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ORIGINAL PAPER

Kink Structure in Nodal Quasiparticle Dispersion in a Boson-Fermion Model of Superconductivity

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Abstract At temperatures below a certain T^*, single unbound electrons in high-T_c cuprates are assumed to coexist with bosonic Cooper pairs (CPs) of electrons emerging *incoherently* from an attractively interacting system of fermions. Due to both simultaneous interfermion attractions (as in BCS theory) and depairings, the conductive electrons at temperatures higher than the T_c of a Bose-Einstein condensation of preformed CPs fluctuate unceasingly between single-fermionic states and states of two bound electrons considered actual bosonic objects. We explore how these interactions in a background of such "frustrated electrons," i.e., those electrons appearing both as unbound (free) electrons and as constituents of the bosonic CPs, affect the dispersion of a fermion moving in the assembly of other electrons and, in particular, how the recently observed "kinks" in the dispersion curves emerge.

Keywords Boson-Fermion mixture · Pseudogap · Kinks · Nodal quasiparticle dispersion

1 Introduction

In contrast with conventional low-T_c superconductors, superconductivity in high-T_c materials is preceded, as absolute temperature T is lowered, by a state of pseudogapped fermions [1]. Widespread belief is that both superconducting and pseudogapped states are phenomena of the same origin and ultimately stem from interfermionic attractions [2]. The ground state made up of wavefunctions of *purely* single-fermionic states was long believed to be less obvious in the description of a system of attractively interacting fermions than is a ground state made up of *both* free-fermion wavefunctions and two-fermionic bound states [3–5]. If an attractive interaction operates between electrons then below some specific T^*, single electrons in high-T_c cuprates are assumed to coexist with CPs emerging incoherently from the attractively interacting system of electrons. The simplest Hamiltonian describing such a system of interacting fermions was first written phenomenologically and was later also extracted from microscopic considerations (see, Refs. in [6]). In terms of creation/annihilation operators for fermions (a operators) and bosons (b operators), the hamiltonian $\mathcal{H}$ introduced to describe a binary boson-fermion (BF) mixture of pairable fermions coexisting with bosonic CPs of total (or center-of-mass) momentum wavevector $\mathbf{K}$ is

$$\mathcal{H} = \sum_{\mathbf{k},\sigma} \xi_{\mathbf{k}} a^+_{\mathbf{k}\sigma} a_{\mathbf{k}\sigma} + \sum_{\mathbf{K}} \mathcal{E}_{\mathbf{K}} b^+_{\mathbf{K}} b_{\mathbf{K}} + \frac{f}{N^{1/2}} \sum_{\mathbf{q},\mathbf{K}} \left(\phi_{\mathbf{q}} b^+_{\mathbf{K}} a_{\mathbf{q}+\mathbf{K}/2\uparrow} a_{-\mathbf{q}+\mathbf{K}/2\downarrow} + h.c. \right) \quad (1)$$

Here, N is the number of unit cells in a lattice while fermion energies $\xi_{\mathbf{k}}$ and boson energies $\mathcal{E}_{\mathbf{K}}$ are measured from μ and 2μ, respectively, where the electron chemical potential μ is fixed by the constancy of the total electron number. The number of bosons N_B corresponding to (1) is not a fixed number. Not the number operator N_e of unpaired fermions but the *total* fermion number operator

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$N = N_e + 2N_B$ commutes with (1). Therefore, the total N which consists of both the numbers of unpaired fermions and twice of N_B of bosons is an invariant of motion [7]. Thus, it is expected that, as temperature varies, there will be continual transitions between the fermionic and bosonic subsystems leading to variations in the compositions of each of them. Also, in (1), a so-called anisotropy factor $\phi_{\mathbf{q}}$ is introduced to account for the directional dependence of *two-fermion→boson* and conversely transitions associated with the symmetry of background fermions. Being normalized to unity $\phi_{\mathbf{q}}$ describes the distribution of transitions around an average value f as in $f_{\mathbf{q}} = f\phi_{\mathbf{q}}$. Thus, the effect of pair formations/disintegrations in (1) appears in a form compatible with the symmetry of the electronic structure. To obtain BCS theory as a special case, the boson-fermion (BF) vertex coupling constant f has been identified [8] with $\sqrt{2\hbar\omega_D V}$ where V and ω_D are respectively the usual BCS model-interaction net attraction (i.e., including Coulomb repulsions) and the Debye frequency (for details see ref. [6]). The Hamiltonian describing the system of carriers interacting with localized pairs of fermions *in coordinate space* had been originally suggested by J. Ranninger [9]. The idea of a mixture state of unbound fermions and *movable* bosons which emerges from a gas of strongly attractively interacting itinerant fermions led to the Friedberg- T.D. Lee [3] phenomenological hamiltonian (1) modified later to include both anisotropy as well as Coulomb repulsions. A scenario with CPs *gradually* emerging from the attractively interacting system of fermions, which relied on an *unfamiliar positive-energy* solution of the familiar Cooper problem [10], was developed by us (see Refs. in [11]). There, it was shown that owing to two-electron positive-energy resonances (converting single-particle states of attractively interacting fermions into correlated states of two fermions above the Fermi sea and vice versa, a mixture state of pairable/unpaired fermions coexisting with two-fermionic entities becomes energetically *more favorable* than the state of unbound fermions. The presence in a system of attractively interacting fermions of *two-fermionic resonanses* with energies *above* the Fermi sea was discussed, e.g., in refs. [41] and [42] cited in ref. [11] where a proof was given for the statement that such resonating correlations become feasible already in the well-known Cooper problem of two fermions interacting via an attractive potential $-V$ in a filled Fermi sea. Processes of formation of two-fermionic resonanses (followed by their disintegrations into two unbound fermions and vice versa, as given by the last term in (1)) were decisive to obtain, upon cooling, a continuous decrease of μ with respect to the value E_F associated with unbound fermions thus leading, below a specific T^*, to a lower-energy binary mixture state wherein the original system of fermions separate into two interacting subsystems, namely, pairable/unpaired fermions and two-fermionic entities (or preformed pairs) considered as actual bosons (see, e.g, in [6]). This scenario enables one to derive analytical expressions for T^* (defined as the onset temperature for the formation of two-fermionic composites considered as actual bosons but *without* coherence) and the BEC-onset temperature T_c for a *coherent* fluid in [11] which have been applied to explain the qualitatively different behaviors of T^* and T_c as function of doping concentration, specifically, an everywhere decreasing T^* in sharp contrast to the well-known *dome-shaped* T_c behaviour reported in experiments on HTSCs. Introducing a Coulomb interaction along with an anisotropic BF binary gas mixture model allows one to predict in [6], even with phonon-mediated electron dynamics, the presence in the energy-momentum dispersion relation of HTSCs of a line of nodal points along which the gap in the single-particle spectrum vanishes, and hence, gives rise to the so-called Fermi arcs as reported [12] from ARPES experiments. Besides the physics of pseudogapped and superconducting states seen in HTSCs, the Hamiltonian (1) is also applied to understand the physics of the BCS-BEC crossover in superconductors and ultracold atoms [4]. Success of the BF model encouraged us in the present work to explore how the dispersion of single-fermionic states is affected by the presence of carriers appearing as constituents of the bosonic CPs.

2 Main Formulae

The distribution of free carriers can be determined by starting from the occupation numbers $\left\langle a^+_{\mathbf{k},\sigma} a_{\mathbf{k},\sigma} \right\rangle$ obtained, e.g., from an infinite chain of equations for two-time retarded Green functions (GFs) $\left\langle\!\left\langle A(t) \mid B(t') \right\rangle\!\right\rangle$ as defined in ref. [13] for dynamical operators $a_{\mathbf{k}\sigma}(t)$ and $a^+_{\mathbf{k}'\sigma}(t')$. Fourier transform $\langle\langle A| \, B\rangle\rangle_\omega$ of $\left\langle\!\left\langle A(t) \mid B(t') \right\rangle\!\right\rangle$ in ω-space, with $\hbar\omega$ in energy units, satisfies the chain of equations (see, e.g., ref. [14]. Specifically

$$\hbar\omega \, \langle\langle A \mid B\rangle\rangle_\omega = \left\langle [A, B]_\eta \right\rangle_{\mathcal{H}} + \left\langle\!\left\langle [A, \mathcal{H}]_- \mid B \right\rangle\!\right\rangle_\omega \tag{2}$$

where square brackets $[A, B]_\eta \equiv AB + \eta BA$ of operators A and B denote the commutator if $\eta = -1$ or anticommutator if $\eta = +1$. Here, we summarize how to get a *closed* expression for the first-order retarded GFs the poles of which determine the spectra of elementary excitations in the system. The details of this technique are found, e.g., in refs. [11, 14]. Choosing $A \equiv a_{\mathbf{k}\uparrow}$ and $B = a^+_{\mathbf{k}'\uparrow}$ in (2), and using the relations $[a^+_{\mathbf{k},\sigma}, a_{\mathbf{k}',\sigma'}]_+ = \delta_{\mathbf{k},\mathbf{k}'}\delta_{\sigma,\sigma'}$, $\left[a_{\mathbf{k},\sigma}, a_{\mathbf{k}',\sigma'}\right]_+ = 0$, and $[a^+_{\mathbf{k},\sigma}, a^+_{\mathbf{k}',\sigma'}]_+ = 0$ to calculate commutators $\left[a_{\mathbf{k},\uparrow}, \mathcal{H}\right]_-$, leads to the first equation relating $\left\langle\!\left\langle a_{\mathbf{k}\uparrow} \mid a^+_{\mathbf{k}'\uparrow} \right\rangle\!\right\rangle$ on the lhs of (2) with GFs $\left\langle\!\left\langle a^+_{\mathbf{k}\downarrow} \mid a^+_{\mathbf{k}'\uparrow} \right\rangle\!\right\rangle$ and $\left\langle\!\left\langle b_{\mathbf{K}} a^+_{-\mathbf{k}+\mathbf{K}\downarrow} \mid a^+_{\mathbf{k}'\uparrow} \right\rangle\!\right\rangle$ on the rhs. Then, applying (2)

again but now for $A \equiv a^+_{\mathbf{k}\downarrow}$ and for the same $B = a^+_{\mathbf{k}'\uparrow}$ gives a new equation which relates GFs $\left\langle\left\langle a^+_{\mathbf{k}\downarrow} \mid a^+_{\mathbf{k}'\uparrow}\right\rangle\right\rangle$ with GFs $\left\langle\left\langle a_{\mathbf{k}\uparrow} \mid a^+_{\mathbf{k}'\uparrow}\right\rangle\right\rangle$ and $\left\langle\left\langle b^+_{\mathbf{K}} a_{-\mathbf{k}+\mathbf{K}\uparrow} \mid a^+_{\mathbf{k}'\uparrow}\right\rangle\right\rangle$. Thus, one gets a couple of equations relating the first-order GFs $\left\langle\left\langle a_{\mathbf{k}\uparrow} \mid a^+_{\mathbf{k}'\uparrow}\right\rangle\right\rangle$ and $\left\langle\left\langle a^+_{\mathbf{k}\downarrow} \mid a^+_{\mathbf{k}'\uparrow}\right\rangle\right\rangle$ with *higher-order* GFs $\left\langle\left\langle b_{\mathbf{K}} a^+_{-\mathbf{k}+\mathbf{K}\downarrow} \mid a^+_{\mathbf{k}'\uparrow}\right\rangle\right\rangle$ and $\left\langle\left\langle b^+_{\mathbf{K}} a_{-\mathbf{k}+\mathbf{K}\uparrow} \mid a^+_{\mathbf{k}'\uparrow}\right\rangle\right\rangle$. The higher-order GFs containing bosonic operators $b_{\mathbf{K}}$ and $b^+_{\mathbf{K}}$ are rewritten correspondingly as

$$\langle b_{\mathbf{K}}\rangle \left\langle\left\langle a^+_{-\mathbf{k}+\mathbf{K}\downarrow} \mid a^+_{\mathbf{k}'\uparrow}\right\rangle\right\rangle + \left\langle\left\langle (b_{\mathbf{K}} - \langle b_{\mathbf{K}}\rangle) a^+_{-\mathbf{k}+\mathbf{K}\downarrow} \mid a^+_{\mathbf{k}'\uparrow}\right\rangle\right\rangle$$

and

$$\left\langle b^+_{\mathbf{K}}\right\rangle \left\langle\left\langle a_{-\mathbf{k}+\mathbf{K}\uparrow} \mid a^+_{\mathbf{k}'\uparrow}\right\rangle\right\rangle + \left\langle\left\langle \left(b^+_{\mathbf{K}} - \left\langle b^+_{\mathbf{K}}\right\rangle\right) a_{-\mathbf{k}+\mathbf{K}\uparrow} \mid a^+_{\mathbf{k}'\uparrow}\right\rangle\right\rangle$$

where, following Bogoliubov [15], we retain only terms proportional to $\langle b_K\rangle$ and $\left\langle b^+_K\right\rangle$. This allows one to truncate the chain of equations. Some manipulations then lead to the single-fermion GF

$$\left\langle\left\langle a_{\mathbf{k}\uparrow} \mid a^+_{\mathbf{k}'\uparrow}\right\rangle\right\rangle_\omega = \delta_{\mathbf{k}\mathbf{k}'} \left[\hbar\omega - \xi_{\mathbf{k}} - \Sigma_{\mathbf{k}}(\omega)\right]^{-1} \tag{3}$$

with a self energy (or so-called mass operator) $\Sigma_{\mathbf{k}}(\omega)$ found as

$$\Sigma_{\mathbf{k}}(\omega) = \frac{f^2}{N} \sum_{\mathbf{K}} \phi^*_{-\mathbf{k}+\mathbf{K}/2} \phi_{\mathbf{k}-\mathbf{K}/2} \frac{\langle b_{\mathbf{K}}\rangle_{\mathcal{H}} \left\langle b^+_{\mathbf{K}}\right\rangle_{\mathcal{H}}}{\hbar\omega + \xi_{-\mathbf{k}+\mathbf{K}}}. \tag{4}$$

Defined on the ω complex plane, both (3) and (4) contain real and imaginary parts. For the interaction $f\phi_{\mathbf{q}}$ in (1) of d-wave-like symmetry, $\phi_{\mathbf{k}}$ crosses zero along some specific directions $\mathbf{k_n}$ of the Brillouin zone. Along these directions, one immediately gets

$$\hbar\omega = \xi_{\mathbf{k_n}} + \frac{f^2}{N} \sum_{\mathbf{K}>\mathbf{0}} \left|\phi_{\mathbf{k_n}-\mathbf{K}/2}\right|^2 \frac{N_{B\mathbf{K}}}{\hbar\omega + \xi_{\mathbf{k_n}-\mathbf{K}}} \tag{5}$$

where $\xi_{\mathbf{k}} \equiv \xi_{-\mathbf{k}}$ by symmetry. However, along directions $\mathbf{k}$ *different from* $\mathbf{k_n}$, the separation of terms with $\mathbf{K} = \mathbf{0}$ and those with $\mathbf{K} > \mathbf{0}$ in the (4) the denominator of (3) equated to zero gives

$$\frac{(\hbar\omega)^2 - E^2_{\mathbf{k}}}{\hbar\omega + \xi_{\mathbf{k}}} = \frac{f^2}{N} \sum_{\mathbf{K}>\mathbf{0}} \left|\phi_{\mathbf{k}-\mathbf{K}/2}\right|^2 \frac{N_{B\mathbf{K}}}{\hbar\omega + \xi_{\mathbf{k}-\mathbf{K}}} \tag{6}$$

where

$$E_{\mathbf{k}} \equiv \sqrt{\xi^2_{\mathbf{k}} + E^2_{g\mathbf{k}}} \qquad E_{g\mathbf{k}} \equiv f\phi_{\mathbf{k}} \frac{N^{1/2}_{B\mathbf{0}}}{N^{1/2}} \tag{7}$$

and

$$N_{B\mathbf{K}} \equiv \langle b_{\mathbf{K}}\rangle_{\mathcal{H}} \left\langle b^+_{\mathbf{K}}\right\rangle_{\mathcal{H}} = \left\langle b_{\mathbf{K}} b^+_{\mathbf{K}}\right\rangle_{\mathcal{H}}, \tag{8}$$

see ref. [16].

The poles $\hbar\omega(\mathbf{k})$ of (3) appear gapless along the $\mathbf{k_n}$, i.e., change in a continuous manner particularly in directions perpendicular to the Fermi surface. On the other hand, the roots $\hbar\omega(\mathbf{k})$ found by equating the denominator of (3) to zero leads to a discontinuity at $\mathbf{k}_\mu$ of the Fermi surface for directions $\mathbf{k}$ *different from* $\mathbf{k_n}$. To investigate how the presence of an assembly of uncondensed paired states, i.e., the states with $\mathbf{K} \neq 0$, affect the single-particle spectrum, one should estimate the sum in the rhs of (4) which can be evaluated as an integral over $\mathbf{K}$. We are interested in occupied electronic states below μ, as dealt with in angle-resolved photoemission spectroscopy (ARPES) experiments [17], i.e., with $k \leq k_\mu$ where k_μ is the wavenumber corresponding to the fermion chemical potential μ of an interacting BF mixture. For the quadratic energy dispersion of holes $\xi_{\mathbf{k}} = \mu - \hbar^2 k^2/2m$, one puts

$$\xi_{\mathbf{k}-\mathbf{Q}} = \xi_{\mathbf{k}} - \mu k_\mu^{-2} \left(Q^2 - 2Qk\cos\theta\right) \tag{9}$$

in (4). Here, (9) is energy of fermions appearing below μ by exciting electrons and θ is a polar angle defining the direction of $\mathbf{Q}$ with respect to $\mathbf{k}$. In the present study, we consider renormalizations of $\xi_{\mathbf{k}}$ occurring along nodal lines *only*. Furthermore, to obtain qualitative results, without loss of generality, the anisotropy factor in (5) will be replaced by its average value $\phi_{\mathbf{k}} = 1$. Then, in spherical coordinates, by fixing the z-axis along the vector $\mathbf{k}$, one has

$$\Sigma_{\mathbf{k}}(\omega) = \frac{f^2}{N} \frac{L^3}{(2\pi)^2} \int_0^\infty N_{BQ} K_{\mathbf{k},\omega}(Q) Q^2 dQ \tag{10}$$

where

$$K_{\mathbf{k},\omega}(Q) \equiv \int_0^\pi \frac{\sin\theta d\theta}{\hbar\omega + \xi_{-\mathbf{k}+\mathbf{Q}}}. \tag{11}$$

For an explicit expression for (5), one needs (10) which is easy to calculate. The operator identity $(x + i\varepsilon)^{-1} \equiv P\left(x^{-1}\right) - i\pi\delta(x)$ applied to (11) leads to the full expression

$$K_{\mathbf{k},\omega}(Q) \equiv -\frac{k^2_\mu}{2\mu Qk} \left(P \int_{y_-}^{y_+} \frac{dy}{y-b} - i\pi \int_{y_-}^{y_+} \delta(y-b)dy\right) \tag{12}$$

with P denoting the principal value of integral (11) and b defined as $b \equiv \hbar\omega + \xi_{-\mathbf{k}}$. The limits of integration y_- and y_+ in (12) are given as $y_\pm \equiv \mu k_\mu^{-2}\left(Q^2 \pm 2Qk\right)$. The integrals in parenthesis are easy to find. The first integral may be expressed in terms of the roots $Q_1 \equiv k - k_\mu\sqrt{J(\omega)}$ and $Q_2 \equiv k + k_\mu\sqrt{J(\omega)}$ of the quadratic form $Q^2 - 2kQ - k^2_\mu\mu^{-1}\left(\hbar\omega + \xi_{\mathbf{k}}\right)$ with $J(\omega) \equiv 1 + \hbar\omega/\mu$. For it, one readily obtains

$$P \int_{y_-}^{y_+} \frac{dy}{y - \hbar\omega - \xi_{-\mathbf{k}}} = \ln\left(\left|\frac{(Q+Q_1)(Q+Q_2)}{(Q-Q_1)(Q-Q_2)}\right|\right). \tag{13}$$

The second integral in (12) differs from zero only if the region of integration includes the point b where the

argument of the δ-function vanishes. In terms of the Heaviside step function, $\theta(x) \equiv 1$ *for all positive* x and *otherwise* 0, leads to

$$\int_{y_-}^{y_+} \delta(y - \hbar\omega - \xi_{-\mathbf{k}})dy = \theta(Q - |Q_1|)\theta(Q_2 - Q) \qquad (14)$$

so that (13) and (14) written in (12) which is then accounted for (10) yields

$$\Sigma_{\mathbf{k}}(\omega) = \frac{f^2}{N}\frac{L^3}{(2\pi)^2}\frac{k_\mu^2}{2\mu k}\int_0^\infty dQ N_B(Q) Q \left(\ln\frac{|(Q-Q_1)(Q-Q_2)|}{|(Q+Q_1)(Q+Q_2)|} + i\pi\theta(Q-|Q_1|)\theta(Q_2-Q)\right) \qquad (15)$$

The coefficient before (15) may be rewritten as

$$f^2\frac{N_{B0}}{N}\frac{L^3}{2\mu k}\frac{3}{4}\left(\frac{2\frac{4\pi}{3}k_\mu^3}{(2\pi)^3}\right)\frac{1}{k_\mu} \equiv \lambda\hbar\omega_D\frac{N_{B0}}{N}\frac{\mathcal{N}(\mu)}{\mathcal{N}(E_F)}\frac{1}{k_\mu k}$$

where (1) the term in the large round brackets on the lhs is replaced by the number density n_e of free fermions filling a Fermi sphere of radius k_μ, (2) the electronic density of states (EDOS) for each spin $\mathcal{N}(\mu) = (3n_e/4\mu)$ at the chemical potential μ is introduced [18], and then (3) the BF vertex coupling constant f^2 is assumed [8] to be $2\hbar\omega_D V$ where $V \geq 0$ is the familiar net attractive interaction between electrons of BCS theory [10] and $\lambda \equiv \mathcal{N}(E_F)V$ with $\mathcal{N}(E_F)$ the EDOS at $\mu = E_F$ of interactionless fermions. In the following, the abbreviation C is introduced for $\lambda\hbar\omega_D\frac{N_{B0}}{N}\frac{\mathcal{N}(\mu)}{\mathcal{N}(E_F)}$.

Integrals over Q may formally be performed *exactly* by the fact that the numbers $N_{B\mathbf{Q}}$ in (15) never become larger as Q increases. Indeed, $N_{B\mathbf{Q}} \simeq [\exp(E_{\mathbf{Q}}/2k_BT) - 1]^{-1}$ (see ref. [11]) rapidly decreases with Q justifying the above suggestion. Then, according to the mean-value theorem, for any $f(x)$ and $g(x)$ with nonincreasing $g(x)$ within the interval (a, b) of integration, one may write $\int_a^b f(x)g(x)dx = g(a)\int_a^c f(x)dx$ where c is some value within (a, b) and $g(a)$ is the value of $g(x)$ at the lower integration limit [19]. Thus, the integrals in (15) can be dealt with as

$$\int_0^\infty (...)N_{BQ}QdQ = N_{BQ=0}\int_0^{\bar{Q}} (...)QdQ$$

where the yet undefined $\bar{Q}$ is some characteristic bosonic wavenumber from the entire interval of allowable Q. After some algebra, the real and imaginary parts of (15) are found to be

$$Im\,\Sigma_{\mathbf{k}}(\omega) = 2\pi C\sqrt{1 - |\hbar\omega|/\mu}. \qquad (16)$$

$$\mathrm{Re}\,\Sigma_k(\omega) = \lambda\hbar\omega_D\frac{N_{B0}}{N}\frac{\mathcal{N}(\mu)}{\mathcal{N}(E_F)}\frac{1}{k_\mu k} \left\{\frac{\bar{Q}^2 - Q_1^2}{2}\ln\frac{|\bar{Q}-Q_1|}{|\bar{Q}+Q_1|} + \frac{Q_2^2-\bar{Q}^2}{2}\ln\frac{|\bar{Q}+Q_2|}{|\bar{Q}-Q_2|} - 2\bar{Q}k\right\} \qquad (17)$$

where (16) and (17), used in Figs. 1 and 2, determine the self energy of fermions moving in a mixture of quasiparticles consisting of unpaired fermions and paired states of two fermions. According to (16), close to the Fermi surface free fermionic excitations are very dissipative with a minimum lifetime at $|\hbar\omega| = 0$. The most short-lived fermionic states appearing are those on the Fermi surface. As the single-particle levels dip deeper into the Fermi sea, dissipation of quasiparticles *decreases*. For fermions sufficiently deep in the Fermi sea, i.e., for those states less affected by the pairing interaction, lifetimes change insignificantly leaving the single-particle excitations as they were for interactionless fermions.

3 Discussion

In Fig. 1, the imaginary part of fermionic self-energies below μ (i.e., holes) fluctuating between single-fermionic and bosonic hole pairs is shown as a function of a binding energy of fermions $\hbar\omega$ in units of $\hbar\omega_D$. Note that $Im\,\Sigma_{\mathbf{k}}(\omega)$ differs from zero only if some fraction of fermions are paired into so-called *preformed pairs*. In the numerical calculations of Fig. 1 we put $\mathcal{N}(\mu)/\mathcal{N}(E_F) = 1$ implying that the EDOS does not vary around E_F and $\lambda = 1$. Moreover, we take $\hbar\omega_D/E_F = 0.35$ ignoring the difference between

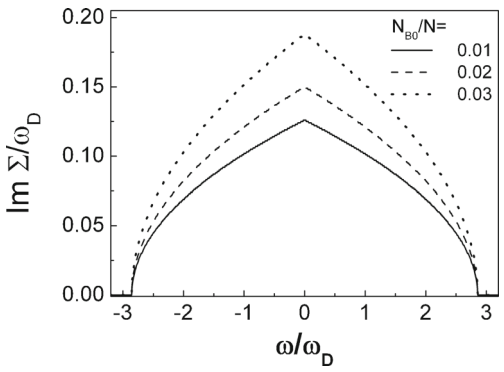

Fig. 1 Imaginary part of self energy of holes moving in background of particles which emerge at different instants both as single fermions or as bound entities made up of two fermions, as function of binding energy of fermions $\hbar\omega$ in units of $\hbar\omega_D$

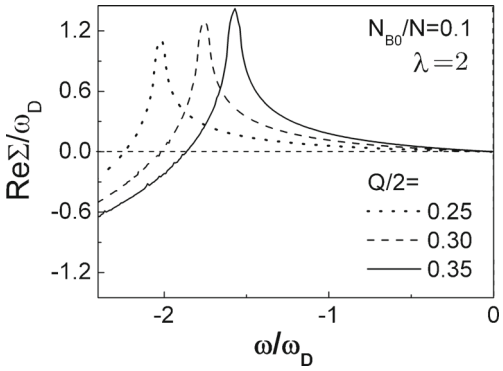

Fig. 2 Real part of self energy of fermions interacting with a background of "frustrated electrons" as function of binding energy of fermions $\hbar\omega$ in units of $\hbar\omega_D$

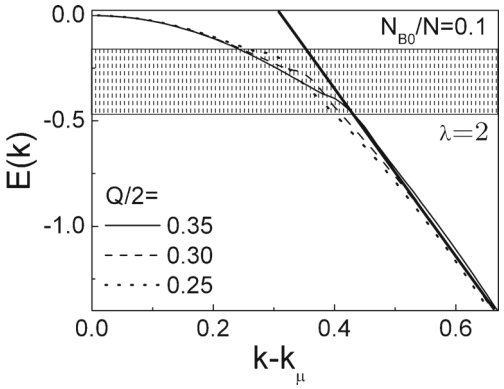

Fig. 3 Single-particle energy-momentum relation along the nodal lines for fermions in an assembly of quasiparticles participating in continual pair formations and their subsequent disintegrations. Here $E(k)$ and k are in units of $\hbar\omega_D$ and k_μ, respectively. Dotted straight line is guide to the eye

μ and E_F in obtaining Fig. 1. As to the fraction N_{B0}/N of CPs in a unit cell, for a qualitative description, we put there $N_{B0}/N = 0.01$ (solid curve), 0.02 (dashed curve), and 0.03 (dotted curve) assuming that only a very small fraction of fermions are paired. The function $Im\,\Sigma_{\mathbf{k}}(\omega)$ is *maximum* at the Fermi level (defined as the locus of points where $k = k_\mu$). Increasing the ratio $|\omega|/\omega_D$ (i.e., upon increasing the deviation of k from the k_μ), $Im\,\Sigma_{\mathbf{k}}(\omega)$ decreases. Actually, *as a function of* k , it decreases within the annulus (of width defined by the ratio $\hbar\omega_D/\mu$) centered at k_μ. Outside this annulus $Im\,\Sigma_{\mathbf{k}}(\omega)$ becomes zero, meaning that switching on pairing interaction affects only the lifetimes of those fermions in the immediate vicinity of the Fermi surface. The $Im\,\Sigma_{\mathbf{k}}(\omega)$ does not depend on the parameter $\bar{Q}$ introduced above.

For different values of a fitting parameter $\bar{Q}$ Fig. 2 shows the real part vs ω/ω_D of the self energy of fermions moving in a background of *frustrated* fermions (i.e., in a background of particles which may emerge at different instants both as single fermions *or* as entities made up of two fermions). As the magnitude of binding energy $|\hbar\omega|$ rises, $Re\,\Sigma_{\mathbf{k}}(\omega)$ first increases gradually reaching its maximum value and then sharply decreases, changing sign to become negative at energies approximately of order $2\hbar\omega_D$. The real and imaginary parts of $\Sigma_{\mathbf{k}}(\omega)$ are related through the Kramers-Kronig relation. Thus, the shapes of $Im\,\Sigma_{\mathbf{k}}(\omega)$ given in Fig. 1 predetermine the graphs in Fig. 2. Spectral-peak positions and energy-momentum renomalizations are defined by the behavior of the corresponding $Re\,\Sigma_{\mathbf{k}}(\omega)$.

Figure 3 illustrates the single-particle energy-momentum relation along the nodal lines, i.e., in directions of the Brillioun zone along which the dispersion remains gapless at all temperatures. In Fig. 3, we used the values $\lambda = 2$, $\hbar\omega_D/E_F = 0.35$, assumed that the EDOS is constant near the E_F and ignored the difference between μ and E_F. The fraction N_{B0}/N of CPs in a unit cell is assumed to be $N_{B0}/N = 0.1$. Different curves in Fig. 3 correspond (in units of k_μ) to the values $\bar{Q}/2 = 0.25$ (dashed curve), 0.3 (dotted curve) and 0.35 (full curve), meaning that the phonon spectrum contributing to the pairing interaction is cut off at the maximum wavenumber $\bar{Q}/2$. Increasing the fitting parameter $\bar{Q}$ leads to stronger changes observed as *kinks* in the slope of the E vs k dispersion curve. As seen in Fig. 3, when the processes of continual pair formation and their subsequent disintegration into two unpaired electrons are included, the spectrum of single fermions then displays the aforementioned *kinked structure*. Kinks become more pronounced for larger λ. By bosonization, the chemical potential μ dips below the E_F of interactionless fermions. It cannot be excluded that such an immersion of μ may bring it close to the saddle points [20] of EDOS enhancing the ratio $\mathcal{N}(\mu)/\mathcal{N}(E_F)$ and thus promoting the appearance of kinks even for smaller values of $\mathcal{N}(E_F)V$ of the BCS theory. An extreme example of how λ changes drastically when the chemical potential dips below the Fermi energy by doping is discussed in ref. [21]. The low-energy kink positioned in the vicinity of $50 - 80$ meV where sudden changes are observed at different doping levels and temperatures for different superconducting cuprates in experimental dispersion curves [22] appears to be in agreement with the findings of the present very simplified viewpoint which predicts the presence of such abrupt changes at the low-energy scale of ω/ω_D.

4 Conclusions

It was shown that the spectrum of single-fermionic excitations in a binary BF mixture consisting of unbound electrons and incoherent bosonic CPs displays a kinked structure with sudden slope changes of the energy-momentum dispersion. Those kinks occur at energies ξ within the annulus

$\mu - \hbar\omega_D < \xi < \mu + \hbar\omega_D$. As in BCS theory, switching on a net attractive interaction of strength V between fermions within that annulus produces two-fermionic resonances above the Fermi sea of interactionless fermions which become, on lowering T, actual bosonic CPs with energies below the Fermi sea [6]. Emergence of preformed CPs in an attractively-interacting fermion gas of a binary BF mixture state occurs at and below temperatures T^* much higher than the superconducting BEC T_c.

Hence, *both* superconductivity (as a BEC below T_c of preformed CPs unceasingly fluctuating between single-fermionic states and states of two bound electrons) *and* kinks observed in experiments (below an onset temperature T^* below which the evolution of the gas of free fermions into a binary BF mixture state just begins) emerge as an intrinsic property of an attractively-interacting fermion gas evolving on lowering T from pseudogapped to superconducting states.

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