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Abstract – The effects of a Planck-scale deformation of the Minkowski energy-momentum dispersion relation on the phenomenology of non-trapped Bose-Einstein condensates (BECs) are examined. Such a deformation is shown to cause a shift in the condensation temperature T_c of the BEC and, for a specific functional form of deformation, this shift can be as large as the current measured precision on T_c. For a $^{85}_{37}$Rb cold-atom BEC with a particle density $n \simeq 10^{12}$cm^{-3} we find a fractional shift of order 10^{-4}, but this can be much larger for even more dilute BECs. We discuss the possibility of planning specific experiments with BECs that might provide phenomenological constraints on Planck-scale physics. These corrections to T_c are found to be extremely small for ultrarelativistic BECs implying that, in some cases, Planck-scale effects may be more important in low- rather than high-energy processes.

Lively interest has recently emerged to experimentally probe quantum-gravitational Planck-scale effects [1] due to deformations of the standard Minkowski free-particle energy-momentum dispersion relation of special relativity. Such deformations are a general feature of quantum-gravity theories, as, *e.g.*, loop quantum gravity [2–8] or noncommutative geometries [9–12]. Further, it is also of great relevance in the general context of Lorentz-symmetry breaking [13]. Quite generally, a deformed dispersion relation can be written as

$$E(p) \equiv E_0(p) + \delta E(p, m, M_P), \qquad (1)$$

where $E_0(p) \equiv \sqrt{p^2c^2 + m^2c^4}$ is the familiar Minkowski dispersion relation, with p the particle momentum, m its rest mass and c the speed of light. Here $\delta E(p, m, M_P)$ represents the deviation from $E_0(p)$ which in addition is a function of the Planck mass M_P while its explicit form depends on the details of the quantum-gravity model used. The relation (1) is assumed to be universal, *i.e.*, it is the same for all the elementary particles, including composite particles such as nucleons or atoms when internal degrees of freedom are negligible.

The dependence on the Planck mass M_P is introduced since one expects that departures from Lorentz symmetry become important at Planck scales, but one should require that this symmetry be restored when $p \ll cM_P$. Moreover, one would like to preserve the interpretation of m as the particle rest mass. In general, therefore, one should impose the two conditions

$$\delta E(p=0, m, M_P) = 0,$$
$$\delta E(p, m, M_P) \xrightarrow[M_P \longrightarrow \infty]{} 0. \qquad (2)$$

Initially, attempts to constrain the functional form of $\delta E(p, m, M_P)$ were in an astrophysical context where particles are in the ultrarelativistic (UR) regime, $p \gg mc$ [14]. In this limit the dispersion relation deformation can be parameterized quite generally, regardless of the explicit model under study. Due to the extremely large value of $M_P \simeq 10^{19}$ GeV$/c^2$ a series expansion of $\delta E(p, m, M_P)$ in inverse powers of M_P might prove useful along with the leading term in $1/M_P$ as first

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approximation. Accordingly, $\delta E(p, m, M_P)$ would be written [15,16] as

$$\delta E(p, m, M_p) \simeq \frac{1}{2M_p} \left(\eta_1 p^2 + \eta_2 mcp + \eta_3 m^2 c^2 \right) \qquad (3)$$

with the three real parameters: η_1 associated with the leading term, η_2 to the next leading term, and η_3 to the next-to-next leading term. As pointed out in many studies [14,17–20], astrophysical data could be sensitive to a leading-order deformation with $|\eta_1| \lesssim 1$. A preliminary analysis of the Fermi Space Telescope data [21–25] is currently underway to constrain η_1. Very recently [15,16], constraining the functional form of (1) in the nonrelativistic (NR) regime $p \ll mc$ based on ultra-precise cold-atom recoil frequency experiments was proposed. In the NR limit, a more appropriate parametrization of $\delta E(p, m, M_P)$ is [15,16]

$$\delta E(p, m, M_P) \simeq \frac{1}{2M_P} \left(\xi_1 mcp + \xi_2 p^2 + \xi_3 \frac{p^3}{mc} \right) \qquad (4)$$

with parameter bounds obtained as $-6.0 < \xi_1 < 2.4$ for the leading-order deformation parameter and $-3.8 \times 10^9 < \xi_2 < 1.5 \times 10^9$ for the next-to-leading order, both within a 95% confidence level [15,16].

The low- and high-energy bounds just mentioned are perfectly complementary. To illustrate this fact, consider the following deformation: $\delta E(p, m, M_P) = -\eta \left[m^3 c^4 / \sqrt{m^2 c^4 + p^2 c^2} - m^2 c^2 \right] / M_P$ [15]. In the low-energy NR limit this dispersion relation corresponds to (4) with $\xi_2 = -\eta$, $\xi_1 = \xi_3 = 0$, namely the next-to-leading order. In the high-energy UR limit, on the other hand, this corresponds to (3) with $\eta_3 = 2\eta$, $\eta_1 = \eta_2 = 0$ or to the next-to-next leading order. This example also exhibits how the the deformation δE can be more important in the NR than in the UR limit.

It might be objected that (4) can be ruled out for macroscopic objects when $\xi_1 \sim 1$. One has $p^2/2m \lesssim \delta E$ for $p \lesssim p_0 \equiv \xi_1 m^2 c / M_P$ so that the deformation δE dominates over the Minkowski kinetic term. Standard-model particles with $m \lesssim 10^{-16} M_P$ makes δE dominate in the extreme NR limit $p \lesssim p_0 \sim 10^{-16} \xi_1 mc$. However, for macroscopic objects one can easily have $m \sim M_P$ and so that $p^2/2m \lesssim \delta E$ for $p \lesssim p_0 \sim \xi_1 mc$, *i.e.*, the deformation δE dominates in the entire NR regime. But this would contradict the familiar dynamics of classical NR bodies. Note, however, that (4) is merely the small-p asymptotic expansion of the full deformation $\delta E(p, m, M_P)$, and is thus valid for all momenta up to some p_λ, where p_λ depends on the explicit functional form of δE. For example a deformation

$$\delta E(p, m, M_P, p_\lambda) = \xi_1 \frac{mcp}{2M_P} \exp(-p/p_\lambda) \qquad (5)$$

behaves as $\delta E = \xi_1 mcp/2M_P$ for $p \lesssim p_\lambda$ and $\delta E \simeq 0$ for $p \gg p_\lambda$. Therefore, to apply (5) to macroscopic bodies one should measure the momenta of extended objects with $p \lesssim p_\lambda$ and this is impossible for sufficiently small p_λ below the lowest measurable momentum for extended bodies. Since one supposes that this is always the case for p_λ, the relation (4) cannot be ruled out for classical macroscopic bodies. We also emphasize how (4) is commonly accepted in the literature [15,16].

This letter addresses the effect of a deformed dispersion relation on the critical temperature T_c of a spatially uniform non-trapped Bose-Einstein condensate (BEC). This effect can be compared with the current precision in T_c measurements that allows constraining the leading order parameter of the NR deformation (4) up to $|\xi_1| \lesssim 10^2$.

After languishing for seven decades as a mere academic exercise in textbooks, BEC was finally observed in the laboratory in laser-cooled, magnetically trapped ultracold bosonic clouds of $^{87}_{37}$Rb atoms [26], $^{7}_{3}$Li [27], $^{23}_{11}$Na [28], $^{1}_{1}$H [29], $^{85}_{37}$Rb [30], $^{4}_{2}$He [31], $^{41}_{19}$K [32], $^{133}_{55}$Cs [33], $^{174}_{70}$Yb [34] and $^{52}_{24}$Cr [35]. The relativistic BEC *including* antibosons as well as bosons has been reported [36], but for simplicity we neglect antibosons even in the UR limit in what follows. A generalization including antibosons is straightforward. Nowadays there exist very accurate measurements of BEC critical temperatures T_c so that the aforementioned constraints can be addressed. Specifically, in ref. [37] very accurate measurements of T_c shifts due to strong interboson interactions are reported for $^{39}_{19}$K. Such high-precision measurements might be useful in constraining Planck-scale dispersion relation deformations introduced in (1) in feasible experiments.

Neglecting interboson interactions in the BEC we find that the leading-order deformation in (4) produces a shift $\Delta T_c/T_c^0 \propto \xi_1 m^2 c / \hbar M_P n^{1/3}$ where $\Delta T_c \equiv T_c - T_c^0$ with T_c the critical temperature of the gas with the deformed dispersion relation, T_c^0 that same temperature in the undeformed Minkowski case, n the boson number density and m the boson mass. This shift can be unexpectedly high. For example, we obtain $\Delta T_c/T_c^0 \sim 10^{-4} \xi_1$ for $^{85}_{37}$Rb with a particle number density $n \simeq 10^{12} \text{ cm}^{-3}$. Since in high-precision measurements of $\Delta T_c/T_c^0$ in $^{39}_{19}$K due to interboson interactions is $\Delta T_c/T_c^0 \sim 5 \times 10^{-2}$ within a 1% error [37], the deformation parameter can be constrained up to $|\xi_1| \lesssim 10^2$. Moreover, since $\Delta T_c/T_c^0 \propto n^{-1/3}$ one could enlarge the temperature shift merely by reducing n (without however making it so small that it invalidates the thermodynamic limit), and therefore the bound on the deformation parameter ξ_1 can be improved using dilute (small n) BECs.

Based on such an unexpected relevant result, it behooves one to propose experiments with dilute BECs with the aim of measuring a Planck-scale induced shift of T_c. Of course, one should eventually generalize our result concerning $\Delta T_c/T_c^0$ to trapped BECs but this is not trivial. The main task there would be to calculate the energy levels of the trapped bosons which in the case of free bosons corresponds to the deformed dispersion relation (4), with the result expected to depend on the

specific quantum-gravity framework used. Additionally, in planning a feasible experiment one should include effects currently measurable experimentally [37] due to interboson interactions and compare them Planck-scale contributions to $\Delta T_c/T_c^0$. These caveats are currently under scrutiny [38] (see also refs. [39,40]).

We also analyze the effect of the next-to-leading-term deformation in (4) and show that one obtains $\Delta T_c/T_c^0 = \xi_2 m/M_P$ which, being an extremely small and constant shift, cannot constrain the deformation parameter ξ_2 with T_c measurements alone. Therefore, the next-to-leading-order deformation in (4) cannot be excluded nor even bounded. Lastly, we derive the effect of a Planck-scale dispersion relation deformation in the UR limit (3) and find the remarkable result that this effect is always negligibly small. All this suggests that Planck-scale corrections to T_c can be important for NR BECs rather than UR ones.

The general procedure to calculate $\Delta T_c/T_c^0$ is first sketched. We write the general form

$$\delta E(p) = x f(p) \tag{6}$$

for the dispersion relation deformation $\delta E(p)$ where $x \ll 1$ is a dimensionless deformation parameter. For the dispersion-relation deformation defined in (6) the condensation temperature $T_c(x)$ is a function of the deformation parameter x and is obtained by extracting $T_c(x)$ from its defining implicit relation

$$2\pi^2\hbar^3 n = \int_{p_0}^{\infty} \left[\exp\left[\frac{E_0(p) + x f(p) - mc^2}{k_B T_c(x)} \right] - 1 \right]^{-1} p^2 \mathrm{d}p, \tag{7}$$

where $p_0 \equiv \pi\hbar/L$ for free particles in a box of volume L^3. Clearly, for $x = 0$ one recovers the usual Minkowski value $T_c(0) = T_c^0$ that in the NR limit $E_0(p) \simeq mc^2 + p^2/2m$ gives the familiar BEC formula

$$T_c^0 = T_c^{NR} \equiv \frac{2\pi}{\zeta(3/2)^{2/3}} \frac{\hbar^2 n^{2/3}}{k_B m}. \tag{8}$$

Since the lhs of (7) is independent of x one has $\partial_x n = 0$. After some algebra one readily obtains

$$\partial_x T_c(x)/T_c(x) = \int_{p_0}^{\infty} f(p)g(p,x)p^2\mathrm{d}p / \int_{p_0}^{\infty} p^2\mathrm{d}p$$
$$\times \left[E_0(p) + x f(p) - mc^2 \right] g(p,x), \tag{9}$$

where we have defined

$$g(p,x) \equiv \left(\exp\left[\frac{E_0(p) + x f(p) - mc^2}{k_B T_c(x)} \right] - 1 \right)^{-2}$$
$$\times \exp\left[\frac{E_0(p) + x f(p) - mc^2}{k_B T_c(x)} \right]. \tag{10}$$

This expression is quite useful to calculate the shift in T_c due to the dispersion relation deformation. In fact, since $x \ll 1$, one can take

$$\frac{\Delta T_c}{T_c^0} = \frac{T_c(x) - T_c(0)}{T_c(0)} \simeq x \left(\frac{\partial_x T_c(x)}{T_c(x)} \right)_{|x=0} \tag{11}$$

and the last term can be evaluated by use of (9). Let us consider the leading term of the NR deformation given in (4), i.e., $\delta E = \xi_1 mcp/2M_P$. This corresponds to $x = \xi_1 m/2M_P$ and $f(p) = cp$. In the NR limit one can write $E_0(p) \simeq mc^2 + p^2/2m$ and $T_c(0) = T_c^{NR}$, so that (11) becomes

$$\frac{\Delta T_c}{T_c^0} \simeq 0.1 \left(\frac{m^2 c}{\hbar M_P\, n^{1/3}} \right) \xi_1 \ln(N), \tag{12}$$

where $N \equiv nL^3$ is the total number of particles. This shift can be evaluated for a $^{85}_{37}$Rb BEC with number density $n \simeq 10^{12}\,\mathrm{cm}^{-3}$ [41] and boson mass $m \simeq 150 \times 10^{-27}\,\mathrm{kg}$ and $N = 10^9$. Hence $\Delta T_c/T_c^0 \simeq 8.6 \times 10^{-5}\xi_1$. Since one expects $\xi_1 \sim 1$ in quantum-gravity theories, this can be extremely large when compared to the strength of the deformation in (4) which is of order $\delta E/E \simeq \xi_1 p/2cM_P \ll \xi_1\, m/M_p \sim 10^{-17}$. Evidently, the temperature shift (12) being $\propto n^{-1/3}$ can be enlarged for BECs with sufficiently small n, but not so small to render the thermodynamic limit inapplicable. This fact is vital since one can then seek low-density BECs with a correspondingly large $\Delta T_c/T_c^0$ in order to constrain and perhaps even measure the deformation parameter ξ_1. Moreover, from (12) another way of enlarging $\Delta T_c/T_c^0$ is to consider more massive bosons.

As mentioned initially, if one is to deal with actual laboratory measurements of $\Delta T_c/T_c^0$ one should generalize (12) to the case of a trapped gas. However, such a generalization is a delicate matter well beyond the intent of this paper and that will be presented elsewhere [38]. Here we merely stress that, for non-trapped BECs the shift $\Delta T_c/T_c^0$ due to Planck-scale effects can be as large as $\sim 10^{-4}\xi_1$ and even larger for dilute BECs. Since we expect a comparable effect for trapped BECs, it makes sense to compare such a shift with empirical values of $\Delta T_c/T_c^0$.

In high-precision measurements of $\Delta T_c/T_c^0$ in $^{39}_{19}$K [37] due to interboson interactions, the order of magnitude is $\Delta T_c/T_c^0 \simeq 5 \times 10^{-2}$ with at most a 1% error so that such measurements may be sensitive to Planck-scale effects. As seen above, the Planck-scale-induced shift in the condensation temperature is $\Delta T_c/T_c^0 \simeq 10^{-4}\,\xi_1$ for a $^{85}_{37}$Rb BEC, which allows to constrain the deformation parameter up to $|\xi_1| \lesssim 10^2$. Moreover, since the temperature shift can be enhanced for even more dilute and/or more massive BECs, even better bounds on ξ_1 are obtainable.

Note also that (12) may theoretically suggest the exclusion of the leading term in (4) as it would cause an unbounded shift for very small n. Even though the temperature shift becomes arbitrarily large for small n the critical temperature vanishes as $n^{1/3}$ when $n \to 0$, since one has

$$T_c \simeq \frac{2\pi}{\zeta(3/2)^{2/3}} \frac{\hbar^2 n^{2/3}}{k_B m} \left[1 + 30 \left(\frac{m^2 c}{\hbar M_P\, n^{1/3}} \right) \xi_1 \right]. \tag{13}$$

Thus, a nonzero deformation parameter ξ_1 is not inconsistent. Remarkably therefore, a measured nonzero shift of T_c in a low-density BEC, but unrelated to interboson interactions, would imply a leading-order dispersion relation deformation.

Let us briefly examine the next-to-leading-order term in (4), corresponding to $\xi_2 \neq 0$ and $\xi_1 = \xi_3 = 0$ with the bound $|\xi_2| \lesssim 10^9$ [15,16]. This implies the energy dispersion relation

$$E \simeq mc^2 + p^2/2m + \xi_2 p^2/2M_p \equiv mc^2 + p^2/2m^{\xi_2}, \quad (14)$$

where $m^{\xi_2} \equiv mM_p/(\xi_2 m + M_p)$. This gives $T_c = T_c^0[1 + \xi_2(m/M_P)]$ so that the temperature shift becomes

$$\frac{\Delta T_c}{T_c^0} = \xi_2 \frac{m}{M_p}. \quad (15)$$

Since $m/M_P \ll 1$ such a shift is extremely small and therefore the next-to-leading term in (4) cannot be excluded nor bounded even for very large values of ξ_2.

So far our focus has been limited to the NR limit of the deformed dispersion relation (4). We now show that Planck-scale corrections to T_c are negligibly small as a result of the leading term in the UR limit (3). In this case $\delta E(p) = \eta_1 p^2/2M_P$ and the temperature shift can be calculated via (9)–(11) with $x = \eta_1 m/2M_P$ and $f(p) = p^2/m$. Moreover, one has $E_0(p) \simeq cp$ and $T_c(0) = T_c^{UR}$, where T_c^{UR} is the UR condensation temperature,

$$T_c^{UR} \equiv \hbar c \pi^{2/3} n^{1/3}/k_B \zeta(3)^{1/3}. \quad (16)$$

The resulting temperature shift is thus

$$\frac{\Delta T_c}{T_c^0} \simeq 1.8 \eta_1 \left(\frac{k_B T_c^{UR}}{c^2 M_P} \right) \quad (17)$$

and such a shift can be appreciable only for extremely high densities n such that $k_B T_c^{UR} \sim c^2 M_P \sim 10^{19}$ GeV. This result is significant as being counterintuitive since one expects that Planck-scale effects be appreciable in UR rather than NR phenomena. In fact, the situation is quite the opposite: Planck-scale corrections to BEC critical T_c's are appreciable in NR BECs (12) but are extremely small for UR BECs (17).

Lastly, we briefly discuss the cosmological consequences of our analysis. It was recently proposed [42–64] that dark matter in the universe might consist of a BEC phase due to some boson. Since the condensation temperature determines the epoch of formation of such a condensate, a shift in T_c may delay or anticipate the condensation of dark-matter particles, thus affecting the phenomenology of the model. This motivates exploring whether Planck-scale deformation can in fact affect BECs. At any rate, one is far from a direct observation of such a cosmological condensate so that at present its existence is speculative, even though one can search for its indirect cosmological traces. Indeed, this suggests an interesting avenue of research.

To conclude, we have determined the effects of Planck-scale deformation of the dispersion relation on the condensation temperature of a non-trapped BEC. In particular, assuming a nonrelativistic leading-order dispersion relation defined in (4) one finds that such an effect may be comparable with the precision measurement of T_c [37] and that one can bound the leading-order deformation parameter up to $|\xi_1| \lesssim 10^2$. We thus argue that one should generalize this result to the case of trapped BECs [38] in order to propose feasible experiments sensitive to Planck-scale physics. We have also shown that the next-to-leading correction in (4) causes an extremely small shift that is unobservable with any experiment so such a deformation can neither be excluded nor fixed outside presently existing bounds. Finally, it is noteworthy that Planck-scale corrections to T_c are extremely small for UR condensates. This leads one to the conclusion that Planck-scale physics may be relevant for cold-atom NR BECs rather than UR BECs.

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