Energy 66 (2014) 750–756

Contents lists available at ScienceDirect

Energy

journal homepage: www.elsevier.com/locate/energy

# Heat transfer and entropy generation in the parallel plate flow of a power-law fluid with asymmetric convective cooling

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ARTICLE INFO

*Article history:*
Received 27 June 2013
Received in revised form
9 December 2013
Accepted 18 December 2013
Available online 17 January 2014

*Keywords:*
Heat transfer
Entropy generation
Power-law fluid
Minimum entropy generation

ABSTRACT

The heat transfer and entropy generation in the parallel plate flow of a power-law fluid are analyzed. Asymmetric convective cooling is included in the analysis by considering thermal boundary conditions of the third kind. Using the known velocity profile, the temperature field is analytically derived. Conditions for minimum entropy generation are determined.



## 1. Introduction

Due to their widespread presence in many practical situations, heat transfer problems in fluid systems have attracted the attention of researchers for a long time. While the original focus was on Newtonian fluids (a classical reference is the work by Shah and London [1]), more recently interest in non-Newtonian fluids has increased [2–8]. This has been provoked by the importance for many industries (including polymer processing and the food industry) of thermal conditions in the flow of such systems through pipes, ducts and devices of different shapes. For instance, when dealing with polymeric materials, a key factor for the quality of the final product is a proper temperature control. Since the actual operating conditions are in general very complex, the analysis of relatively simple but tractable problems is usually taken as a useful resource to gain some insight. So it is not surprising that many such developments have appeared in the literature, even quite recently, which include fluid flow and heat transfer in cylindrical conduits or between parallel plates under different thermal boundary conditions [9–18]. Another tool which has become rather popular in recent times (see for instance Refs. [14–19]) for the analysis of these problems is the use of the second law of thermodynamics, in particular in what concerns the generation of entropy within the system. This generation is caused by the various irreversibilities present in the process under investigation and so a detailed knowledge of the parameters determining such irreversibilities may prove crucial for specifying the best operating conditions. In fact, as Bejan has pointed out [20], good engineering heat transfer design in problems where either heat transfer augmentation or thermal insulation are required usually involves the minimization of entropy generation. Interestingly enough the same approach has been recently used in different contexts by several authors, see for instance Ref. [21].

A few years ago, using the Entropy Generation Minimization method [20,22] two of us [23] showed that the entropy generation in the viscous flow between parallel plates with asymmetric convective cooling displayed a minimum for given values of the ambient temperature and the upper and lower plate Biot numbers. The same effect has also been found for other problems [24] that involve the flow of Newtonian fluids. The question then naturally arises as to whether the consideration of a non-Newtonian fluid will affect, and if so to what extent, the features that stem out of previous investigations. In fact, the analysis [25] of the heat transfer problem in the zero-mean oscillatory flow of a Maxwell fluid flowing between parallel plates with convective cooling suggests

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0360-5442/$ – see front matter 
http://dx.doi.org/10.1016/j.energy.2013.12.046

that the effects of viscoelasticity may produce heat transfer enhancement with respect to that of a Newtonian fluid under similar operating conditions. To further address the question posed above, in this paper we have examined the problem of heat transfer and entropy generation in the fully developed parallel plate flow of a power-law fluid with asymmetric convective cooling. The choice of the power-law fluid to carry out this analysis is due to two reasons. On the one hand, it includes the Newtonian fluid as a special case. On the other hand, although it has also been used to analyze heat transfer and entropy generation in different kinds of flow [26–47], the parallel plate flow under asymmetric convective cooling has not been examined to our knowledge so far.

The paper is organized as follows. In the next Section, we provide a brief description of the power-law fluid and of the assumptions under which our problem will be set, including all the governing equations. This is followed in Sec. 3 by the explicit determination of the velocity and temperature fields and hence of the corresponding local and global entropy generation and their analysis for both pseudoplastic and dilatant fluids The paper is closed in Sec. 4 with some further discussion and concluding remarks.

## 2. The model fluid and the governing equations

In this Section we will start by stating the problem under consideration. This involves writing down the equations that determine the velocity and temperature fields in the fully developed parallel plate flow of a power-law fluid with asymmetric convective cooling. A power-law fluid is a type of generalized Newtonian fluid for which the shear stress $\tau$ is given by Ref. [48]

$$\tau \equiv -m\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}, \tag{1}$$

where $y$ is the transversal coordinate, $u$ the axial fluid velocity, $m$ is the flow consistency index, $\partial u/\partial y$ is the shear rate or the velocity gradient perpendicular to the plane of shear and $n$ is the flow behavior index. If $n<1$ the fluid is pseudoplastic (*e.g.* styling gel) while if $n>1$ it is dilatant (rarely encountered, *e.g.* an uncooked paste of cornstarch and water). If $n=1$, then it is the Newtonian fluid. In Fig. 1 we show a schematic diagram of the system we want to examine.

For the sake of deriving analytical results, we will take the following simplifying assumptions. We consider a steady laminar flow of an incompressible power-law fluid that takes place between parallel rigid plates separated by a distance $b=2a$. The flow is driven by a constant pressure gradient, $\partial p/\partial x$, in the axial $x$-

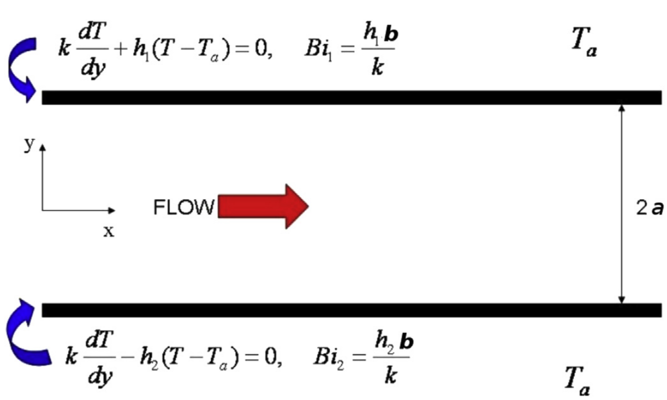


**Fig. 1.** Schematic representation of the problem under analysis.

direction. We assume that the parallel plates are infinite so that border effects are neglected and the velocity and temperature profiles are fully developed. For the solution of the momentum balance equation we assume that the velocity satisfies the no slip condition at the plates. In turn, the heat transfer equation is solved using boundary conditions of the third kind that indicate that the normal temperature gradient at any point in the boundary is assumed to be proportional to the difference between the temperature at the surface and the external ambient temperature $T_a$, which is assumed constant. With these conditions the amount of heat entering or leaving the system depends on the external temperature as well as on the convective heat transfer coefficient. A fundamental assumption in this problem is that the heat transfer coefficient of each plate is different and therefore, we have an asymmetric convective cooling. We also assume that natural convection is absent and that the thermal conductivity of the fluid, $k$, is constant.

Given the previous assumptions, let us now express the balance equations for momentum and energy along with their boundary conditions. In dimensionless terms, upon substitution of the expression for the shear stress as given in Eq. (1), the momentum balance equation turns out to be the following

$$\frac{d}{dy^*}\left(\left|\frac{du^*}{dy^*}\right|^{n-1}\frac{du^*}{dy^*}\right) = 1, \tag{2}$$

where the dimensionless velocity $u^*$ is normalized by the maximum axial fluid velocity $u_0 = ((1/m)(\partial p/\partial x))^{1/n} b^{n+1/n}$, while the dimensionless transversal coordinate is given by $y^*=y/b$. The solution of Eq. (2) must satisfy the no slip boundary conditions

$$u^*(1/2) = u^*(-1/2) = 0. \tag{3}$$

In turn, the energy balance equation results

$$\frac{d^2T^*}{dy^{*2}} + \left|\frac{du^*}{dy^*}\right|^{n-1}\left(\frac{du^*}{dy^*}\right)^2 = 0. \tag{4}$$

where the dimensionless temperature is defined as $T^* = k(T-T_a)b^{n-1}/mu_0^{n+1}$.

According to our assumptions, we now consider boundary conditions of the third kind for the thermal problem. As mentioned earlier, this has to do with the fact that the amount of heat going into or out of the system depends on the external temperature as well as on the (effective) convective heat transfer coefficients. The latter include both the thermal resistance of the plates and the external convective heat transfer coefficients. Therefore, the boundary conditions associated to our heat transfer problem [*c.f.* Eq. (4)] are given by

$$\frac{dT^*}{dy^*} + \mathrm{Bi}_1 T^* = 0, \text{ at } y^* = 1/2 \tag{5}$$

and

$$\frac{dT^*}{dy^*} - \mathrm{Bi}_2 T^* = 0, \text{ at } y^* = -1/2. \tag{6}$$

In Eqs. (5) and (6) the Biot numbers $\mathrm{Bi}_1 = (h_{\mathrm{eff}})_1 b/k$ and $\mathrm{Bi}_2 = (h_{\mathrm{eff}})_2 b/k$ are the dimensionless expressions of the effective convective heat transfer coefficients of the upper and lower plates, $(h_{\mathrm{eff}})_1$ and $(h_{\mathrm{eff}})_2$, respectively, which, due to our former assumptions, turn out to be different and $k$ is the heat conductivity of the fluid. The effective heat transfer coefficients are defined as

$$\left(h_{\rm eff}\right)_j = \frac{1}{\frac{\delta_w}{k_w} + \frac{1}{(h_e)_j}}, \quad j = 1,2 \tag{7}$$

where $\delta_w$ and $k_w$ are the thickness and thermal conductivity of both plates which have further been assumed to be the same. In addition, $(h_e)_1$ and $(h_e)_2$ are the external heat transfer coefficient of the upper and lower walls, respectively.

As a final point in this section and due to our interest in performing a second-law analysis, we have to specify the entropy generation rate. In general, the derivation of the form of the entropy generation of a simple fluid within the framework of irreversible thermodynamics goes as follows (for details see Refs. [49–52]). One starts from the assumption of local equilibrium and the local (in space and time) Gibbs equation for the material time derivative of the local entropy density. In this latter, apart from the local temperature and the local pressure, the material time derivatives of the mass density and the internal energy density also appear. By eliminating these material time derivatives (using the continuity equation and the energy equation) and comparing with the alternative form for the time derivative of the entropy density (written as a balance equation involving the divergence of the entropy flux given by the quotient of the heat flux divided by the local temperature and the local entropy generation), one ends up with the following expression for the local entropy generation rate

$$\sigma = -\mathbf{q}\cdot\frac{\mathrm{grad\ T}}{\mathrm{T}^2} - \tau^T : \frac{\mathrm{grad}\ \mathbf{v}}{\mathrm{T}}, \tag{8}$$

where $\mathbf{q}$ is the heat flux, $\tau$ is the (shear) stress tensor (the superindex $T$ denoting its transpose) and $\mathbf{v}$ the (vectorial) velocity field. The first term on the right-hand side is due to heat conduction and the second one to fluid friction. Note that Eq. (8) holds irrespectively of whether the fluid is Newtonian or non-Newtonian. It contains the sum of the usual flux times force terms of irreversible thermodynamics [50], namely, the fluxes are $\mathbf{q}$ and $\tau$ and the corresponding forces are grad $T/T^2$ and grad $\mathbf{v}/\mathbf{T}$, respectively. The global entropy generation is finally obtained from integration over the spatial and time domains. Of course the above expression for the local entropy generation is only formal and is useless until one specifies the constitutive equations for both $\mathbf{q}$ and $\tau$. Here we will take Fourier's law for the former ($\mathbf{q} = -k$ grad $T$) and the stress tensor for the power-law fluid. Hence, the dimensionless form of the local entropy generation rate reads

$$s^*\left(y^*\right) = \frac{1}{\left(T^*(y^*) + T_a^*\right)^2}\left(\frac{\mathrm{d}T^*}{\mathrm{d}y^*}\right)^2 + \frac{1}{T^*(y^*) + T_a^*}\left|\frac{\mathrm{d}u^*}{\mathrm{d}y^*}\right|^{n-1}\left(\frac{\mathrm{d}u^*}{\mathrm{d}y^*}\right)^2, \tag{9}$$

where $s^* = \sigma b^2/k$, $T_a^* = kb^{n-1}/mu_0^{n+1}T_a$ and the global entropy generation rate is

$$< s^* > = \int_{-1/2}^{1/2} s^* \mathrm{d}y^*. \tag{10}$$

As is clear from the above, if one can determine the velocity and temperature fields by solving Eqs. (2) and (4), together with the boundary conditions Eqs. (3), (5) and (6), substitution of the results in Eqs. (9) and (10) will allow us to use the Entropy Generation Minimization method for the parallel-plate flow of the power-law fluid with asymmetric convective cooling. This will be presented in the next Section.

## 3. Results

The solution to Eq. (2) with the boundary conditions (3) is a dimensionless generalization of the Poiseuille flow velocity profile given by (see Eq. (22) in Ref. [43])

$$u^*\left(y^*\right) = \frac{2^{-\frac{1+n}{n}}n}{1+n}\left(1 - 2^{\frac{1+n}{n}}\left|y^*\right|^{\frac{1+n}{n}}\right), \tag{11}$$

to which it reduces for the Newtonian fluid, that is when $n = 1$ [48]. It is important to remark that care must be taken in reporting results for power-law fluids in this geometry, since some authors (see for instance Ref. [45]) take the velocity profiles without the absolute value which, for some values of $n$ (in particular $n = 1/2$) do not satisfy the 'no slip' boundary conditions. Various dimensionless velocity profiles both for pseudoplastic ($n < 1$) and dilatant ($n > 1$) fluids are displayed in Fig. 2, where the dimensionless Poiseuille flow profile for the Newtonian fluid has also been included for comparison. Note that in all cases the profiles are symmetrical with respect to $y^* = 0$ and vanish at $y^* = \pm 1/2$ as they should.

Using Eq. (11) to compute $\mathrm{d}u^*/\mathrm{d}y^*$ and subsequent substitution of the result in Eq. (4) leads to a second order linear differential equation whose solution has to satisfy the boundary conditions (5) and (6). We have solved such an equation using Mathematica and obtained the following dimensionless temperature field

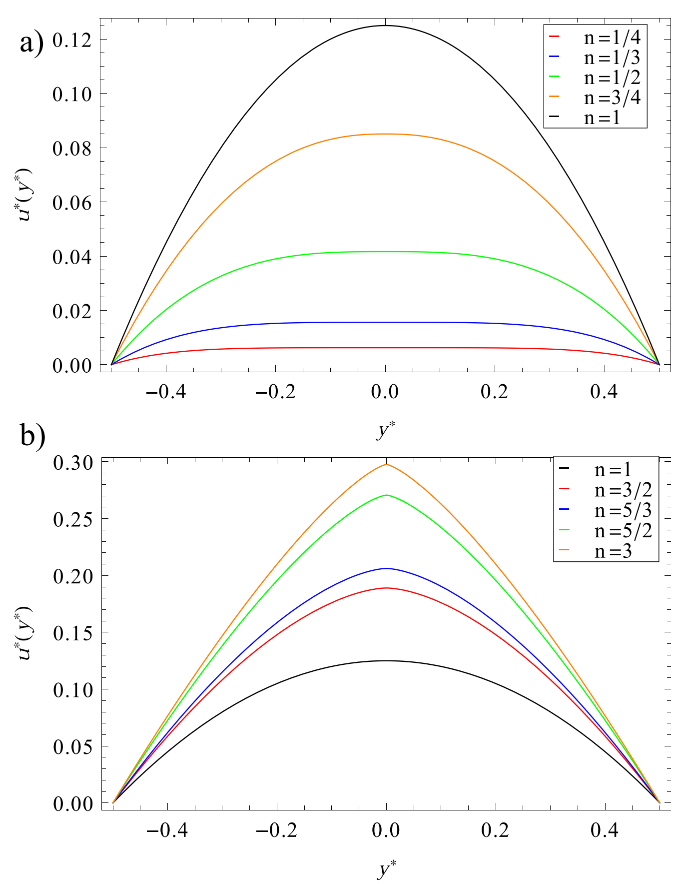


**Fig. 2.** Dimensionless velocity profiles. a) Different pseudoplastic fluids. b) Different dilatant fluids. The Newtonian fluid corresponds to the flow behavior index $n = 1$.

$$T^*\left(y^*\right) = 4^{-\frac{1+n}{2n}} \frac{n(4+\mathrm{Bi}_2+12n+4\mathrm{Bi}_2 n+\mathrm{Bi}_1(1+(4+\mathrm{Bi}_2)n))}{4(\mathrm{Bi}_1+\mathrm{Bi}_2+\mathrm{Bi}_1\mathrm{Bi}_2)(1+2n)(1+3n)} + \frac{\left(-2+4^{-\frac{1+n}{2n}}\mathrm{Bi}_2-\mathrm{Bi}_1\left(2+4^{-\frac{1+n}{2n}}+2\mathrm{Bi}_2\right)\right)ny^*}{2(\mathrm{Bi}_1+\mathrm{Bi}_2+\mathrm{Bi}_1\mathrm{Bi}_2)(1+2n)} + \frac{ny^*\left(1-\frac{ny^{*\frac{2+n}{n}}}{2+2n}\right)}{2+n}. \quad (12)$$

Again, as it should be, we recover the result of Ref. [23] in the Newtonian fluid case, namely $n = 1$. Note that Eq. (12) is valid for any value of $\mathrm{Bi}_1$ and $\mathrm{Bi}_2$ except for the case $\mathrm{Bi}_1 = \mathrm{Bi}_2 = 0$ (insulating plates) in which the mathematical problem is ill-posed. Illustrative examples of the dimensionless temperature field for given values of the Biot numbers $\mathrm{Bi}_1$ and $\mathrm{Bi}_2$ in the case of various pseudoplastic and dilatant fluids, together with a comparison with the case of the Newtonian fluid are presented in Figs. 3 and 4, respectively.

As the above graphs indicate, for fixed (but different) values of $\mathrm{Bi}_1$ and $\mathrm{Bi}_2$, the dimensionless temperature fields are not symmetrical with respect to $y^* = 0$ and their values increase as $n$ increases. Also, for $\mathrm{Bi}_2 > \mathrm{Bi}_1$, and for every power-law fluid, the values of $T^*(1/2)$ are higher than those of $T^*(-1/2)$. Although not shown, the converse is true if $\mathrm{Bi}_2 < \mathrm{Bi}_1$. Only when $\mathrm{Bi}_1 = \mathrm{Bi}_2$ the temperature fields become symmetrical around the origin and then of course $T^*(1/2) = T^*(-1/2)$. On the other hand, fixing $\mathrm{Bi}_1 = 1$ for both pseudoplastic and dilatant fluids one finds that the values of the dimensionless temperature field decrease as $\mathrm{Bi}_2$ (taken to be greater than 1) increases (*c.f.* Fig. 4).

Eqs. (11) and (12) allow us to compute $du^*/dy^*$ and $dT^*/dy^*$, respectively. In turn, use of such results in Eqs. (9) and (10) yields the global (dimensionless) entropy generation rate per unit length in the axial direction $<s^*>$ as a function of the flow behavior index $n$, the external dimensionless ambient temperature $T_a^*$, and the two Biot numbers $\mathrm{Bi}_1$ and $\mathrm{Bi}_2$. The explicit expression is not very illuminating and so will be omitted. Suffice it to state here that it is readily amenable for numerical evaluation. Without loss of generality and since in our previous study for the Newtonian fluid [23] we considered mainly the case where $T_a^* = 7$, we will in what follows also fix this value for the dimensionless external ambient temperature. As can be clearly seen in Figs. 5–8, the qualitative trends already observed for Newtonian fluids (*c.f.* Ref. [23]) are also present for all power-law fluids. To begin with, although not shown we find that when the two Biot numbers are equal the global entropy generation does not display a minimum value with respect to Biot number. On the

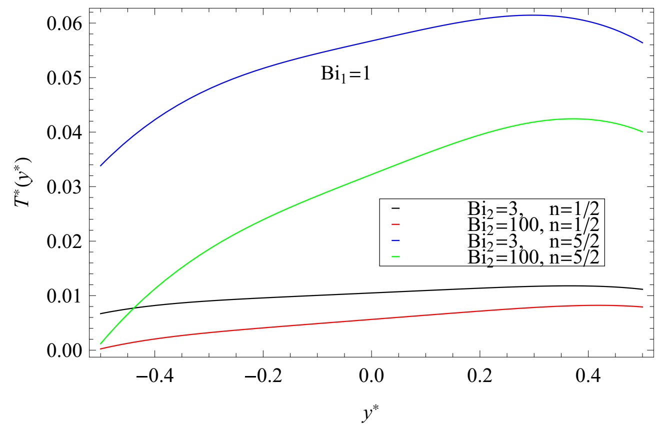


**Fig. 4.** Dimensionless temperature fields for pseudoplastic and dilatant fluids when $\mathrm{Bi}_1 = 1$ and two values of $\mathrm{Bi}_2$.

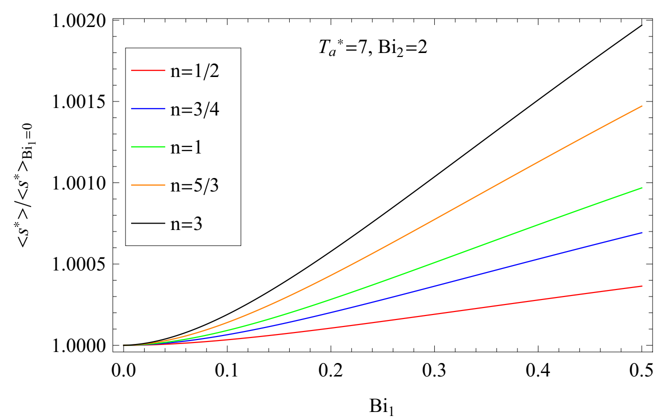


**Fig. 5.** Dimensionless global entropy generation rate (normalized with its value at $\mathrm{Bi}_1 = 0$) for pseudoplastic and dilatant fluids as a function of $\mathrm{Bi}_1$ when $\mathrm{Bi}_2 = 2$ and $T_a^* = 7$. The result for the Newtonian fluid corresponds to the flow behavior index $n = 1$.

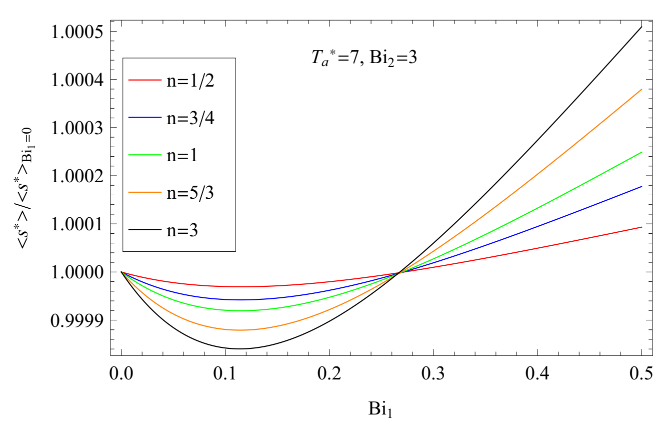


**Fig. 6.** Dimensionless global entropy generation rate (normalized with its value at $\mathrm{Bi}_1 = 0$) for pseudoplastic and dilatant fluids as a function of $\mathrm{Bi}_1$ when $\mathrm{Bi}_2 = 3$ and $T_a^* = 7$. The result for the Newtonian fluid corresponds to the flow behavior index $n = 1$.

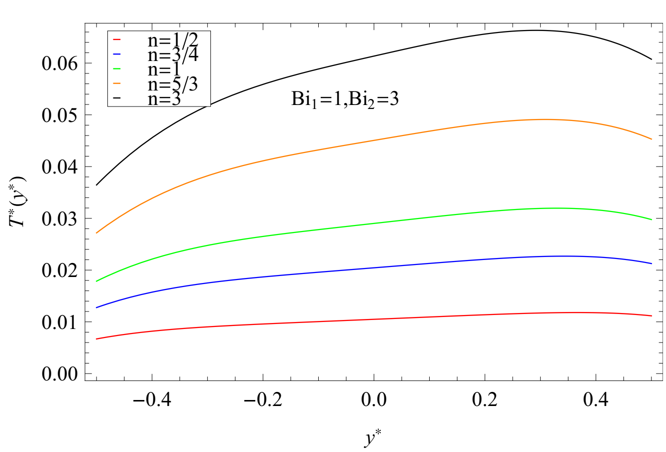


**Fig. 3.** Dimensionless temperature fields for different pseudoplastic and dilatant fluids when $\mathrm{Bi}_1 = 1$ and $\mathrm{Bi}_2 = 3$. The Newtonian fluid corresponds to the flow behavior index $n = 1$.

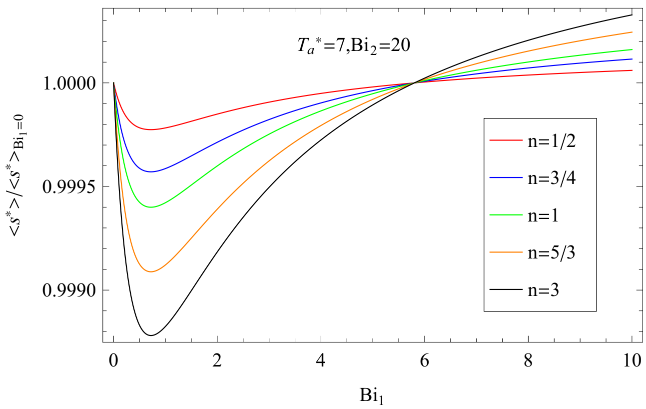


**Fig. 7.** Dimensionless global entropy generation rate (normalized with its value at $Bi_1 = 0$) for pseudoplastic and dilatant fluids as a function of $Bi_1$ when $Bi_2 = 20$ and $T_a^* = 7$. The result for the Newtonian fluid corresponds to the flow behavior index $n = 1$.

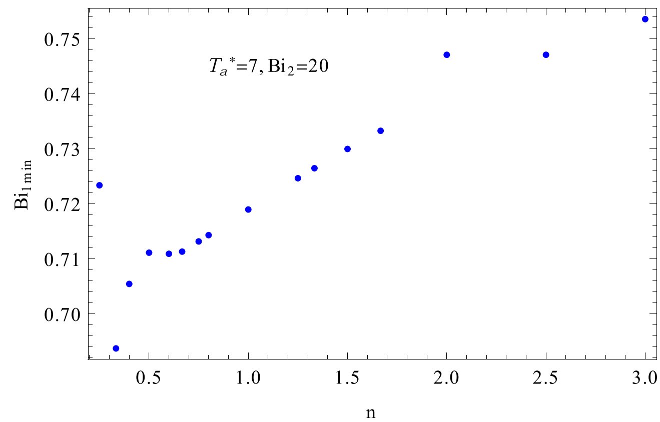


**Fig. 9.** Optimum Biot number $Bi_{1min}$ as a function of the power-law index $n$ when $Bi_2 = 20$ and $T_a^* = 7$.

other hand, when the convective cooling is asymmetric there is a threshold value of the Biot number of the upper plate (which, except for the Newtonian case that was dealt with in Ref. [23], we have not been able to determine analytically for all $n$ but, as follows from Figs. 5 and 6 it should be between $Bi_2 = 2$ and $Bi_2 = 3$) beyond which a minimum of the global entropy generation rate is always present. Given fixed values of the lower plate Biot number and the external ambient temperature, the corresponding minimum provides the conditions under which the irreversibilities due to viscous dissipation and/or heat flow are minimized. As in the Newtonian case, *c.f.* Figs. 6 and 8, for both dilatant and pseudoplastic fluids the depth of the minimum increases and its position moves to the right on the lower plate Biot number $Bi_1$ axis as $Bi_2$ increases.

For the sake of illustration we now take $T_a^* = 7$ and $Bi_2 = 20$, and display in Fig. 9 the value of the Biot number corresponding to the minimum of the global entropy generation rate $Bi_{1min}$ as a function of the flow behavior index $n$. In this case, no systematic trend may be discerned.

The availability of the velocity and temperature fields as given by Eqs. (11) and (12) also allow us to consider the global Nusselt number at the upper plate. In this case it is given by

$$\mathrm{Nu} = \frac{\left.\frac{dT^*}{dy^*}\right|_{y^*=1/2}}{2\left(T^*(1/2) + T_a^* - T_b^*\right)} = \frac{Bi_1 T^*(1/2)}{2\left(T^*(1/2) + T_a^* - T_b^*\right)}, \tag{13}$$

where the dimensionless bulk temperature (the cross-section averaged dimensionless temperature of the stream) is defined as

$$T_b^* = \frac{\int_{-1/2}^{1/2} u^*\left(T^* + T_a^*\right) dy^*}{\int_{-1/2}^{1/2} u^* dy^*}. \tag{14}$$

In Fig. 10 we display the behavior of the global Nusselt number at the upper plate (computed in each case with the value of the lower plate Biot number $Bi_{1min}$ corresponding to the minimum of the global entropy generation) as a function of the upper plate Biot number $Bi_2$ with $T_a^* = 7$ and different values of the power-law index $n$.

As the above figure indicates, the behavior of the global Nusselt number at the upper plate $Nu(Bi_{1min})$ as a function of $Bi_2$ is similar irrespective of whether the fluid is pseudoplastic, Newtonian or

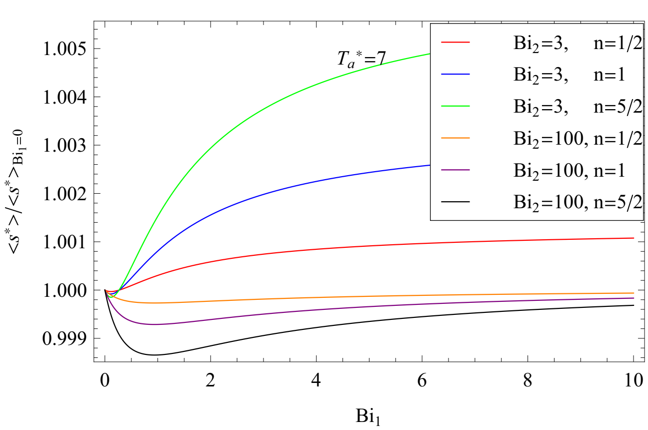


**Fig. 8.** Dimensionless global entropy generation rate (normalized with its value at $Bi_1 = 0$) for pseudoplastic and dilatant fluids as a function of $Bi_1$ when $Bi_2 = 3100$ and $T_a^* = 7$. The result for the Newtonian fluid corresponds to the flow behavior index $n = 1$.

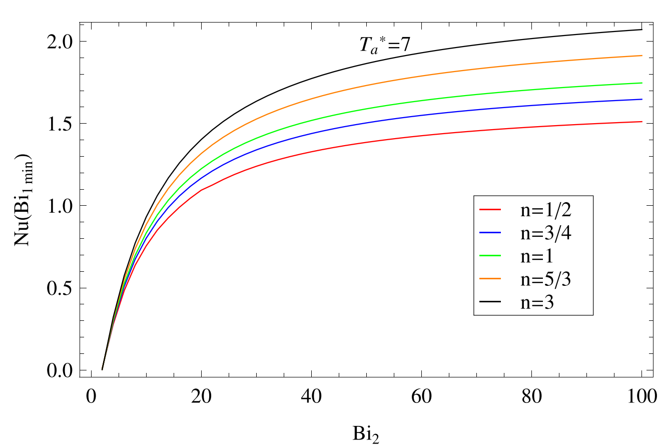


**Fig. 10.** Global Nusselt number at the upper plate $Nu(Bi_{1min})$ for pseudoplastic and dilatant fluids as a function of the upper plate Biot number $Bi_2$ when $T_a^* = 7$. The result for the Newtonian fluid corresponds to the flow behavior index $n = 1$.

dilatant. As a general trend, for fixed $Bi_2$ its value increases as the power-law index increases and this increase is more pronounced the higher the value of $Bi_2$. Although not shown, we find that the effect of varying the external ambient temperature on the value of the global Nusselt number at the upper plate $Nu(Bi_{1min})$ is relatively small and again more pronounced when both $n$ and/or $Bi_2$ increase.

## 4. Concluding remarks

In this paper we have presented a study of heat transfer and entropy generation in the parallel plate flow of a power-law fluid with asymmetric convective cooling. The main contributions of our work are the analytic results for the temperature profiles (*c.f.* Eq. (12)) and the computation of the global entropy generation rate and global Nusselt number at the upper plate under various conditions. From the analysis of our results one may conclude that the qualitative features of the effect of asymmetric convective cooling on the fully developed flow of a Newtonian fluid between parallel plates that we had found before [23] remain for power-law fluids. In particular, if $Bi_1 = Bi_2$ or if both $Bi_1 \to \infty$ and $Bi_2 \to \infty$, the global entropy generation displays no minimum. In contrast, when the convective cooling is asymmetric, there is a threshold value of the Biot number of the upper plate (which unfortunately we were not able to determine analytically for any given $n$) beyond which a minimum of the global entropy generation rate is always present. Therefore, we have shown that for a power-law fluid minimum entropy generation rates can be reached by extracting heat in the system in an asymmetric way. Although this provides the conditions under which the irreversibilities due to viscous friction and/or heat flow are minimized, the heat transfer at the wall presents an asymptotic behavior but is not necessarily optimized. This has to do with the fact that, as pointed out by Shah and Skiepko [53], minimum entropy generation is not always related to the best heat transfer performance. Addressing this point for the present problem seems certainly worthwhile but lies beyond the scope of this paper. In any case, what we have found in connection with heat transfer at the wall is that the local Nusselt number for minimum entropy generation conditions displays a monotonic behavior as $Bi_2$ increases and reaches a limiting value as $Bi_2 \to \infty$. Our illustrative calculations strongly suggest that, in order to reduce the irreversibilities due to viscous dissipation and/or heat flow in devices using the parallel-plate geometry, the consideration of power-law fluids instead of Newtonian fluids under similar operating conditions may prove advantageous. As pointed out above, good engineering heat transfer design involves minimizing the losses due to irreversible behavior. Therefore, the possibility of reaching a minimum in the entropy generation rate using both asymmetric convective cooling and this or other non-Newtonian fluids, might be useful also to optimize operating conditions of different kinds of heat transfer devices.

## Acknowledgments

Financial support from CONACYT, Mexico, through Project 131399 is gratefully acknowledged.

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