5. Model Applied and Discussed

The generalized gap eq. (20) differs from zero only within a (thin) energy shell around k_F such that $k_{max} \leq k_F \leq k_{min}$. For any deviation of k from k_F one expects that $E_{g\mathbf{k}}$ approaches rapidly to zero. Outside this thin shell the normal distribution of charge carriers prevails. The behavior of $E_{g\mathbf{k}}$ as a function of wavevector $\mathbf{k}$ such that $k_{max} \leq k \leq k_{min}$ is expressed in terms of dimensionless factors $\phi_{\mathbf{k}}$ whose explicit expression requires a microscopic treatment. However, to extract qualitative conclusions on the effect of the actual anisotropic interaction, one may approximate $\phi_{\mathbf{k}}$ by modelling it in line with common symmetry requirements. Namely, we assume that as $\mathbf{k}$ changes the function $\phi_{\mathbf{k}}$ varies in accordance with the symmetry of a 2D square Brillouin zone. It can thus be fitted as

$$\phi_{\mathbf{k}}^{\alpha} = \frac{1}{1-\alpha/2}(1-\alpha\sin^2 2\varphi) \tag{26}$$

with the prefactor chosen so as to normalize to unity the mean value of $\phi_{\mathbf{k}}^{\alpha}$ over the azimuthal angle φ determining the direction in k-space of the 2D vector $\mathbf{k}$. Here $0 \leq \alpha \leq 1$ with $\alpha = 0$ for isotropic superconductors. Varying α from 0 to 1 spans all ranges from weak to very strong anisotropy. Note that $\phi_{\mathbf{k}}$ (26) modulates the angular dependence of the anisotropic BF interaction strength $f_{\mathbf{k}} \equiv f\phi_{\mathbf{k}}$ which is assumed to be distributed within the interval $f - \Delta f \leq f_{\mathbf{k}} \leq f + \Delta f$ of width $2\Delta f$ around its average value f in eq. (1).

In Fig. 2 the anisotropy factor $\phi_{\mathbf{k}}$ with $\alpha = 0.25$ in eq. (26) and the dimensionless uniform screened Coulomb potential [second factor $(\Omega_0/2\hbar\omega_D)(U_0/V)$ in parenthesis in eq. (20)] are sketched as a function of angle φ. Wavevectors $\mathbf{k}$ satisfying the condition $\phi_{\mathbf{k}} \leq (\Omega_0/2\hbar\omega_D)(U_0/V)$ are distributed within the shaded areas in the figure. On the 2D Fermi surface (not shown in Fig. 2) the arrowheads of wavevectors $\mathbf{k}$ trace out so-called "Fermi arcs".[31–34] The energy-momentum relation of electronic excitations with $\mathbf{k}$ varying along these Fermi arcs behaves like the dispersion of a normal metal $\xi_{\mathbf{k}}$ in eq. (1). As shown in Fig. 2, at temperatures below T^* and while the Coulomb factor $[\Omega_0(\lambda, T)/2\hbar\omega_D](U_0/V)$ is in the range between $\phi_{\min}$ and $\phi_{\max}$, instead of the full Fermi surface there exist only disconnected Fermi arcs in a BF mixture with anisotropic (2) and uniform (3) Coulomb interactions.

Fermi arcs along which the normal distribution of fermions occurs are separated by regions (BF arcs in 2D or BF "islands" in 3D) where the opposite condition of $\phi_{\mathbf{k}} > (\Omega_0/2\hbar\omega_D)(U_0/V)$ or

$$\frac{\hbar\omega_D}{\Omega_0}\phi_{\mathbf{k}} > \frac{U_0}{2V} \tag{27}$$

holds. On such islands the energy-momentum relation of single fermions is not the dispersion $\xi_{\mathbf{k}}$ of a normal metal but is gapped with the generalized gap eq. (20) in a Bogoliubov-type dispersion picture. That is, by satisfying eq. (27) which contains the large ratio $U_0/2V$ in its rhs, the system of attractively-interacting fermions separates into unpaired fermions coexisting with bosonic pairs *of* fermions. We stress that the condition (27) becomes possible owing to the effects of boson energy renormalization. Indeed, the relation (27) is hardly feasible if there were *no* softening of boson energies, i.e., if

$$\lim_{T\to T_c} \Omega_0(T) = 0. \tag{28}$$

The single-electron spectrum gap eq. (20) is largest along the directions of maximum $\phi_{\mathbf{k}}$. But for any fixed λ and T as the vector $\mathbf{k}$ deviates from the axes along which the gap is maximum, $E_{g\mathbf{k}}(\lambda, T)$ decreases and turns to zero at points where the graph of $\phi_{\mathbf{k}}$ and circle of a temperature-dependent radius $R \equiv (\Omega_0/2\hbar\omega_D)(U_0/V)$ in Fig. 2 cross each other. Shaded regions in Fig. 2, where $\phi_{\mathbf{k}} < (U_0/2V)(\Omega_0/\hbar\omega_D)$ alternate with areas where $\phi_{\mathbf{k}} > (U_0/2V)(\Omega_0/\hbar\omega_D)$. Thus, there emerge a) BF regions and b) Fermi arcs on the 2D Fermi surface, namely different segments of directions in a mixture with anisotropic BF interaction (2) for which pair formation/disintegration preferably occurs and is otherwise forbidden if $\phi_{\mathbf{k}} \leq (U_0/2V)(\Omega_0/\hbar\omega_D)$. (For an isotropic BF mixture such a separation occurs if the stricter condition $U_0/2V < \hbar\omega_D/\Omega_0$ holds). The extent of these regions or arcs is not rigid but varies with temperature. Indeed, because of Ω_0 decreasing as it does upon cooling, the efficiency of the Coulomb factor diminishes. This is accompanied by a reduction in the circle radius R, i.e., broadening of the regions in Fig. 2 where $\phi_{\mathbf{k}} > (U_0/2V)(\Omega_0/\hbar\omega_D)$. BF arcs in 2D (or islands in 3D) in a $\mathbf{k}$-space where paired, albeit incoherent, states exist become larger and larger as temperature drops and join together at the points where the screened Coulomb factor (depicted in Fig. 2 as a dotted circle of radius $\phi_{\min}$) passes through the minima of $\phi_{\mathbf{k}}$.

When the temperature-dependent Coulombic factor R becomes equal or less than the minimum of the anisotropy factor $\phi_{\mathbf{k}}$ then the Fermi arcs, i.e., areas swept out by $\mathbf{k}$ with normal distribution of fermions disappear entirely and the attractively-interacting Fermi gas becomes an *anisotropic* mixture of coexisting bosons and fermions converting into each other.

The presence of disconnected Fermi arcs in HTSC films is well-established experimentally.[31–34] Specifically, that at temperatures $T_c \leq T \leq T^*$ the electronic dispersion behaves, depending on the direction of wavevector $\mathbf{k}$ within the Brillouin zone, in a different manner, namely, as if the sample were a normal bad metal in the underdoped (i.e., pseudogapped) phase, or as if superconducting. The idea of the presence of various charge-carrier groups in HTSCs has been extensively explored in the literature (see, e.g., ref. 35). In particular, a phenomenological model with: a) Fermi lines (on a 2D square-like Fermi surface) and b) regions of momentum space where bosons (originating from fermions paired into a d-wave symmetry state) are formed, was proposed in ref. 36. However, as noted ref. 28 in most BF-model studies like refs. 36 and 20 the temperature, coupling and the boson number density dependences of boson energies which are so important for any BF model, were missed.

6. Conclusions

Introducing a uniform Coulomb interaction in an anisotropic boson–fermion gas mixture model reveals, in the pseudogap phase, along with BF regions where the quasiparticles exhibit a Bogoliubov-dispersion behavior, i.e., *Fermi arcs* with *no* pseudogap or segments in momentum space where a *normal* distribution of free fermions prevails. On cooling the extent of these Fermi arcs diminish. On the other hand, such BF regions grow and merge into each other at temperatures when Fermi arcs disappear entirely. This happens because of a specific weakening in the pair-breaking ability of a uniform Coulomb repulsion, which in turn occurs due to the softening of the boson energies as one approaches the BEC.

The present work predicts the presence of a line of nodal points in the pseudogap phase *even* in phonon-mediated mechanisms of HTSCs, i.e., lines in momentum space along which the generalized gap eq. (20) vanishes. This provides an explanation for the origin of Fermi arcs reported in numerous references.[31–34]

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