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Dirichlet boundary condition for the Ginzburg-Landau equations

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Abstract. As is well known, the Ginzburg Landau phenomenological theory described with a good accuracy the thermodynamic properties of a superconducting material. The system of two coupled nonlinear differential equations is completed with the usual Neumann boundary condition as long as is considered a superconductor insulator interface. In this paper, we solve the Ginzburg Landau equations for a circular geometry containing a half-circular pillar defect and considering the unusual superconducting Dirichlet boundary condition. This choice, leading to take the extrapolation de Gennes length equal to zero. Our results point that, the thermodynamic critical fields, magnetization, free energy and vorticity, depend on the chosen boundary condition.

1. Introduction

Physics properties in a conventional superconducting material has generate much activity in the wide scientific community. Several topologies as samples with surface roughness or surface defects have attracted attention as potential new components for low-temperature electronics physics [1, 2, 3, 4, 5]. Also, many phenomena of current interest are directly related to the non-monotonic interaction between vortices, being a possible explanation for highly debated unusual vortex configurations observed in MgB_2 and $Ba(Fe_{0.95}Co_{0.05})_2As_2$ [6, 7]. The transparent physical ideas behind the Landau theory, as well as the long history of its successful applications, have led to a general belief that captures the physics of the superconducting matter in a much wider range of physically plausible situations. By solving the first non-linear time dependent Ginzburg-Landau equation considering the Dirichlet boundary condition for the superconducting pseudo-function wave, we calculate the thermodynamic critical fields, magnetization, Gibbs free energy and vorticity for a thin disk containing a half-circular pillar defect in presence of a external magnetic field.

2. Theoretical formalism

Let us consider a thin superconducting disk with a half-circular pillar defect, immersed in an insulating medium in the presence of a perpendicular uniform magnetic field $\mathbf{H_0}$. Superconducting matter is described in the Ginzburg-Landau theory by the order parameter ψ, $|\psi|^2$, describes the spatial variation of the superconducting electrons, and the potential vector $\mathbf{A}$, related to the magnetic induction as $\mathbf{h} = \nabla \times \mathbf{A}$, in absence of external currents and using

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the London gauge ($\nabla \cdot \mathbf{A} = \mathbf{0}$), electrical potential ($\varphi = 0$) (for more details, see [8, 9, 10]) as:

$$\frac{\partial \psi}{\partial t} = -(i\nabla + \mathbf{A})^2\psi + (1-T)\psi(1-|\psi|^2) \quad (1)$$

$$\frac{\partial \mathbf{A}}{\partial t} = (1-T)\mathrm{Re}\left[\bar{\psi}(-i\nabla - \mathbf{A})\psi\right] - \kappa^2\nabla\times\mathbf{h} \quad (2)$$

The dynamical equations are complemented with the appropriate boundary conditions for the order parameter (See [11]):

$$(-i\nabla - \mathbf{A})\psi\cdot\mathbf{n} = \frac{i\hbar}{b}\psi|_n \Rightarrow \begin{cases} b\to\infty \Rightarrow \nabla\psi|_n = 0 & \text{Neumann Boundary Condition} \\ b = 0 \Rightarrow \psi|_n = 0 & \text{Dirichlet Boundary Condition} \end{cases} \quad (3)$$

$\hat{n}$ is the unity vector perpendicular to the surface of the superconductor. b is the deGennes parameter, (see [1]). We will neglect the z-dependence on the order parameter. This is reasonable for thickness of the disk d much smaller than the coherence length. Then $\mathbf{H_0} = \nabla\times\mathbf{A_0}$ inside the disk and the equations 1 can be written as:

$$\frac{\partial \psi}{\partial t} = -(i\nabla + \mathbf{A_0})^2\psi + (1-T)\psi(1-|\psi|^2) \quad (4)$$

$b \to \infty$ leads to the usual Neumann boundary condition for superconducting materials, i.e. $\hat{n}\cdot\nabla\psi = 0$, and $\psi|_n \neq 0$ typical condition for a superconducting/vacuum interface, it will be assumed that normal current density not vanishes at the interface. Now, positives (negatives) values for b simulates a superconductor/metal (superconductor/superconductor) interfaces, for these conditions we have always $\nabla\psi \neq 0$ and $\psi \neq 0$ in the surface of the sample. $b = 0$ simulates a high density of defects in the interface, its lead to take $\psi|_n = 0$ in equation 3, the usual Dirichlet boundary condition in quantum mechanics but unusual for Ginzburg Landau equations for the superconductivity [11, 12].

3. Results and discussion

The parameters used in our numerical simulations for a thin film of pure Al of thickness d (assuming $\xi(0) = 1.2\ \mu m$, $T_c = 1.7K$, $T = 0.85$, the radius of the disk was $R = 8.0$ with a internal radius $r_i = 0.3$, the defect position was $P(r,\theta) = 1.45$ for $\pi \leq \theta \leq 2\pi$ and $r = R/2$ ([13, 14]). Figures 1(c, g) shown the contour plot of the order parameter for several vorticities

Table 1. Thermodynamics values for Dirichlet and Neumann conditions.

B. Condition	$-G(H_{01})$	$-4\pi M(H_{01})$	$N\Phi_0(H_{01})$	H_{01}	H_{02}	H_{03}
Dirichlet	0.815	0.820	2.0	0.650	0.820	1.029
Neumann	0.705	0.712	1.0	1.077	1.05	1.740

(in logarithmical scale (a, e)) and its phase (b, f), also the magnetic induction (d) and (h) the super-current for a disk with a defect for a magnetic field $H_0 = 0.888$ with $N = 6$ vortices (up) and $H_0 = 0.652$, $N = 4$ vortices (down); $N = 2$ vortices remains in the superconductor region in both cases, while four (up), and two (down) sit in the center of the disk. Although they are not visible in the contour plot of the magnitude of order parameter, there is a change in the phase around the hole equal to $\Delta\Theta = 12\pi$ (figure 1(b)), and $\Delta\Theta = 8\pi$ (figure 1(f)). In these figures we can appreciate the half-circular pillar defect in the lower section of the disk, in this place never sit any vortices, because it acts like an anti-pinning defect. We can appreciate in

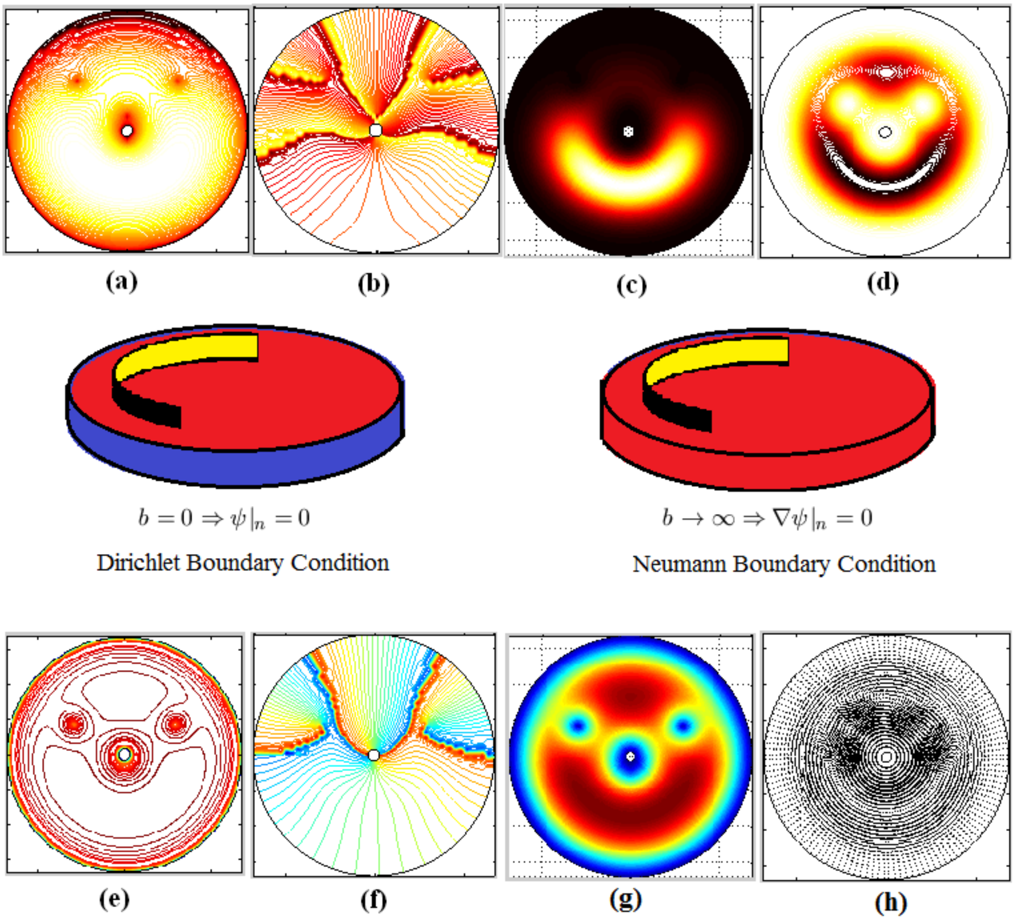

Figure 1. Square modulus of the order parameter $|\psi|^2$ (c, g) (in logarithmical scale (a, e)) and its phase $\Delta\Theta$ (b, f). Magnetic induction $\vec{\mathbf{h}}$ (d), and super-current $\vec{\mathbf{J}}$ (h), for a disk with a defect at $H_0 = 0.888$ with $N = 6$ vortices (up) and $H_0 = 0.652$ $N = 4$ (down), using the Dirichlet condition.

the table the Gibbs free energy G, the magnetization $-4\pi M$, vorticity $N\Phi_0$ (calculated at H_{01}), and the critical fields H_{01}, H_{02}, H_{03}, dependents of the chosen boundary condition. The growth of the magnetization and free energy with Dirichlet condition means that the less the metallic boundary is $(b \to \infty)$, the more diamagnetic the material is. Also, when we take the condition $\psi = 0$ in the surface allow the superconductivity be destroyed and nucleated at lower magnetic fields.

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