Computer simulations of the collapse of columns formed by elongated grains

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A numerical investigation of the collapse of granular columns has been conducted. In particular, we address the effect of the grain shape on the properties of the collapse. We show that the final runout and height of the deposits scale as a power law of the initial aspect ratio of the column, a, independently of the elongation of the grains used. We describe this process in terms of an energy balance, and construct an "inertial number" that can be used to describe the flow in terms of a recently proposed granular rheology. We argue that an effective friction that results from this dimensionless quantity explains why the shape of the grains is irrelevant for the final properties of the collapse.

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I. INTRODUCTION

The dynamic behavior of granular materials is fundamental in many natural and industrial phenomena where the flow and accumulation of particles is of central concern. Many anthropogenic and natural processes, like road construction and landslides, involve a granular phase which displays a behavior that differs from that of ordinary fluids. Despite their ubiquity, the understanding of granular materials is still a developing subject that requires the collaboration of many fields of knowledge. Much of what is known of granular matter is the result of recent experimental studies and numerical simulations that complement the empirical knowledge obtained by many years of experience and use of these materials.

A simple experiment that has received attention in recent years is that of the sudden collapse of vertical granular columns, where the flow is driven only by gravity. The original experiments [1,2] released granular materials from a cylindrical container into a flat horizontal surface and studied the flow and properties of the final deposits. In a different setup, the cylindrical column was replaced by a rectangular step [3–5]. In both cases, it was found that the final state of the deposits, characterized by the total distance traveled in the radial (or horizontal) direction and the final height of the deposited mass, scale with the column aspect ratio [1–7]. The aspect ratio, a, is defined as the ratio of the initial height h_0 to the initial radius r_0 (or the initial horizontal extension x_0) of the column: $a = h_0/x_0$.

Numerical studies that take into account the discrete nature of the system have been able to reproduce experimental observations using different numerical schemes [3,6,7]. So far, these simulations have considered only spherical particles or disks to model grains. Real granular materials involve a large collection of particles with different properties, including different shapes and sizes, that can have an effect on the collective behavior of the grains. It is interesting to note that the results of Refs. [2,4] suggest that the scaling properties of the collapsed columns do not depend strongly on the shape of the grains used. In most experiments, a single type of elongated grains were used having similar friction properties as the round particles. The same scaling law was observed. Because these works were not explicitly looking for the effect of particle shape on the final properties of the collapses, this question remained unanswered. Recently, a systematic experimental study has addressed this issue [8].

Whereas the effect of the shape of the grains has not been fully explored in the column collapse problem, it has been addressed in other configurations. Elongated granular materials, like rods, show a phase transition from a disordered to an ordered state on a preferential direction exclusively as a result of the elongation of the grains [9,10]. Rods and elongated grains have a larger surface area on which friction acts modifying the dynamics observed on circular grains. While circular grains can have high packing fraction values, elongated particles tend to leave many voids, decreasing the packing fraction, which will also cause different dynamics when the grains flow [8].

In this paper we present results of the scaling properties of the final deposits of the collapse of 2D granular columns made of grains with different elongations obtained with numerical simulations. Individual circular grains are modeled as soft disks that interact under prescribed elastic and viscous contact forces in the normal and tangential directions, while elongated grains are created by "gluing" two or more circular grains and constraining them to a linear geometry. Elongated grains made of k circular grains are denoted by N_k; for example, N_1 refers to circular grains, N_2 to elongated grains made of two circular grains, and so on.

II. NUMERICAL SCHEME AND PARAMETERS

Numerical simulations of the collapse of granular columns were performed using a discrete element method similar to that used in Ref. [7], as an idealization of the experimental studies of Refs. [1,2].

Details of the numerical algorithm, which integrates Newton's equations of motion for each grain of the system, can be found in Ref. [7]. It is based on a second-order velocity-explicit Verlet method [11,12] in which grains interact only at contact by prescribed elastic and viscous forces in the normal and tangential direction. The friction between grains is modeled using a Coulomb type sliding friction determined by the

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value of the friction coefficient μ. The only external force on the grains is gravity. To create an elongated grain an extra force, determined by evaluating the Lagrange multipliers, is necessary to maintain k circular grains constrained to a linear geometry at all times. This method is implemented and described in full detail in Ref. [12].

Columns are constructed by randomly placing a total number $N_t = 2000$ of grains of type N_k between two walls whose half separation determines the initial length, x_0 of the column. The simulation parameters used here correspond to glass particles of diameter $\bar{d} = 0.3$ mm with a 10% size distribution, except for the friction coefficient between grains, which was chosen equal to $\mu = 0.5$ to directly compare with the results reported in Ref. [7]. Once the grains have settled under gravity and reached an equilibrium state, the maximum height of the particles, h_0, is measured and the value of the corresponding parameter a is obtained for the column. In this way we prepare columns with initial aspect ratio values a in the interval [0.3,15] for elongated grains of types: N_1 (circular grains), N_3, and N_5; only a few cases where run for N_8. At $t = 0$ the walls are removed instantaneously, releasing the grains and initiating the collapse. When the grains come to a stop, the final maximum height and extension of the deposit are measured.

We did not perform a systematic study of the dependence of the collapse on the preparation of the columns. Based on previous results [13], it is expected that the initial packing fraction will have an influence on the collapse process. We performed a few test simulations of columns prepared by initially placing elongated grains parallel or transverse to the horizontal surface (on which the collapse occurs) with the most clear difference being that the column may not collapse (results not shown). Clearly, a more in depth study would be needed to assess this issue for the particular case of elongated grains.

III. RESULTS

A. Final runout and height

For each column the normalized final runout distance x_∞, and final height h_∞, measured at the end of the collapse are shown in Fig. 1 as a function of the column aspect ratio a. The results are consistent with the previously observed monotonic increase (decrease) of $x_\infty(h_\infty)$ as a increases (decreases). The general features of the collapse process are captured by our simulations; a good agreement with previous results [6,7] is observed. Surprisingly, the data on Fig. 1 show similar scaling features for all types of grains, with the circular grains (type N_1, red solid circles) having the largest final runout values and the N_5 type grains (black diamonds) having the lower runout values. The final runout measurements for grains of type N_3 (blue squares) and N_8 (green triangles) lie in between these two and remarkably close to each other. A similar behavior can be observed for the final height of the deposits, shown as the corresponding open symbols in the figure. We can, in part, attribute the small differences in the results for different grains to the uncertainty that arises from considering a "small" number of particles.

The results of Fig. 1 allow us to write a scaling law for the final runout distance with the column aspect ratio of the form

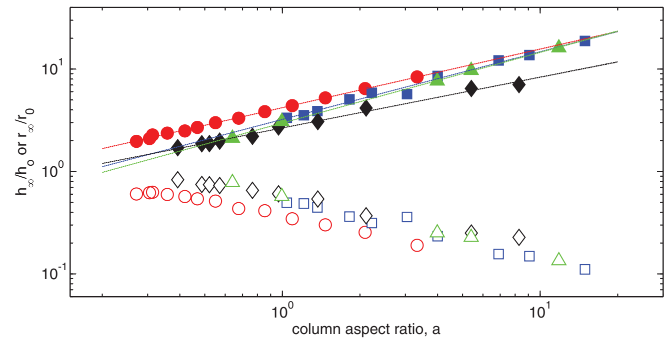


FIG. 1. (Color online) Final runout (filled symbols) and height (empty symbols) as a function of the column's initial aspect ratio a for columns made of long grains of type N_1 (circle), N_3 (square), N_5 (diamond), and N_8 (triangle). The dashed lines correspond to the fit that gives the scaling exponent of Table I.

$$\frac{r_\infty}{r_0} \sim a^{p_k} \tag{1}$$

and a similar expression for the final height h_∞. By performing a fit to the data of Fig. 1, we obtain the values shown in Table I for the exponent p_k. Despite the small differences for each case, these exponents are within the same range as those found in the experiments [1–5] and numerical simulations [6,7] for the collapses of columns for circular grains, reinforcing the hypothesis that the final runout distance of the collapse of granular columns scale similarly and independently of the types of grains used.

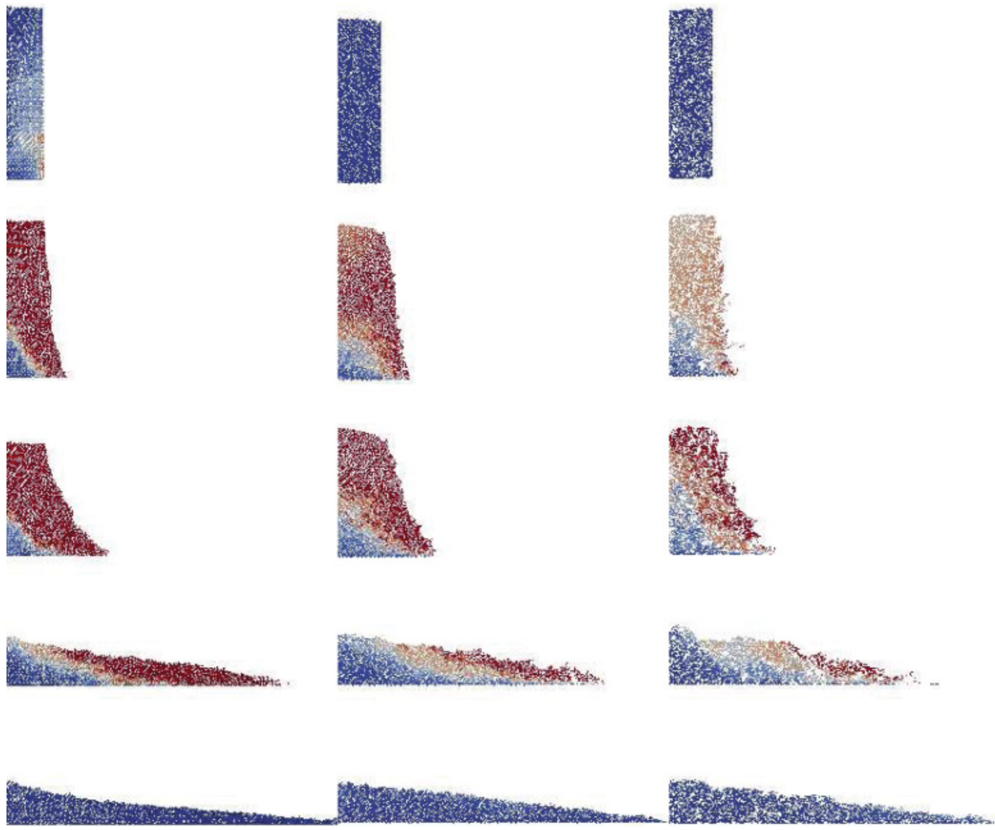
The final height of the columns, h_∞, shown in Fig. 1 as open symbols, indicate that columns made of larger grains collapse into deposits that are slightly taller than columns made of shorter grains. In accordance with Ref. [8] the collapse follows the same scaling law with the column initial aspect ratio.

B. Evolution of collapse

There are known differences in the dynamics of round and elongated particles [9,10]. To us, it is perplexing to observe that such an effect has little influence on the final properties of the deposits that results from column collapses. To further investigate this issue, we now turn our attention to the process that takes place from the initial column state to the final deposit. Figure 2 shows snapshots of the column collapse process at different instants for a particular column aspect ratio ($a \approx 4$). We compare three cases for grains of different lengths (N_1, N_3, and N_8) at dimensionless times $\tau \approx 0.0, 0.3, 0.6, 1.5$, and 3.0 where $\tau = t/\sqrt{2h_0/g}$.

TABLE I. Scaling exponents of the final runout distance as given in expression (1). The values correspond to the slopes of the straight lines shown in Fig. 1.

Grain type	p_k
N_1	0.57 ± 0.02
N_3	0.66 ± 0.07
N_5	0.50 ± 0.05
N_8	0.69 ± 0.03



(a) N_1 (b) N_3 (c) N_8

FIG. 2. (Color online) Snapshots of the column collapse process for a column aspect ratio $a \approx 4$. Each row shows a snapshot taken at the dimensionless time $\tau \approx 0.0, 0.3, 0.6, 1.5$, and 3.0 (from top to bottom) as indicated in Fig. 3. Light gray (blue online) shading to dark gray (red online) shading colors indicate increasing grain speed. Note that only the right side of the collapses are shown as they are very symmetric.

For the three columns shown, at all times the collapse process looks similar. During the collapse, the height and extension of the columns are indistinguishable from the figure. Regions of static and moving grains that change in shape as the collapse occurs can be observed. At $\tau \approx 0.3$ a nearly triangular region of static grains of type N_1 can be observed at the bottom of the column. For longer grains this static region appears less regular in shape and it covers a larger area. We can also notice that at this instant of the collapse there is a larger portion of circular grains that have acquired kinetic energy (color indicates speed), whereas for longer grains (type N_8) fewer grains appear to have a lower energy, as can be seen by the difference in color.

At a later time $\tau \approx 0.6$, a surface of flowing grains develops in all three columns, but the region of static grains has increased in size, most noticeable for longer grains. The height of the static region is about half the maximum height of the column at that instant, whereas for circular grains the static region is much smaller for the same time. Longer grains have less mobility as they compact due to their shape. At this instant, which coincides with the time for which the maximum kinetic energy is reached (see below), about $2/3$ of the circular grains are participating in the flow while for longer grains only half of the grains are moving. Until this time, the shape of the collapsed columns still looks similar for all grain shapes and, except for the aspects we just discussed, no significant differences can be observed.

At $\tau \approx 1.5$ the collapses are near the end of the avalanche phase and moving grains flow over a bed of static grains below. During this period of time grains slow down, dissipating energy mostly by friction. Notice that the region of static grains has increased in extension in all three cases but we can still observe that more circular grains participate in the flow. The deposit of larger grains has more irregularities on the surface than that for circular grains and a smaller portion of grains are moving. When the grains have dissipated all their energy, the final deposits have a different shape due to the irregularity created by the longer grains: The column made of circular grains ends up with a deposited mass that has a well-defined maximum tip, while the long grain deposits show a more rounded surface profile. On closer inspection, the surface for grains of type N_8 reveal a more irregular surface than for the other columns. Nevertheless, the overall final extent of the deposits is similar for all cases, as shown in Fig. 1.

Some of the dynamic differences among the collapses for different grains can be quantified if we consider an energy balance. Following Ref. [7], we plot the dimensionless mean potential and kinetic energy of the grains (normalized by the

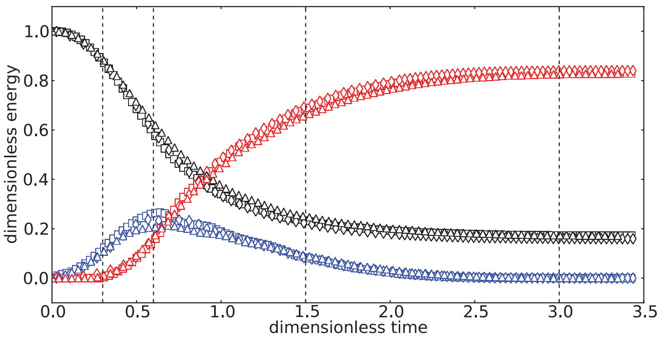


FIG. 3. (Color online) Dimensionless energies as a function of time for columns with initial aspect ratio of $a \approx 4$. Symbols are the same as in Fig. 1 for grains of type N_3 (square), N_5 (diamond), and N_8 (triangle). The vertical lines indicate the times at which the snapshots of Fig. 2 where taken. Black curves show E_p^*, blue E_k^*, and red E_{dis}^*.

initial total energy of the column), E_p^* and E_k^*, together with the cumulative dissipated energy during the collapse,

$$E_{\mathrm{dis}}^* = 1 - E_p^* - E_k^*.$$

Figure 3 shows the energy balance for the columns shown in Fig. 2. The vertical lines on this plot indicate the instants of time at which the snapshots of Fig. 2 where taken. The kinetic energy of the grains increases monotonically to a maximum value that depends on the type of grains used; the shorter grains gain more kinetic energy. One can expect longer grains to accelerate at a slower rate since their surface area is greater; friction forces between these grains act through a longer distance. After this acceleration phase, grains decelerate and E_k^* decreases monotonically, again, at a slower rate for longer grains. Finally, by the end of the collapse, the kinetic energy E_k^* of all columns reaches a zero asymptotic value when they come to rest after dissipating their energy; columns made of shorter grains dissipate energy at a faster rate than columns made of longer grains, as can be seen from the red curves for the cumulative dissipated energy E_{dis}^*. The potential energy E_p^* of the grains, which can be related to the maximum height of the column, also behaves similarly for all columns: It decreases monotonically as time progresses until it reaches a finite value. For the columns made with longer grains, the rate at which the grains convert potential energy to kinetic energy is slower than that for circular grains. The intermediate case of N_3 type grains falls in between. Surprisingly, all three columns reach nearly the same final value of E_p^*, supporting the observation that the final properties of the collapse scale similarly with the parameter a and do not depend on the type of grain used [2,4,8].

IV. DISCUSSION AND CONCLUSIONS

Numerical simulations of the collapse of 2D columns capture the main features of laboratory experiments very well. For columns made with grains of different elongations, we have found that the final distances that characterize the collapse scale with the initial aspect ratio, a, similarly to columns made of circular grains. For all types of grains used, a monotonical increase (decrease) of the final runout (height) with a is observed. The scaling exponent seems to depend weakly on the elongation of the grains. Energy calculations like those shown in Fig. 3 indicate that the final properties of the deposits can be understood in terms of the energy conversion of the system as the collapse occurs: Columns with similar values of initial aspect ratio a transform the initial potential energy in such a way that the final properties of the deposit are similar, irrespective of the type of grains used.

Recently, significant advances have been reached in the rheological description of granular flow [14]. A single dimensionless number, the inertial number I, has been used to describe flow properties, in particular for the transient flow in the column collapse problem [15,16]. Since this approach has proven quite successful we have used some of these ideas to explain our results. We construct a quantity equivalent to the inertial number, which is the ratio of two characteristic times of the flow during the columns collapse.

The use of the the available energy for spreading does not depend on the type of grain used but does depend on the value of a, as with circular grain columns [6]. In the energy plots of Fig. 3 we notice that the conversion of the available energy from potential to kinetic is very similar for all grains. When the deposit experiences horizontal deformation (not during the initial phase of the collapse) the rate of deformation can be approximated by

$$\dot{\gamma} \approx \frac{x(t) - x_0}{t} \frac{1}{h_0 - h(t)}.$$

On the other hand, we can construct a characteristic time from the pressure of the whole column at the base $P = \frac{Mg}{x - x_0}$, where M is the total mass of the column. Considering an effective diameter $d_{\mathrm{eff}} = \bar{d}\sqrt{k}$ (k being the number of circular grains of mean diameter $\bar{d}$ that make an elongated grain of type N_k) we can construct a pressure time scale as

$$T_P = d_{\mathrm{eff}} \frac{(x - x_0)^{1/2}}{\sqrt{c_0 A g}},$$

where $c_0 A$ is the initial area occupied by the grains in the column. Comparing the pressure time scale with the effective shear rate, the inertial number can be written as

$$I = \frac{d_{\mathrm{eff}}(x - x_0)^{3/2}}{h - h_0} \frac{1}{t\sqrt{c_0 A g}}. \quad (2)$$

In agreement with what has been discussed in Ref. [14], large values of I indicate that the pressure of the material

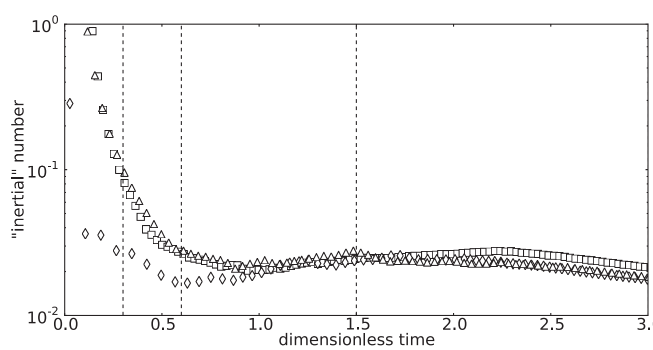


FIG. 4. Time evolution of the inertial number I from Eq. (2) during collapse for the columns of Fig. 2. Symbols and vertical lines are the same as in Fig. 3.

above dominates over horizontal deformation of the deposit, as expected for short times. As times advances, I decreases as the shear deformation becomes important

Figure 4 shows the time evolution of I for the data shown in Fig. 3. We see that I quickly decreases for $\tau < 1$. Then I reaches a value close to 0.02 for all cases, regardless of the grain type. Following Ref. [14] we can argue that the effective friction is approximately the same for all grain types considered: The shape of the grains does not modify the effective friction in a significant manner. Therefore, it is not surprising that the runout is similar for all grain types.

In summary, our numerical results are in close agreement with previously reported results for the column collapse problem [2,4,8]. We have shown that some differences can be observed during the collapse for different grain length; however, such differences do not affect the global friction coefficient in a significant manner. To us it remains a challenge to understand why the grain shape does not influence the global behavior of the collapse.

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