PHYSICAL REVIEW D **92,** 104026 (2015)

One-loop effective potential in nonlocal scalar field models

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(Received 14 August 2015; published 10 November 2015)

In this paper we apply the usual perturbative methodology to evaluate the one-loop effective potential in a nonlocal scalar field theory. We find that the effect induced by the nonlocality of the theory is always very small and we discuss the consequences of this result. In particular we argue that, looking at one-loop corrections for matter fields, it is not possible to find signals of the nonlocality of the theory in cosmological observables since, even during inflation when energies are very high, nonlocality-induced corrections are expected to be very small.

DOI: 10.1103/PhysRevD.92.104026 PACS numbers: 04.60.−m, 03.70.+k, 11.10.−z

I. INTRODUCTION

Higher-derivative field theory models actually attract great attention due to two main reasons. The first one is of cosmological nature. For instance, cosmological models have been considered, which are based on the inclusion of finite higher derivatives (see [1] and references therein), with Lagrangian density $\mathcal{L} = F(R, \Box R, \Box^2 R, \ldots, \Box^m R, \Box^- R, \ldots, \Box^{-m} R)$. Such models are capable to explain inflation and dark energy in a unified framework [2,3] (see also [4] for a complete review). The second reason is connected with the possibility to construct an higher-derivative theory which can be a viable quantum gravity candidate, together with other approaches to this scenario, namely, Loop Quantum Gravity, Strings and Noncommutative Geometries (see [5] for a review).

Among the theories addressing questions related to quantum gravity, one has received a great deal of attention, the so-called Horava-Lifshitz gravity (H-LG) [6]. This assumes a foliation of the spacetime and is characterized by a Lagrangian density which contains higher than first order spatial derivatives and is invariant under foliation-preserving diffeomorphisms. Such a theory is power-counting renormalizable due to the higher spatial derivatives which make the graviton propagator to converge to zero more rapidly than $1/k^2$ at high wave numbers k [7]. Moreover H-LG has been shown to be a viable model of gravitation at the cosmological and astrophysical level [6]. Among different implications of the H-LG, one should mention the consistency with the well-known gravitational solutions, such as black holes [8], Friedmann-Robertson-Walker solutions [9], and Gödel-type metrics [10], and some new cosmological concepts such as bouncing Universe [11] and anisotropic scaling [12], as well as the quantum studies that allow one to discuss the possible construction of a renormalizable gravity theory (see Ref. [13]).

Despite its advantages, H-LG involves an apparently strong problem: the loss of diffeomorphism invariance and then the loss of Lorentz symmetry in flat spacetime, as discussed in [14]. To avoid this problem exploiting the better convergence of the graviton propagator in higher derivative theories, one can consider covariant higher derivative theories of gravitation as in [15], which are invariant under spacetime diffeomorphisms. The main problem here is that such theories usually contain a physical ghost (a state of negative norm) and therefore they violate the unitarity. It is worth mentioning the papers [16], where the higher-derivative extensions of different field theory models were introduced without breaking the unitarity; however, in these theories the higher derivatives are present only in a term proportional to Lorentz-breaking parameters which leads to specific effects like large quantum corrections and fine-tuning.

In order to overcome this problem, a new class of higher derivative theories which manifestly preserve Lorentz invariance, the nonlocal quantum gravity (NLQG) models, has been proposed recently [17,18]. These models were partially inspired by [15], and their construction was done in order to fulfill the following hypotheses: (i) Classical solutions must be singularity-free; (ii) Einstein-Hilbert action must be a good approximation of the theory below the Planck energy scale; (iii) the theory must be perturbatively quantum renormalizable on a flat background; (iv) the

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theory must to be unitary; and (v) Lorentz invariance must be preserved.

Models in the framework of NLQG are both renormalizable and ghost-free [17,18], and therefore, they have no shortcomings of Einstein's gravity related to these points. The typical Lagrangian density for NLQG is a nonpolynomial extension of the renormalizable quadratic Stelle theory [15] and it has the following structure:

$$\mathcal{L} = R - \left(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}\right)\gamma(\Box/\Lambda^2)R^{\mu\nu}, \qquad (1)$$

where the form factor $\gamma(z)$ is a function without poles, $\Box$ is the covariant D'Alembertian operator and Λ is an invariant mass scale, which we expect to be close to the Planck mass, since we want that our theory reduces to Einstein's theory in the low energy limit. We stress that $1/\Lambda$ represents the length scale above which the theory is fully nonlocal. Note that the local behavior of the theory is recovered at energies below Λ and that all degrees of freedom present in the action can be embedded in the function $\gamma(\Box/\Lambda^2)$.

It is convenient to express the form factor $\gamma(\Box/\Lambda^2)$ introducing a new form factor $V(\Box/\Lambda^2)$ defined as

$$\gamma(\Box/\Lambda^2) \equiv \frac{V(\Box/\Lambda^2)^{-1} - 1}{\Box}, \qquad (2)$$

so that the propagator of the theory is

$$G(k^2) = \frac{V(k^2/\Lambda^2)}{k^2}\left(P^{(2)} - \frac{P^{(0)}}{2}\right), \qquad (3)$$

where $P^{(0)}$ and $P^{(2)}$ are the spin zero and spin two projectors.[1]

Since one wants to recover Einstein's gravity for small momenta, one needs to impose that $V(0) = 1$ and $V(z) \simeq 1$ for $|z| \ll 1$. It is now evident that, if $V(z)$ has no poles, the only propagating degrees of freedom correspond to the two polarizations of Einstein's theory, and thus the theory does not contain ghosts. As a consequence, the function $\gamma(z)$ cannot be polynomial, otherwise the function $V(z)$ would have a pole at z_i. Therefore, in order to avoid ghosts, we have to pay a price, namely, NLQG must contain derivatives of arbitrary order, which means that it must be nonlocal. Moreover, from (3) it follows that, if $V(z)$ goes to zero for $|z| \gg 1$ sufficiently fast, the theory is super-renormalizable, since this requirement improves the convergence of the integrations over loops (see for instance [17] for details).

The class of nonlocal theories (1) have nice properties. It has been shown in [21] that NLQG has 2 degrees of freedom corresponding to the spin 2 graviton and at most one extra scalar degree of freedom: any further extra degree of freedom in fact would correspond to a ghost or a tachyon, breaking the unitarity of the theory. Moreover, in [21,22] it has been shown that, at the cosmological level, NLQG reduces to the $R + \epsilon R^2$ Starobinsky model [23] with the identification $\epsilon \equiv 1/\Lambda^2$ up to corrections of order $1/\Lambda^4$. Therefore, NLQG gives a viable inflation in agreement with Planck data [24] for $\Lambda \sim 10^{-5} M_P$ [21,22].

A further relevant feature of NLQG is that, due to higher derivatives, it can be free from the singularities which affect Einstein's gravity. In fact, in NLQG the linearized equations for gravitational perturbations of Minkowski background typically reads $\exp(\Box/\Lambda^2)\Box h_{\mu\nu} = M_P^{-2}\tau_{\mu\nu}$ [18], where $\tau_{\mu\nu}$ is the stress energy tensor of matter. For a pointlike source of mass m this gives a Newtonian potential $h_{00} \sim erf(r\Lambda/2)m/rM_P^2$, where $erf(z)$ is the error function of argument z, which is finite for all $r \geq 0$. This shows how black hole singularities of general relativity can be removed in NLQG (see also [25]). With similar arguments one can show that also the big bang singularity can be removed and a nonsingular bouncing cosmology can be obtained [26].

The problem with NLQG is that, due to the high complexity of the theory, it is very difficult to perform explicit calculations. This is why, in order to understand the basic properties of nonlocal models, it is interesting to consider toy models, e.g. nonlocal scalar field theory. The properties of nonlocal scalar fields has been first studied by Efimov in a series of seminal papers [27–29]. Specifically, the quantization scheme has been described in [27], the unitarity of the theory has been demonstrated in [28] and the causality has been discussed in [29]. Here we also mention that a nonlocal version of QED has been studied in [30] and nonlocal vector field theory has been introduced in [31].

More recently, in [32] has been considered the case of a nonlocal scalar field with specific self-interactions which have been chosen to present the same symmetries of the NLQG; a toy model depiction of a nonlocal gravity. The form factor in (2) has been chosen in such a way that $V(z) = \exp(-z)$ and the graviton propagator in (3) has an exponential suppression, so that it is asymptotically free. In particular, the two-point function is still divergent, but it can be renormalized by adding appropriate counterterms, so that the ultraviolet behavior of all other one-loop diagrams as well as the two-loop, two-point function remains finite.

In this paper we consider a nonlocal scalar field model with generic potential and calculate the one-loop corrections to this quantity. We show that the corrections induced by the nonlocality of the theory, with respect to the local one, are very small even in the strong field limit, i.e., when the typical energies are of the order or much greater than Λ.

[1] The projectors are defined by [19,20] $P^{(2)}_{\mu\nu,\rho\sigma}(k) = \frac{1}{2}(\theta_{\mu\rho}\theta_{\nu\sigma} + \theta_{\mu\sigma}\theta_{\nu\rho}) - \frac{1}{D-1}\theta_{\mu\nu}\theta_{\rho\sigma}$, $P^{(1)}_{\mu\nu,\rho\sigma}(k) = \frac{1}{2}(\theta_{\mu\rho}\omega_{\nu\sigma} + \theta_{\mu\sigma}\omega_{\nu\rho} + \theta_{\nu\rho}\omega_{\mu\sigma} + \theta_{\nu\sigma}\omega_{\mu\rho})$, $P^{(0)}_{\mu\nu,\rho\sigma}(k) = \frac{1}{D-1}\theta_{\mu\nu}\theta_{\rho\sigma}$, $\bar{P}^{(0)}_{\mu\nu,\rho\sigma}(k) = \omega_{\mu\nu}\omega_{\rho\sigma}$, $\theta_{\mu\nu} = \eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}$, $\omega_{\mu\nu} = \frac{k_\mu k_\nu}{k^2}$, where D is the dimension of the spacetime.

This implies that it turns out to be very hard to find traces of the nonlocal nature of the theory looking at the one-loop corrections to the bare potential. For instance, one might hope to find nonlocality signatures in very energetic contexts, for instance in inflation. If the inflaton field, which is responsible of the nearly exponential of the early universe, is actually nonlocal, one can seek traces of the nonlocality of the theory in cosmological observables. However, since the one-loop corrections to the inflaton potential would be very small, it is impossible to detect the effect of one-loop corrections to the cosmological observables, even using the most precise measurements currently available, as the cosmic microwave background radiation temperature and polarization measurements made by Planck [24]. This suggests us that one should look for the signals of nonlocality studying other physical effects.

This paper is organized as follows: In Sec. II we introduce the nonlocal scalar field action and discuss its properties. In Sec. III we calculate the one-loop correction to the potential for different nonlocal field actions. Finally we leave to Sec. IV our conclusions and most relevant remarks.

II. NONLOCAL SCALAR FIELD

In this paper we consider nonlocal scalar fields with a Lagrangian density given by

$$\mathcal{L} = -\frac{1}{2}\phi F(\Box/\Lambda^2)\phi - V(\phi), \tag{4}$$

where $V(\phi)$ is the scalar field potential which describes self-interactions [for instance $V(\phi) = \lambda\phi^4/4!$] and $F(z)$ is a nonpolynomial analytic function, which in most cases can be represented by a series expansion

$$F(z) \equiv \sum_{n=0}^{\infty} f_n z^n, \tag{5}$$

with $f_n \neq 0$ for any $n > n_0$, with $n_0 \in N$. By definition $F(\Box/\Lambda^2)$ contains derivatives of arbitrary order, which makes the theory nonlocal.

From Eq. (4) it is immediate to recognize that the free field propagator of the theory is

$$G(k^2) = -\frac{1}{F(-k^2)} \tag{6}$$

and this expression makes evident that to each zero of the function $F(z)$ corresponds a pole in the propagator, and therefore a physical particle.

This conclusion is made more evident by the following considerations (see [33]). Suppose that $F(z)$ has a finite number M of zeros. Due to the Weierstrass factorization theorem, we can decompose it as

$$F(z) = f(z)\prod_{a=1}^{M}(z\Lambda^2 + m_a^2)^{r_a}, \tag{7}$$

where the function $f(z)$ has no zeros and no poles and r_a corresponds to a positive integer number. We can therefore use a field redefinition,

$$\varphi \equiv f(\Box/\Lambda^2)^{1/2}\phi, \tag{8}$$

so that the Lagrangian given by Eq. (4) becomes

$$\mathcal{L} = -\frac{1}{2}\varphi\prod_{a=1}^{M}(\Box + m_a^2)^{r_a}\varphi - V(\Gamma(\Box/\Lambda^2)^{-1/2}\varphi). \tag{9}$$

At this point one can introduce the M independent fields φ as

$$\varphi^a \equiv \prod_{b\neq a}(\Box/\Lambda^2 + m_b^2)^{r_b}\varphi, \qquad a = 1, 2\ldots M. \tag{10}$$

and express the field ϕ as a superposition of the fields φ^a as

$$\phi = f(\Box/\Lambda^2)^{-1/2}\sum_{a=1}^{M}\eta_a\varphi^2, \tag{11}$$

where the coefficients η_a satisfy the relation

$$\sum_{a=1}^{M}\frac{\eta_a}{(z + m_a^2)^{r_a}} = \left(\prod_{a=1}^{M}(z + m_a^2)^{r_a}\right)^{-1}. \tag{12}$$

Therefore the Lagrangian density (9) can be expressed as

$$\mathcal{L} = -\sum_{a=1}^{M}\frac{1}{2}\eta_a\varphi^a(\Box + m_a^2)^{r_a}\varphi^a - V\left(\Gamma(\Box/\Lambda^2)^{-1/2}\sum_{b=1}^{M}\eta_b\varphi^b\right). \tag{13}$$

Restricting to the case in which all zeros of $F(z)$ are simple, i.e., $r_a = 1\ \forall\ a$, one has that the theory contains M interacting constituent scalar fields. Since the η_a have alternating sign, thus some of these constituent fields are ghosts. We also note the following: from Eq. (13) it is evident that, in the new variables, the nonlocality of the theory is contained only in the potential V, and therefore it follows that a noninteracting nonlocal theory is in fact local.

Since from the previous considerations it follows that the unique ghost-free case is that with $M = 1$, from now on we limit our interest to functions of the type $F(z) = f(z)(z - m^2)$, so that

$$\mathcal{L} = -\frac{1}{2}\phi f(\Box/\Lambda^2)(\Box + m^2)\phi - V(\phi), \tag{14}$$

from which we can write the propagator, which is given by

$$G(k^2) = \frac{1}{f(-k^2/\Lambda^2)(k^2 - m^2)}. \tag{15}$$

From the knowledge of the propagator it is possible to obtain the one-loop correction to the scalar field potential, as is explained in the next section

III. ONE-LOOP CORRECTIONS

In this section we calculate the one-loop effective potential for this theory. To do it, we generalize the formula (9–119) of [34], to find the one-loop correction to the scalar field potential

$$V^{(1)} = -\frac{i}{2}\int \frac{d^4k}{(2\pi)^4}[\ln[1 - G(k^2)V''] + G(k^2)V'' + \frac{1}{2}(G(k^2)V'')^2]. \tag{16}$$

The second and third terms in (16) are added, as in the standard local case, to comply with the prescription of normal ordering, which avoids the inclusion of tadpole diagrams, and corresponds to the one-loop diagrams of the two- and four-point functions [34].

Performing a Wick rotation, we obtain

$$V^{(1)} = \frac{1}{2}\int \frac{d^4k_E}{(2\pi)^4}[\ln[1 - G(-k_E^2)V''] + G(-k_E^2)V'' + \frac{1}{2}(G(-k_E^2)V'')^2]. \tag{17}$$

Therefore, to calculate the one-loop correction to the potential we only need to know the form of the propagator $G(k^2)$. In what follows we make this calculation explicitly in the case of some specific choice of the function $F(z)$ which reduces to the local model at $\Lambda \to \infty$.

A. Case 1

We first consider the theory described by the Lagrangian density,

$$\mathcal{L} = -\frac{1}{2}\phi(\exp(\Box/\Lambda^2)\Box + m^2)\phi - V(\phi), \tag{18}$$

where we have considered the same exponential nonlocal factor used in [32]. This Lagrangian provides the scalar propagator

$$G(k^2) = \frac{1}{k^2 \exp(-k^2/\Lambda^2) - m^2}. \tag{19}$$

From Eq. (17) and after integration over the solid angle, one has

$$V^{(1)} = \frac{1}{(4\pi)^2}\int_0^\infty dk_E F(k_E, \Lambda, m, V''), \tag{20}$$

where

$$F(k_E, \Lambda, m, V'') \equiv k_E^3\left[\ln\left[1 + \frac{V''}{k_E^2\exp[k_E^2/\Lambda^2] + m^2}\right] - \frac{V''}{k_E^2\exp[k_E^2/\Lambda^2] + m^2}\frac{1}{2}\left(\frac{V''}{k_E^2\exp[k_E^2/\Lambda^2] + m^2}\right)^2\right], \tag{21}$$

From the last expression we deduce the following: the departure from the standard local result is given by integration at high momenta $k_E \gtrsim \Lambda$, where the exponential $\exp(k_E^2/\Lambda^2)$ is significatively different from unity. Therefore, if $V''/(k_E^2\exp(k_E^2/\Lambda^2) + m^2) \ll 1$ at such high momenta, the integrand is very small and the effect of nonlocality is negligible. Thus we expect two significantly different regimes: the strong field regime $V'' \gtrsim \Lambda$, where the effect of nonlocalities is stronger, and the weak field regime $V'' \ll \Lambda$, where a similar effect is weaker.

Before proceeding with the explicit calculation of (20), let us recall the one-loop correction in the case of a local model. The local model is obtained in the limit $\Lambda \to \infty$, and it is given by

$$V^{(1)}_{\text{local}} = \frac{1}{(4\pi)^2}\int_0^\infty dk_E F_\infty(k_E, m, V'') = \frac{1}{(8\pi)^2}\left[(V'' + m^2)^2\ln\left[1 + \frac{V''}{m^2}\right] - V''\left(\frac{3}{2}V'' + m^2\right)\right], \tag{22}$$

where we have defined

$$F_\infty(k_E, m, V'') = \lim_{\Lambda\to\infty} F(k_E, \Lambda, m, V'') = k_E^3\left[\ln\left[1 + \frac{V''}{k_E^2 + m^2}\right] - \frac{V''}{k_E^2 + m^2} + \frac{1}{2}\left(\frac{V''}{k_E^2 + m^2}\right)^2\right]. \tag{23}$$

It is useful for our proposal, to express Eq. (20) as

$$V^{(1)} \simeq \frac{1}{(4\pi)^2}\int_\Lambda^\infty dk_E[F_0(k_E, \Lambda, V'') - F_\infty(k_E, m, V'')] + V^{(1)}_{\text{local}} = V^{(1)}_{\text{local}} + \delta V^{(1)}, \tag{24}$$

where

$$\delta V^{(1)} \equiv V_a^{(1)} + V_b^{(1)} \tag{25}$$

represents the deviation from the result obtained in the local theory and where we have defined

$$V_a^{(1)} \equiv -\frac{1}{(4\pi)^2}\int_\Lambda^\infty dk_E F_\infty(k_E, m, V'') = -\frac{1}{(8\pi)^2}\left[\ln\left[1+\frac{V''}{\Lambda^2+m^2}\right] \times (V''(V''+2m^2)+m^4-\Lambda^4) - \frac{V''}{2(\Lambda^2+m^2)}(3V''m^2+2m^4+V''\Lambda^2-2\Lambda^4)\right] \tag{26}$$

and

$$V_b^{(1)} \equiv \frac{1}{(4\pi)^2}\int_\Lambda^\infty dk_E F_0(k_E,\Lambda,V''), \tag{27}$$

with

$$F_0(k_E,\Lambda,V'') \equiv F(k_E,\Lambda,m=0,V'') = k_E^3\left[\ln\left[1+\frac{V''}{k_E^2\exp[k_E^2/\Lambda^2]}\right] - \frac{V''}{k_E^2\exp[k_E^2/\Lambda^2]} + \frac{1}{2}\left(\frac{V''}{k_E^2\exp[k_E^2/\Lambda^2]}\right)^2\right]. \tag{28}$$

While $V_a^{(1)}$ is calculated exactly, it is not possible to find an analytic expression for $V_b^{(1)}$, and we have to perform its expression by making some simple assumption, namely, considering the strong and weak field limits.

1. Strong field limit

Let us proceed with the calculation of (27) in the strong field limit $V'' \gg \Lambda^2$. First of all let us divide the integration region in two intervals: $I_1 \equiv [\Lambda, \bar{k}]$ and $I_2 \equiv [\bar{k}, \infty[$, where $\bar{k} \equiv \Lambda\sqrt{|X|}$ and X is the solution of the equation $X\exp[X] = V''/\Lambda^2$, which is approximately given by

$$X \simeq \ln[V''/\Lambda^2] - \ln[\ln[V''/\Lambda^2]]. \tag{29}$$

Therefore, since $V''/\Lambda \gg 1$ one has $\bar{k} > \Lambda$.

The contribution to $V^{(1)}$ coming from the integration over I_1 is therefore

$$V_{I_1}^{(1)}(\bar{k}) \simeq \frac{1}{(4\pi)^2}\int_\Lambda^{\bar{k}} dk_E F_0(k_E,\Lambda,V'') = \frac{1}{6(8\pi)^2}[6V''^2(EI[-2\bar{k}^2/\Lambda^2]-EI[-2]) + 3(\bar{k}^4-\Lambda^4)+2(\bar{k}^6-\Lambda^6)/\Lambda^2 + 12V''\Lambda^2(\exp[-\bar{k}^2/\Lambda^2]-1) - 6\Lambda^4\ln[V''/\Lambda^2]], \tag{30}$$

where $Ei(z) \equiv -\int_{-z}^\infty dt\exp[-t]/t$ is the exponential integral function and $Ei(-2) \simeq -0.0489$. Note that in the calculation we have neglected 1 with respect to $V''/k_E^2\exp[k_E^2/\Lambda^2]$.

Let us calculate the contribution arising from the integration over I_2. Since in this region $V''/k_E^2 \exp[k_E^2/\Lambda^2] \ll 1$, we can expand the logarithm in $F_0(k_E,\Lambda,V'')$ in power series and then integrate, so that we obtain

$$V_{I_2}^{(1)}(\bar{k}) \simeq \frac{1}{(4\pi)^2}\int_{\bar{k}}^\infty dk_E F_0(k_E,\Lambda,V'') = \Lambda^4\sum_{n=3}^\infty C_n(\bar{k})\left(\frac{V''}{\Lambda^2}\right)^n, \tag{31}$$

where

$$C_n(\bar{k}) \equiv \frac{(-1)^{n+1}}{2(4\pi)^2}\frac{1}{n^{3-n}}\Gamma(2-n, n\bar{k}/\Lambda^2) \tag{32}$$

with $\Gamma(a,x)$ being the incomplete Gamma function. Therefore one has

$$V_b^{(1)} = V_{I_1}^{(1)}(\bar{k}) + V_{I_2}^{(1)}(\bar{k}). \tag{33}$$

To evaluate the effect of nonlocality we can estimate the value of $\delta V^{(1)}/V_{\rm local}^{(1)}$ in the strong field limit. From (22), (30), (31) and (26), one has

$$V_{\rm local}^{(1)} \simeq \left(\frac{V''}{8\pi}\right)^2\ln[1+V''/m^2], \tag{34}$$

$$V_a^{(1)} \simeq -\left(\frac{V''}{8\pi}\right)^2\ln[1+V''/\Lambda^2], \tag{35}$$

$$V_{I_2}^{(1)} \simeq \left(\frac{V''}{8\pi}\right)^2|Ei[-2]|. \tag{36}$$

So,

$$V_{I_3}^{(1)} < \left(\frac{V''}{8\pi}\right)^2\ln[1+V''/\bar{k}^2] \simeq \left(\frac{V''}{8\pi}\right)^2\ln[1+V''/\Lambda^2]. \tag{37}$$

The estimations given by Eqs. (34)–(36) are obtained by taking the limit $V'' \gg \bar{k}^2 \gg \Lambda^2 \gg m^2$ in Eqs. (22), (30), (31) and (26). To obtain (37) it is necessary to note that, since the function $g(x) = \ln[1+x] - x + x^2/2$ grows monotonically for any $x > -1$, one has

$$V_{I_3}^{(1)} < \frac{1}{(4\pi)^2}\int_{\bar{k}}^{\infty} dk_E k_E^3[\ln[1+V''/k_E^2] - V''/k_E^2 + (V''/k_E^2)^2/2] = \frac{\bar{k}^4}{(8\pi)^2}[V''/\bar{k}^2(1-V''/2\bar{k}^2) + \ln[1+V''/\bar{k}^2]((V'')^2/\bar{k}^4-1)]. \tag{38}$$

Therefore, from Eqs. (34)–(37) it follows that $\delta V^{(1)} \simeq V_a^{(1)}$ and therefore

$$\frac{\delta V^{(1)}}{V_{\text{local}}^{(1)}} \simeq \frac{\ln[1+V''/\Lambda^2]}{\ln[1+V''/m^2]} \ll 1, \tag{39}$$

which gives a measure of the effect of the nonlocality of the scalar field on the one-loop corrections of the bare potential. The conclusion is that, even in the strong field limit, since $\Lambda \gg m$, one has $\delta V^{(1)}/V_{\text{local}}^{(1)} \ll 1$, and therefore, this effect is very small.

2. *Weak field limit*

Let us consider now the weak field limit in which $V''/\Lambda^2 \ll 1$ but $V'' \gg m^2$. Since in this limit one has $V''/k_E^2 \exp[k_E^2/\Lambda^2] \ll 1$ for any $k_E \geq \Lambda$, one can always expand the logarithm in (27) in power series and then integrate, so that one obtains

$$V_b^{(1)}(\bar{k}) \simeq \frac{1}{(4\pi)^2}\int_{\bar{k}}^{\infty} dk_E F_0(k_E,\Lambda,V'') = \Lambda^4 \sum_{n=3}^{\infty} C_n(\Lambda)\left(\frac{V''}{\Lambda^2}\right)^n, \tag{40}$$

where

$$C_n(\Lambda) \equiv \frac{(-1)^{n+1}}{2(4\pi)^2}\frac{1}{n^{3-n}}\Gamma(2-n,n). \tag{41}$$

Now, taking the limit $\Lambda^2 \gg V'' \gg m^2$ in (22), (26) and (38), we obtain the result given by Eq. (34) and for $V_a^{(1)}$ we get

$$V_a^{(1)} \simeq -\frac{2}{3}\left(\frac{1}{8\pi}\right)^2 \frac{V''^3}{\Lambda^2}. \tag{42}$$

So,

$$V_{I_3}^{(1)} < \left(\frac{1}{8\pi}\right)^2 \frac{V''^3}{\Lambda^2} 2\Gamma(-2,3), \tag{43}$$

which finally gives

$$\frac{\delta V^{(1)}}{V_{\text{local}}^{(1)}} \simeq \frac{V''}{\Lambda^2}\frac{1}{\ln[V''/m^2]} \ll 1. \tag{44}$$

The comparison of (44) with (39) shows that the relative correction to the result obtained in the local theory is much stronger in the strong field limit.

B. Case 2

A second example is easily obtained considering the Lagrangian density

$$\mathcal{L} = -\frac{1}{2}[\phi T(\Box)(\Box + m^2)\phi + V(\phi)], \tag{45}$$

where we can choose $T(\Box) = \exp[-\Box/\Lambda^2]$. Calculations are the same as in the previous case. In fact the one-loop correction is given by

$$V^{(1)} = \frac{1}{(4\pi)^2}\int_0^{\infty} dk_E G(k_E,\Lambda,m,V''), \tag{46}$$

where

$$G(k_E,\Lambda,m,V'') \equiv k_E^3\left[\ln\left[1+\frac{V''\exp[-k_E^2/\Lambda^2]}{k_E^2+m^2}\right] - \frac{V''\exp[-k_E^2/\Lambda^2]}{k_E^2+m^2} + \frac{1}{2}\left(\frac{V''\exp[-k_E^2/\Lambda^2]}{k_E^2+m^2}\right)^2\right]. \tag{47}$$

From the simple observation that

$$G_\infty(k_E,m,V'') = \lim_{\Lambda\to\infty} G(k_E,\Lambda,m,V'') = F_\infty(k_E,m,V''),$$
$$G_0(k_E,\Lambda,V'') \equiv G(k_E,\Lambda,m=0,V'') = F_0(k_E,\Lambda,V'') \tag{48}$$

one immediately realizes that one can repeat the same steps of the previous section and arrive to the same results, especially to Eqs. (39) and (44), for strong and weak approximations.

C. Case 3

A further example in which we expect a different result is given by the nonlocal Lagrangian density

$$\mathcal{L} = -\frac{1}{2}[\phi T(\Box)(\Box + m^2)\phi + V(\phi)], \tag{49}$$

where the factor $T(\Box)$ was introduced in [17], and is given by

$$T(\Box) \equiv \exp[H(-\Box/\Lambda^2)], \tag{50}$$

where

$$H(z) \equiv \frac{1}{2}[\gamma_E + \ln[p_n(z)^2] - Ei(-p_n(z)^2)] = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{2kk!}(p_n(z))^{2k}, \tag{51}$$

and $p_n(z)$ is a polynomial of order n such that $p_n(z) \ll 1$, for any $z \lesssim 1$ and $p_n(z) \simeq z^n$, for $z \gtrsim 1$. These conditions imply that $H(z) \simeq 1$ for $z \lesssim 1$ and $H(z) \simeq z^n$ for $z \gtrsim 1$.

In this case the one-loop correction is given by

$$V^{(1)} = \frac{1}{(4\pi)^2}\int_0^\infty dk_E Q(k_E, \Lambda, m, V''), \tag{52}$$

where now

$$Q(k_E, \Lambda, m, V'') \equiv k_E^3\left[\ln\left[1 + \frac{V''}{\exp[H(k_E^2/\Lambda^2)](k_E^2+m^2)}\right] - \frac{V''}{\exp[H(k_E^2/\Lambda^2)](k_E^2+m^2)} + \frac{1}{2}\left(\frac{V''}{\exp[H(k_E^2/\Lambda^2)](k_E^2+m^2)}\right)^2\right]. \tag{53}$$

It is worth noting that in this case, we have

$$Q_\infty(k_E, m, V'') = \lim_{\Lambda\to\infty} G(k_E, \Lambda, m, V'') = F_\infty(k_E, m, V''),$$
$$G_0(k_E, \Lambda, V'') \equiv G(k_E, \Lambda, m=0, V'') \neq F_0(k_E, \Lambda, V''), \tag{54}$$

and thus, we expect different results compared with previous cases.

The one-loop correction can be expressed again as in Eq. (24), where now

$$\delta V^{(1)} \equiv V_a^{(1)} + V_b^{(1)}, \tag{55}$$

and $V_a^{(1)} \equiv -\frac{1}{(4\pi)^2}\int_\Lambda^\infty dk_E Q_\infty(k_E, m, V'')$ is the same as in (26), while

$$V_b^{(1)} \equiv \frac{1}{(4\pi)^2}\int_\Lambda^\infty dk_E Q_0(k_E, \Lambda, V'') \tag{56}$$

must be calculated.

In what follows we consider the two previous limiting cases.

1. Strong field limit

In the limit $V'' \gg \Lambda^2$, once again, we can divide the integral (56) into two parts:

$$V_{I_2}^{(1)} \equiv \frac{1}{(4\pi)^2}\int_\Lambda^{\tilde{k}} dk_E Q_0(k_E, \Lambda, V'') \tag{57}$$

and

$$V_{I_3}^{(1)} \equiv \frac{1}{(4\pi)^2}\int_{\tilde{k}}^\infty dk_E Q_0(k_E, \Lambda, V''), \tag{58}$$

where the $\tilde{k}$ is defined by the relation $V''\Lambda^{2n}/\tilde{k}^{2+2n} = 1$. One has

$$V_{I_2}^{(1)} = \frac{1}{2(8\pi)^2}\left[4\frac{V''\Lambda^2}{n-1}\left[\left(\frac{\Lambda^2}{V''}\right)^{n-1/n+1} - 1\right] + \frac{V''2}{n}\left[1 - \left(\frac{\Lambda^2}{V''}\right)^{2n/n+1}\right] + (n+1)(\tilde{k}^4 - \Lambda^4) - \Lambda^4 \ln\left[\left(\frac{V''}{\Lambda^2}\right)^2\right]\right] \simeq \frac{1}{2n}\left(\frac{V''}{8\pi}\right)^2 \tag{59}$$

and

$$V_{I_3}^{(1)} = C_\#\left(\frac{\Lambda^2}{8\pi}\right)^2\left(\frac{V''}{\Lambda^2}\right)^{2/n+1}, \tag{60}$$

where $C_\# = \sum_{m=3}^\infty \frac{(-1)^{m+1}}{m[\frac{m}{2}(n+1)-1]} > 0$ is a real number.

As a conclusion, we can say that the estimation given by Eq. (39) remains valid also for the form factor (60).

2. Week field limit

In the weak field limit $V'' \ll \Lambda^2$, one has

$$V_b^{(1)} = \frac{\Lambda^4}{(8\pi)^2}\sum_{m=3}^\infty \frac{(-1)^{m+1}}{m[\frac{m}{2}(n+1)-1]}\left(\frac{V''}{\Lambda^2}\right)^m, \tag{61}$$

and therefore, we obtain the same results as in Eq. (44).

IV. CONCLUSIONS

In this paper we have considered nonlocal scalar field models with generic potential and studied the effect of the nonlocality on the one-loop corrections to the bare scalar field potential. Three different models have been considered. All of them reduce themselves to the local model for the case where $\Lambda \to \infty$. Moreover, in order to estimate the influence of the nonlocality we have considered two different situations: the strong and weak field limits. We have found that the nonlocality effect is stronger in the strong field limit, i.e., $V'' \gg \Lambda^2$, but indeed very small.

The implications of this finding are important and suggest us to ask the question whether nonlocality of fundamental fields might have observable cosmological signatures. In fact, one might suppose that the inflaton field is nonlocal, so that during inflation, at very high energies, corrections due the scalar field potential associated with nonlocality might play some role in the generation of cosmological perturbations. However, due to the extreme smallness of such corrections, we expect that their trace in cosmological observables should not be detected.

Let us make this claim more clear and let us consider the slow roll parameters ϵ and η, the power spectrum of curvature perturbations $P_\zeta(k)$ and the spectral tilt $n(k)$. These quantities are determined by the complete effective potential $V_{\rm eff}$ of the inflaton field, and their expressions are

$$\epsilon = \frac{M_{\rm Pl}^2}{2}\left(\frac{V'_{\rm eff}}{V_{\rm eff}}\right)^2, \tag{62}$$

$$\eta = M_{\rm Pl}^2 \frac{V''_{\rm eff}}{V_{\rm eff}}, \tag{63}$$

$$P_\zeta(k) = \frac{1}{24\pi^2 M_{\rm Pl}^4}\frac{V_{\rm eff}}{\epsilon}, \tag{64}$$

and

$$n(k) - 1 = -6\epsilon + 2\eta, \tag{65}$$

where $M_{\rm Pl}$ is the Planck mass (see [35]).

In the case of a nonlocal inflaton field, the effective scalar field potential will be

$$V_{\rm eff} = \frac{m^2}{2}\phi^2 + V(\phi) + V^{(1)}_{\rm local} + \delta V^{(1)} = V^{\rm local}_{\rm eff} + \delta V^{(1)}, \tag{66}$$

where $V^{\rm local}_{\rm eff} \gg \delta V^{(1)}$ and again $\delta V^{(1)}$ represents the nonlocality-induced corrections to the effective inflaton potential. Since in our case one has that

$$\delta V^{(1)}/V^{\rm local}_{\rm eff} \lesssim (V''/\Lambda^2)/\ln(1 + V''/m^2) \tag{67}$$

is extremely small (assuming $\Lambda \sim M_{\rm Pl}$) and since ϵ, η, $P_\zeta(k)$ and $n(k)$ are measured with nearly percent precision [24], one concludes that there is no chance to detect the effect of $\delta V^{(1)}$ with current experimental precision. This suggests that one should look for other physical effects in order to detect signals of the nonlocal nature of the physical fields.

It is interesting to note that the methodology of so-called coherent states [36] proposed as a manner to implement the noncommutativity alternative to the well-known Moyal product approach, effectively represents itself as an equivalent description of the nonlocal field theory discussed in this paper. Therefore, as a by-product of our study, we arrive at some justification for the coherent states approach.

ACKNOWLEDGMENTS

This work was partially supported by Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq). F. B. thanks CNPq for Grant No. 400.237/2014-8 and PAPIIT-UNAM for Grant No. IN100314.

[1] T. Clifton, P. G. Ferreira, A. Padilla, and C. Skordis, Phys. Rep. **513**, 1 (2012).
[2] S. Nojiri and S. D. Odintsov, Phys. Lett. B **659**, 821 (2008).
[3] S. Capozziello, E. Elizalde, S. 'i. Nojiri, and S. D. Odintsov, Phys. Lett. B **671**, 193 (2009).
[4] S. Nojiri and S. D. Odintsov, Phys. Rep. **505**, 59 (2011).
[5] L. Smolin, *Three Roads to Quantum Gravity* (Weidenfeld & Nicolson, London, United Kingdom, 2000).
[6] P. Hořava, Phys. Rev. D **79**, 084008 (2009); A. E. Gumrukcuoglu, S. Mukohyama, and A. Wang, Phys. Rev. D **85**, 064042 (2012); P. Hořava and C. M. Melby-Thompson, Phys. Rev. D **82**, 064027 (2010); C. Bogdanos and E. N. Saridakis, Classical Quantum Gravity **27**, 075005 (2010); arXiv:0907.1636; Y.-F. Cai and E. N. Saridakis, J. Cosmol. Astropart. Phys. 10 (2009) 020.
[7] F. Briscese, Y. Rodríguez, and G. A. González, Found. Phys. **42**, 1444 (2012).
[8] R. Konoplya, Phys. Lett. B **679**, 499 (2009); R.-C. Cai, L.-M. Cao, and N. Ohta, Phys. Rev. D **80**, 024003 (2009).
[9] G. Calcagni, J. High Energy Phys. 09 (2009) 112; E. Kiritsis and G. Kofinas, Nucl. Phys. **B821**, 467 (2009); H. Lu, J. Mei, and C. N. Pope, Phys. Rev. Lett. **103**, 091301 (2009).
[10] J. B. Fonseca-Neto, A. Yu. Petrov, and M. J. Reboucas, Phys. Lett. B **725**, 412 (2013).
[11] R. Brandenberger, Phys. Rev. D **80**, 043516 (2009).
[12] S. Mukohyama, J. Cosmol. Astropart. Phys. 06 (2009) 001.
[13] G. Giribet, D. L. Nacir, and F. D. Mazzitelli, J. High Energy Phys. 09 (2010) 009; F. D. Mazzitelli, Int. J. Mod. Phys. D **20**, 745 (2011); D. L. Nacir, F. D. Mazzitelli, and L. G. Trombetta, Phys. Rev. D **85**, 024051 (2012).
[14] M. Pospelov and Y. Shang, Phys. Rev. D **85**, 105001 (2012).
[15] K. S. Stelle, Phys. Rev. D **16**, 953 (1977).
[16] C. M. Reyes, Phys. Rev. D **87**, 125028 (2013); M. Maniatis and C. M. Reyes, Phys. Rev. D **89**, 056009 (2014).

[17] L. Modesto, Astron. Rev. **8.2**, 4 (2013); Phys. Rev. D **86**, 044005 (2012); L. Modesto and L. Rachwal, Nucl. Phys. **B889**, 228 (2014).

[18] T. Biswas, E. Gerwick, T. Koivisto, and A. Mazumdar, Phys. Rev. Lett. **108**, 031101 (2012).

[19] A. Accioly, A. Azeredo, and H. Mukai, J. Math. Phys. (N.Y.) **43**, 473 (2002).

[20] P. Van Nieuwenhuizen, Nucl. Phys. **B60**, 478 (1973).

[21] F. Briscese, L. Modesto, and S. Tsujikawa, Phys. Rev. D **89**, 024029 (2014).

[22] F. Briscese, A. Marcianò, L. Modesto, and E. N. Saridakis, Phys. Rev. D **87**, 083507 (2013).

[23] A. A. Starobinsky, Phys. Lett. B **91**, 99 (1980); S. V. Ketov and A. A. Starobinsky, J. Cosmol. Astropart. Phys. 08 (2012) 022.

[24] P. A. R. Ade *et al.* (Planck Collaboration), arXiv:1502.01589; arXiv:1502.02114.

[25] C. Bambi, D. Malafarina, and L. Modesto, Eur. Phys. J. C **74**, 2767 (2014).

[26] G. Calcagni, L. Modesto, and P. Nicolini, Eur. Phys. J. C **74**, 2999 (2014).

[27] G. V. Efimov, Int. J. Theor. Phys. **10**, 19 (1974); Commun. Math. Phys. **7**, 138 (1968).

[28] V. A. Alebastrov and G. V. Efimov, Commun. Math. Phys. **31**, 1 (1973).

[29] V. A. Alebastrov and G. V. Efimov, Commun. Math. Phys. **38**, 11 (1974).

[30] G. V. Efimov, M. A. Ivanov, and O. A. Mogilevsky, Ann. Phys. (N.Y.) **103**, 169 (1977); G. V. Efimov and K. Namsrai, Teor. Mat. Fiz. **22**, 186 (1975); G. V. Efimov, Ann. Phys. (N.Y.) **71**, 466 (1972).

[31] E. T. Tomboulis, arXiv:hep-th/9702146.

[32] S. Talaganis, T. Biswas, and A. Mazumdar, Classical Quantum Gravity **32**, 215017 (2015).

[33] N. Barnaby, Nucl. Phys. **B845**, 1 (2011); A. Pais and G. E. Uhlenbeck, Phys. Rev. **79**, 145 (1950).

[34] C. Itzykson and J. B. Zuber, *Quantum Field Theory* (McGraw-Hill Inc., New York, 1980).

[35] D. Lyth and A. Liddle, *The Primordial Density Perturbation* (Cambridge University Press, Cambridge, 2009).

[36] A. Smailagic and E. Spallucci, J. Phys. A **36**, L467 (2003).